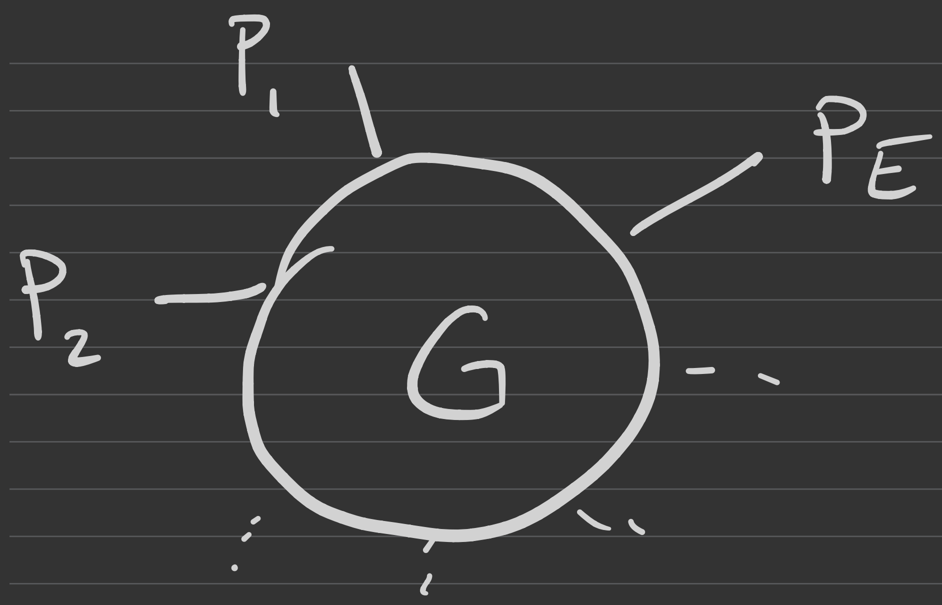


$E = \#$ external legs

$V = \#$ vertices

$I = \#$ internal legs

$L = \#$ loops

$$\sum d_\alpha \equiv d_v$$

$$G_L = \int \prod_{l=1}^{L} d^D k_l \prod_{a=1}^{I} \Delta(p_a) \prod_{\alpha=1}^{V} f_\alpha(p_{a_\alpha})$$

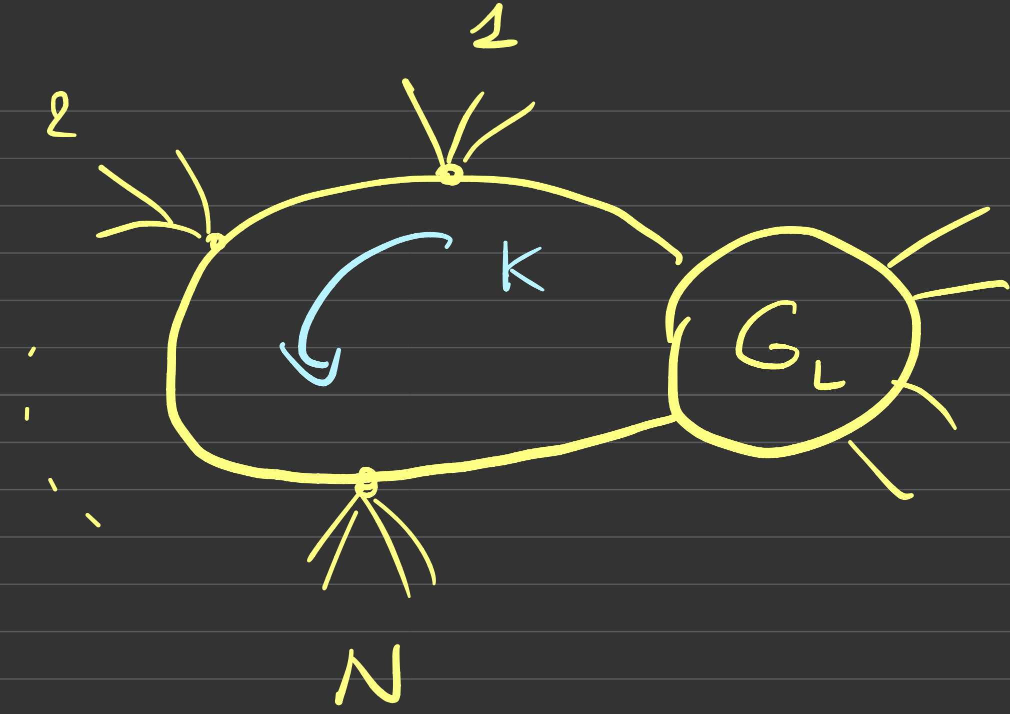
$$\sim \int K^{LD} \frac{dk}{K} K^{-2I} K^{d_v} = \int K^{\delta(G)} \frac{dk}{K}$$

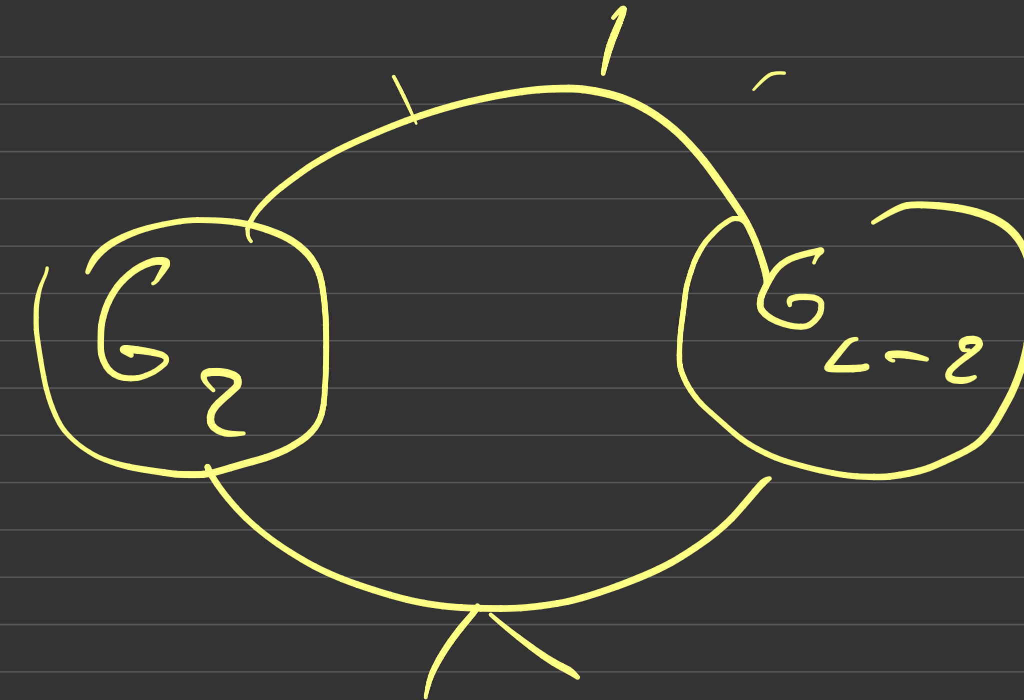
$$\boxed{\delta(G_L) = LD - 2I + d_v}$$

Renormalized $\Rightarrow$ $G_L \sim p^{\delta(G_L)} F_0(\log p) + p^{\delta(G_L)-2} F_1(\log p) + \ldots$

$\downarrow$

$(\log p)^L + (\log p)^{L-1} + \ldots$ by Weinberg Th

• $G_{L+1} =$ 

$E-2$ $+$ 

$$\approx \left(\frac{1}{k^2}\right)^{N+1} k^{\sum_1^N d_i} \left(k^{\delta(G_L)} + k^{\delta(G_L)-2} + \dots\right) d^D k$$

EX

$$= k^{\delta(G_{L+1})} \left(1 + k^{-2} + \dots\right) \frac{dk}{k}$$

• $\partial_P^n G_{L+1} \sim k^{\delta(G_{L+1})-n} \dfrac{dk}{k}$

Reading in Collins

- 5.1 — 5.4
- 5.5.1 (5.5.2 if you want details)
- 5.8

$\Rightarrow$ • $\delta(G_{L+1}) < 0 \quad \Rightarrow$ finite

• $\delta(G_{L+1}) > 0 \quad \Rightarrow \underbrace{\text{divergence} \equiv \text{degree } \delta(G_{L+1}) \text{ polynomial}}_{}$

renormalized by adding a local counterterm

⚠ Structure of needed counter-terms specified by the $\delta(G)$ of the various n-point functions

$$\bullet\ \delta(G) = D - \overbrace{\left(\frac{D-2}{2}\right)}^{\Delta_\phi} E + \sum_\alpha \overbrace{\left(\frac{D-2}{2} n_\alpha + d_\alpha - D\right)}^{-\Delta_\alpha}$$

$$\boxed{\Delta_\alpha = D - d_\alpha - \frac{D-2}{2} n_\alpha} \qquad \Downarrow \ g_\alpha\, \partial^{d_\alpha} \phi^{n_\alpha}$$

Crucial distinction: $\boxed{\Delta_\alpha \geq 0}$ versus $\boxed{\Delta_\alpha < 0}$

1. $\Delta_\alpha \geq 0$: $\bullet\ \delta(G) \geq 0 \implies E \leq \dfrac{2D}{D-2}$

$D=4$: ≤ 4

$D=6$: ≤ 3

$D=3$: ≤ 6

$$\text{C.T} \sim \partial^{\delta(G)} \phi^E \quad \text{with } \delta(G) + \frac{D-2}{2} E \leq D$$

Ex

$$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + g\,\phi^3 \quad \text{in } D=6$$

$$E \leq \frac{12}{4} = 3$$

possible c.T. $\quad \phi^3,\ \phi^2,\ \phi,\ (\partial\phi)^2$

- $\Delta_\alpha \geq 0 \Rightarrow$ finite class of counterterms

$\equiv$ Renormalizable

2. $\Delta_\alpha < 0$ for some vertex

$$\delta(G) = D - \frac{D-2}{2} E - \sum_\alpha \Delta_\alpha$$

- $\forall E$ $\delta(G)$ can be made >0 by sufficient insertions of vertex with $\Delta_\alpha < 0$

C.T. $\quad \partial^{\delta(G)} \phi^{E} \qquad \delta(G) + \frac{D-2}{2} E \overset{= D - \sum_\alpha \Delta_\alpha}{=}$ unbounded

$\Rightarrow$ class of necessary counter-terms is infinite

Non-Renormalizable

Modern perspective

Non-Renormalizable QFT $\equiv$ Effective QFT

- notice $\mathcal{L} \sim g_* \, \partial^d \phi^E$

$$[g_*] = -d - \frac{D-2}{2} E + D < 0$$

$$g_* = \frac{1}{M^{d + \frac{D-2}{2} E - D}} \equiv M^{-\Delta_\alpha}$$

At every E what is the size of effects of g_*?

$$\bar{g}_* = \left(\frac{E}{M}\right)^{d + \frac{D-2}{2} E - D}$$

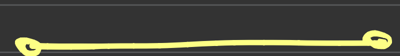
Loop counting

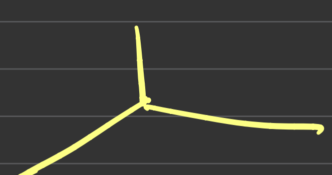
Ex

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{3!}\phi^3$$

$$\phi \longrightarrow \frac{\phi}{g} \implies \mathcal{L} \longrightarrow \frac{1}{g^2}\left[\frac{1}{2}(\partial\phi)^2 + \frac{1}{3!}\phi^3\right]$$

$g^2 \sim \hbar$

 $\propto g^2$

 $\propto 1/g^2$

 $\sim (g^2)^{I-V} = (g^2)^{L-1}$

$\sqrt{\hbar} \sim g$

Ex

$\lambda_4 \sim \hbar$

$$\frac{1}{2}(\partial\phi)^2 + \frac{\lambda_4}{4!}\phi^4 \xrightarrow{\phi \to \frac{1}{\sqrt{\lambda}}\phi} \frac{1}{\lambda_4}\left[\frac{1}{2}(\partial\phi)^2 + \frac{\phi^4}{4!}\right]$$

$$\lambda_4^{I-V} = \lambda_4^{L-1}$$

$$= (g_4^2)^{L-1}$$

$\lambda_4 \equiv g_4^2$

Ex

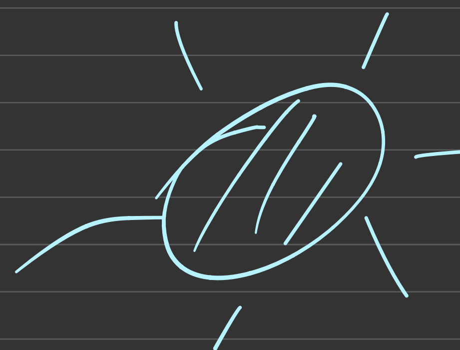
$$\frac{1}{2}(\partial\phi)^2 + \frac{\lambda_6}{6!}\phi^6 \xrightarrow{\phi \to \frac{1}{\lambda^{1/4}}\phi} \frac{1}{\sqrt{\lambda_6}}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{6!}\phi^6\right)$$

$$\sim \lambda_6^{\frac{I-V}{2}} = \lambda_6^{\frac{L-1}{2}} = \frac{1}{\sqrt{\lambda_6}}\lambda_6^{L/2}$$

$$\lambda_6 = g_6^4$$

Ex

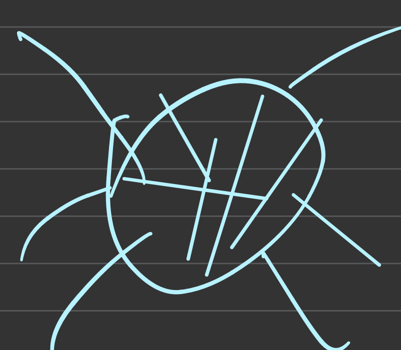
$$\frac{1}{2}(\partial\phi)^2 + \frac{\lambda_5}{5!}\phi^5 \qquad \lambda_5 \sim g_5^3$$



$$\simeq \left(g_5^2\right)^{L-1} = \lambda_5^{\frac{2}{3}(L-1)}$$

Ex

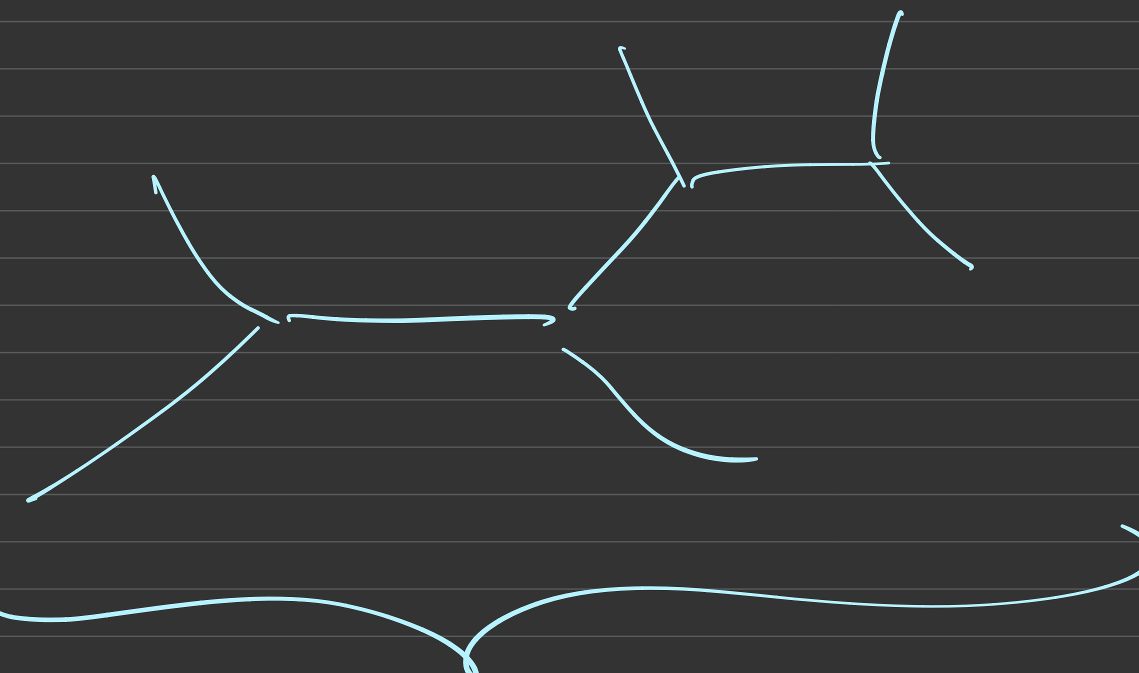
$$\frac{1}{2}(\partial\phi)^2 + \frac{\lambda_n}{n!}\phi^n \qquad \underline{\lambda_n \sim g_n^{n-2}}$$



$$\sim \left(g_n^2\right)^{L-1} = \left(\lambda_n\right)^{\frac{2(L-1)}{n-2}}$$

$$g_n^{\,n-2}\,\Phi_n$$

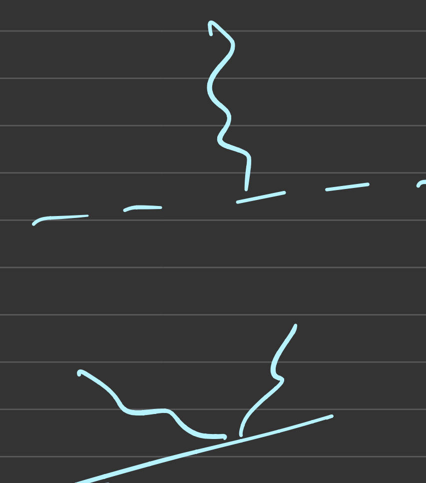
<u>Ex</u> $g\phi^3$



n-legs

$\sim g^{n-2}$

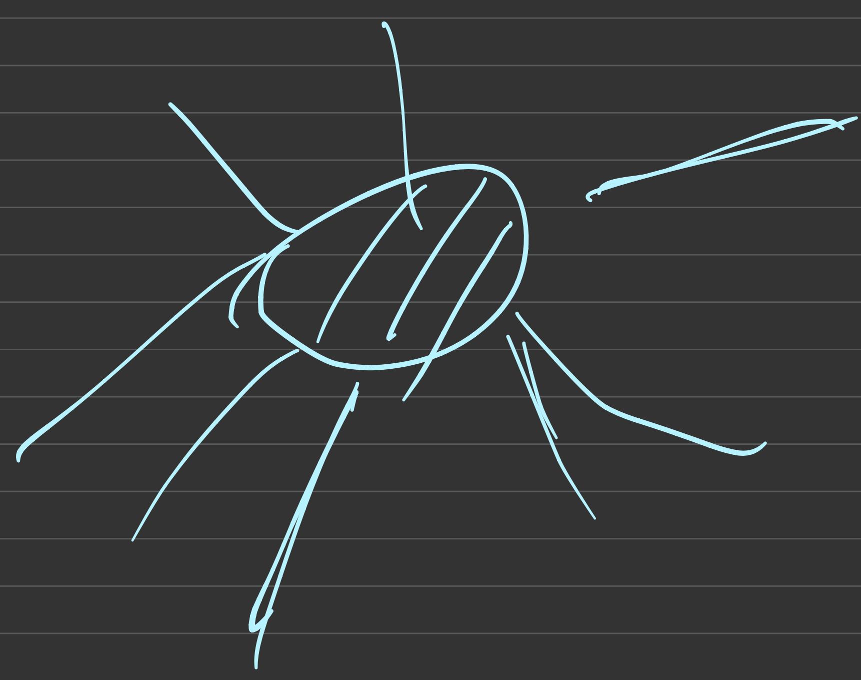
<u>Ex</u> scalar edynamics



$$\left.\begin{array}{l} e \\ e^2 \end{array}\right\} e^{n-2}$$

• <u>Ex</u> in $g\phi^3$ $\qquad \mathcal{M} = \left(\frac{\Pi(p^2-m^2)}{\sqrt{Z}}\right) G$

$$\mathcal{M}_{E\text{-legs}} \sim \frac{1}{g^E} \cdot g^{2E} (g^2)^{I-V} \sim g^{E+2L-2}$$
$$= (g^{E-2})\, g^{2L}$$



$$\langle \phi \phi \rangle \sim \frac{g^2}{p^2} \xrightarrow{\text{cst}} g \sim \sqrt{Z}$$

The Renormalization Group

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT}$$

Ex ϕ^4

$$\mathcal{L}_R = \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

$$\mathcal{L}_{CT} = \frac{\delta Z}{2}(\partial\phi)^2 + \frac{1}{2}\Big[\overbrace{(m^2+\delta m^2)(1+\delta Z) - m^2}^{\Delta m^2}\Big]\phi^2 + \frac{\overbrace{(\lambda+\delta\lambda)(1+\delta Z)^2 - \lambda}^{\Delta\lambda}}{4!}\cdot\phi^4$$

$$\Rightarrow \mathcal{L}_R + \mathcal{L}_{CT} = \frac{Z}{2}(\partial\phi)^2 + \frac{Z}{2}m_0^2\phi^2 + \frac{Z^2}{4!}\lambda_0\phi^4$$

$$\equiv \frac{1}{2}(\partial\phi_0)^2 + \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{4!}\lambda_0\phi_0^4$$

$$Z \equiv 1 + \delta Z$$

$$\phi_0 \equiv \sqrt{Z}\,\phi \qquad m_0^2 \equiv m^2 + \delta m^2 \qquad \lambda_0 \equiv \lambda + \delta\lambda$$

Dim Reg.

$$\lambda_0 = \lambda \mu^{\varepsilon}\left[1 + \frac{a_1(\lambda)}{\varepsilon} + \frac{a_2(\lambda)}{\varepsilon^2} + \dots\right]$$

P. V.

$$\lambda_0 = \lambda\left[1 + \tilde{a}_1(\lambda)\ln\frac{\Lambda}{\mu} + \tilde{a}_2(\lambda)\ln^2\frac{\Lambda}{\mu} + \dots\right]$$

$$a_\ell \sim \ell\text{-loop} + (\ell+1)\text{-loop} + \dots$$

$$= a_{\ell,\ell}\,\lambda^{\ell} + a_{\ell,\ell+1}\,\lambda^{\ell+1} + \dots$$

- Similarly $$m_0^2 = m^2 \left[1 + \frac{b_1(\lambda)}{\varepsilon} + \frac{b_2(\lambda)}{\varepsilon^2} + \dots \right]$$

$$Z = \left[1 + \frac{c_1(\lambda)}{\varepsilon} + \frac{c_2(\lambda)}{\varepsilon^2} + \dots \right]$$

with $$b_\ell \sim \lambda^\ell + \dots$$

$$c_\ell \sim \lambda^\ell + \dots$$

- Basic structure fixed by dimensional analysis

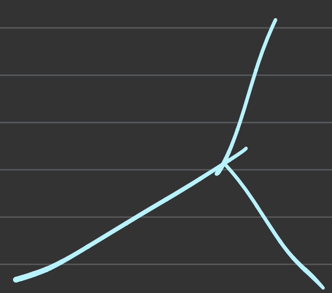
Ex
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \eta\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{3!}g\phi^3 + \frac{1}{4!}\lambda\phi^4$$

$$g_0 = g\mu^{2-D/2}\left[1 + \frac{d_1(\lambda)}{\varepsilon} + \frac{d_2(\lambda)}{\varepsilon^2} + \dots\right]$$

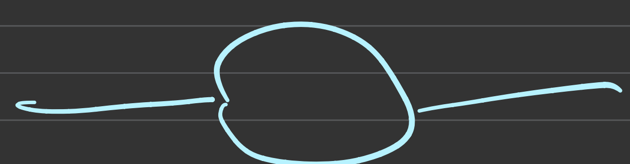
$$m_0^2 = m^2\left[1 + \frac{b_1}{\varepsilon} + \dots\right] + g^2\left[\frac{h_1(\lambda)}{\varepsilon} + \frac{h_2(\lambda)}{\varepsilon^2} + \dots\right]$$

$$\eta_0 = \eta\left[1 + \frac{e_1(\lambda)}{\varepsilon} + \dots\right] + g m^2\left[\frac{f_1(\lambda)}{\varepsilon} + \dots\right] + g^3\left[\frac{m_1(\lambda)}{\varepsilon} + \dots\right]$$

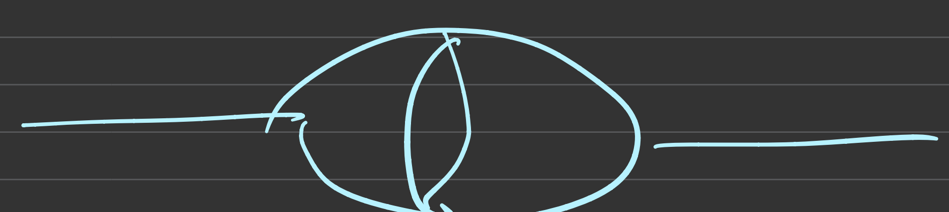
$$g_0 = g\,\mu^{2-D/2}\left[1 + \frac{d_1(\lambda)}{\varepsilon} + \frac{d_2(\lambda)}{\varepsilon^2} + \cdots\right]$$

 +  +  + ...

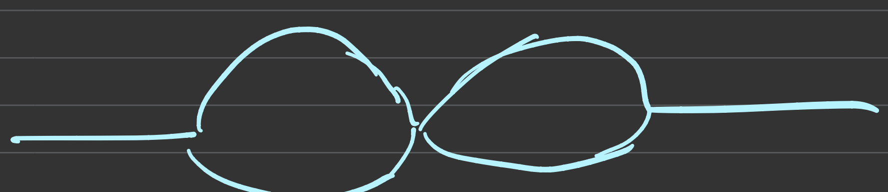
$g \quad \lambda$



$g \quad g \quad \sim \quad g^2 \sim$



$g^2\lambda^2$



$g^2\lambda$

• Amplitudes & Correlators $\equiv \mathcal{O}$

$$\mathcal{O} = \lambda^q \left[\mathcal{O}_0 + \mathcal{O}_1(\lambda) \ln \frac{E}{\mu} + \mathcal{O}_2(\lambda) \ln^2 \frac{E}{\mu} + \cdots \right]$$

$$\mathcal{O}_\ell = \mathcal{O}_{\ell,\ell} \lambda^\ell + \mathcal{O}(\lambda^{\ell+1})$$

Ex

$$\mathcal{M}(2 \to 2) = -\lambda \left[1 + \frac{\lambda}{32\pi^2} \left(3\gamma - 6 + \ln \frac{stu}{(4\pi\mu^2)^3} - i\pi \right) + \cdots \right]$$

$$\sim -\lambda - \frac{6\lambda^2}{32\pi^2} \left(c + \ln \frac{E}{\mu} \right) + \mathcal{O}(\lambda^3)$$

A) $\mathcal{M} \equiv \mu(\lambda, \mu) = -\hat{\lambda} - \frac{6\hat{\lambda}^2}{32\pi^2} \ln E + O(\hat{\lambda}^3)$

$$\cdot\ \hat{\lambda} \equiv \lambda - \frac{6\lambda^2}{32\pi^2} \ln \mu$$

$$(\lambda', \mu') \sim (\lambda, \mu) \quad \text{if}$$

$$\lambda' - \frac{6\lambda'^2}{32\pi^2} \ln \mu' = \lambda - \frac{6\lambda^2}{32\pi^2} \ln \mu$$

$$\Rightarrow \quad \lambda' = \lambda - \frac{3\lambda^2}{16\pi^2} \ln \frac{\mu}{\mu'} + O(\lambda^3)$$

B) $\frac{\lambda}{32\pi^2} \ll 1$ not enough to ensure perturbativity if $\ln E/\mu$ sufficiently large

- $\frac{\lambda}{32\pi^2} \ln\frac{E}{\mu} \gtrsim 1 \implies$ pert. theory breaks down

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT}$$

Renormalization group

- Can this be made more precise?
- Can we still compute when $\frac{\lambda}{16\pi^2} \ln \frac{E}{\mu} \gtrsim O(1)$?

Basic Idea

$$m^2, \lambda, \mu \longrightarrow \mathcal{L}_0 = \frac{Z}{2}(\partial\phi)^2 + \frac{m_0^2}{2} Z \phi^2 + \frac{\lambda_0}{4!} Z^2 \phi^4$$

$$m'^2, \lambda', \mu' \longrightarrow \mathcal{L}_0' = \frac{Z'}{2}(\partial\phi')^2 + \frac{m_0'^2}{2} Z' \phi'^2 + \frac{\lambda_0'}{4!} Z'^2 \phi'^4$$

Require $\left.\begin{array}{l} \lambda_0 = \lambda_0' \\ m_0 = m_0' \end{array}\right\} \Rightarrow \sqrt{Z}\phi = \phi_0$ and $\sqrt{Z'}\phi' = \phi_0'$

identical correlators and can be identified

$$\Rightarrow \mathcal{L}_0 = \mathcal{L}_0'$$

$$\Rightarrow \qquad \mathcal{L} + \mathcal{L}_{CT} = \mathcal{L}' + \mathcal{L}'_{CT} \equiv \mathcal{L}_0$$

$$\frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \qquad\qquad \frac{1}{2}(\partial\phi')^2 + \frac{m'^2}{2}\phi'^2 + \frac{\lambda'}{4!}\phi'^4$$

$\Rightarrow$ the two construction simply correspond to different splittings of the same <u>bare</u> Lagrangian $\mathcal{L}_0$

- $\forall\ \mu \in \mathbb{R}^+$ I can perform such equivalent construction

$$\lambda\ ,\ m\ ,\ \phi \longrightarrow \lambda(\mu)\ ,\ m(\mu)\ ,\ \phi(\mu)$$

- μ-choice optimal to study physics at $E \sim \mu$ where $\log \frac{E}{\mu} \lesssim O(1)$

- <u>Interpretation</u>

$$\mathcal{L}_0 = \mathcal{L}_R(\mu) + \mathcal{L}_{CT}(\mu)$$

$\Downarrow$

compensates quantum fluctuations at $k \gtrsim \mu$

$$\mathcal{L}_0 = \frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

- μ-dependence of $\lambda(\mu)$, $m^2(\mu)$, $\phi(\mu)$ implicitly determined by

$$\mu\frac{d}{d\mu}\lambda_0 = 0 \qquad \mu\frac{d}{d\mu}m_0^2 = 0 \qquad \mu\frac{d}{d\mu}\phi_0 = 0$$

$$\Downarrow$$

Renormalization Group Equations

$$\bullet \quad \mu \frac{d}{d\mu}\lambda_0 = \mu \frac{d}{d\mu}\left[\lambda \mu^{\varepsilon}\left(1 + \frac{a_1}{\varepsilon} + \frac{a_2}{\varepsilon^2} + \dots\right)\right]$$

$$\mu \frac{d}{d\mu}\lambda$$

$$= \left(\hat{\beta}(\lambda) + \varepsilon\lambda\right)\left(1 + \sum_n \frac{a_n}{\varepsilon^n}\right) + \lambda\hat{\beta}\left(\sum_n \frac{a_n'}{\varepsilon^n}\right) = 0$$

$$\Rightarrow \quad \hat{\beta} = -\frac{\varepsilon\lambda + \sum_n \frac{\lambda a_n}{\varepsilon^{n-1}}}{1 + \sum_m \frac{(1+\lambda\partial_\lambda)a_m}{\varepsilon^m}}$$

$$= -\left\{\varepsilon\lambda - \lambda^2 a_1' + \frac{\lambda^2}{\varepsilon}\left[-a_2' + a_1'(a_1 + \lambda a_1')\right] + \dots\right\}$$

⚠ $\mu \to \mu'$, $\lambda(\mu) \to \lambda(\mu')$, $\ldots$ $\equiv$ invariance of UV finite observables

$$\Downarrow$$

$$\hat{\beta}(\lambda) \equiv \text{finite}$$

$\Rightarrow$ all $\frac{1}{\varepsilon}$, $\frac{1}{\varepsilon^2}$, $\ldots$ terms in $\hat{\beta}$ should vanish !!

<u>Ex</u> $-a_2' + a_1'(a_1 + \lambda a_1') = 0$

$a_1 \to a_2$ fixed by diff. eq.

$$Q_1 = c_1 \lambda + c_2 \lambda^2 + \cdots$$

$$Q_1' = c_1 + 2 c_2 \lambda + \cdots$$

$$Q_2' = (c_1 + 2 c_2 \lambda)(c_1 \lambda + c_2 \lambda^2 + c_1 \lambda + 2 c_2 \lambda^2)$$

$$= c_1^2 \lambda + O(\lambda^2)$$

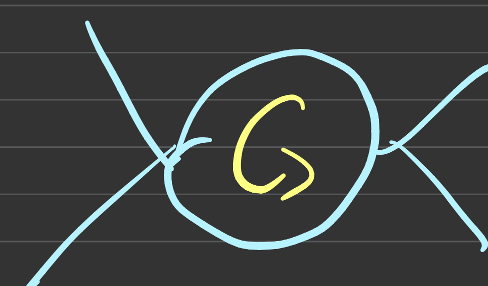
$\longrightarrow$ Known

$$\boxed{Q_2 = \frac{1}{2} c_1^2 \lambda^2 + O(\lambda^3)}$$

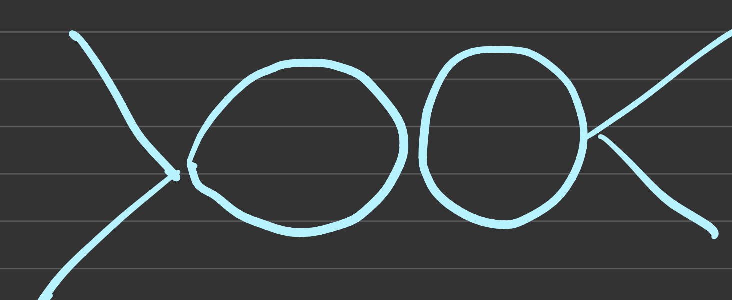
$$\mu \frac{d}{d\mu}\left[\lambda \mu^{\varepsilon}\left(1+\frac{a_1}{\varepsilon}+\frac{a_2}{\varepsilon^2}+\dots\right)\right]$$

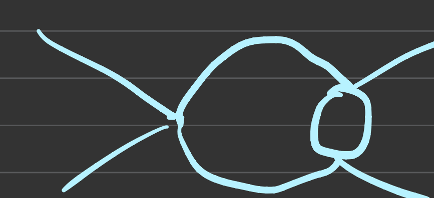
$$\lambda \mu^{\varepsilon}\left(1+\frac{c_1\lambda+\dots}{\varepsilon}+\frac{\frac{1}{2}c_1\lambda^2+\dots}{\varepsilon^2}\dots\right)$$

$\|$ $\|$



$\log \Lambda$





P.V. $\frac{1}{\varepsilon} \rightarrow \ln \Lambda$

$$\Longrightarrow \quad \hat{\beta}(\lambda) = -\varepsilon\lambda + \lambda^2 a_1'$$

$$\equiv -\varepsilon\lambda + \beta(\lambda)$$

- For renormalized quantities $\varepsilon \to 0$ smooth

$$\Longrightarrow \quad \boxed{\mu \frac{d}{d\mu}\lambda(\mu) \overset{d=4}{=} \lambda^2 a_1'(\lambda) \equiv \beta(\lambda)}$$

- Similar story for $m^2(\mu)$:

$$m_0^2 = m^2\left[1 + \frac{b_1(\lambda)}{\varepsilon} + \frac{b_2(\lambda)}{\varepsilon^2} + \cdots\right]$$

$$0 = \mu\frac{d}{d\mu} m_0^2 = \mu\frac{d}{d\mu} m^2\left[1+\sum_n \frac{b_n}{\varepsilon^n}\right] + m^2\left[\sum_n \frac{b_n'}{\varepsilon^n}\right]\left(-\varepsilon\lambda + \lambda^2 a_1'\right)$$

$$\Rightarrow \quad \mu\frac{d}{d\mu}\ln m^2(\mu) = -\frac{\sum_n \frac{b_n'}{\varepsilon^n}\left[-\varepsilon\lambda + \lambda^2 a_1'\right]}{\left[1+\sum_n \frac{b_n}{\varepsilon^n}\right]}$$

$$\equiv -\gamma_m(\lambda)$$

- $$\phi_0 = \sqrt{Z}\,\phi(\mu)$$

$$Z = \left[1 + \frac{c_1(\lambda)}{\varepsilon} + \frac{c_2(\lambda)}{\varepsilon^2} + \cdots\right]$$

$$0 = \mu \frac{d}{d\mu}\phi_0 \Rightarrow \mu \frac{d}{d\mu}\phi = -\gamma(\lambda)\,\phi$$

$$\gamma(\lambda) = \frac{1}{2}\mu\frac{d}{d\mu}\ln Z = \frac{1}{2}\left(\frac{\partial}{\partial\lambda}\ln Z\right)\left(-\varepsilon\lambda + \lambda a_1'\right)$$

- Formal solutions of RG eqs.

- $$\frac{d\lambda}{d\ln\mu} = \beta(\lambda) \implies \ln\frac{\mu'}{\mu} = \int_{\lambda(\mu)}^{\lambda(\mu')} \frac{d\lambda}{\beta(\lambda)}$$

$$d\ln\mu = \frac{d\lambda}{\beta(\lambda)}$$

- $$\frac{d\ln m^2}{d\ln\mu} = -\gamma_m(\lambda) \implies \ln\frac{m^2(\mu')}{m^2(\mu)} = -\int_{\ln\mu}^{\ln\mu'} \gamma_m(\lambda(\tilde{\mu}))\, d\ln\tilde{\mu}$$

$$= -\int_{\lambda(\mu)}^{\lambda(\mu')} \frac{\gamma_m(\lambda)}{\beta(\lambda)}\, d\lambda$$

$$\bullet \quad \frac{d}{d\ln\mu}\phi(\mu) = -\gamma(\mu)\,\phi(\mu)$$

$$\Longrightarrow \quad Z(\mu', \mu) \equiv e^{-\int_{\lambda(\mu)}^{\lambda(\mu')} \frac{\gamma(\lambda)}{\beta(\lambda)}\, d\lambda}$$

$$\begin{cases} \dfrac{d}{d\ln\mu'} Z(\mu', \mu) = -\gamma(\lambda(\mu'))\, Z(\mu', \mu) \\[2ex] Z(\mu, \mu) = 1 \end{cases}$$

$$\Longrightarrow \quad \phi(\mu') = Z(\mu', \mu)\,\phi(\mu) \Longrightarrow \frac{d\,\phi(\mu')}{d\ln\mu'} = -\gamma(\mu')\,\phi(\mu')$$

Examples

- $\lambda\phi^4$

$$\cdot\ \lambda_0 = \lambda\mu^{\varepsilon}\left(1 + \frac{a_1}{\varepsilon} + \dots\right)$$

$$\cdot\ a_1 = \frac{3\lambda}{16\pi^2}\lambda + O(\lambda^2)$$

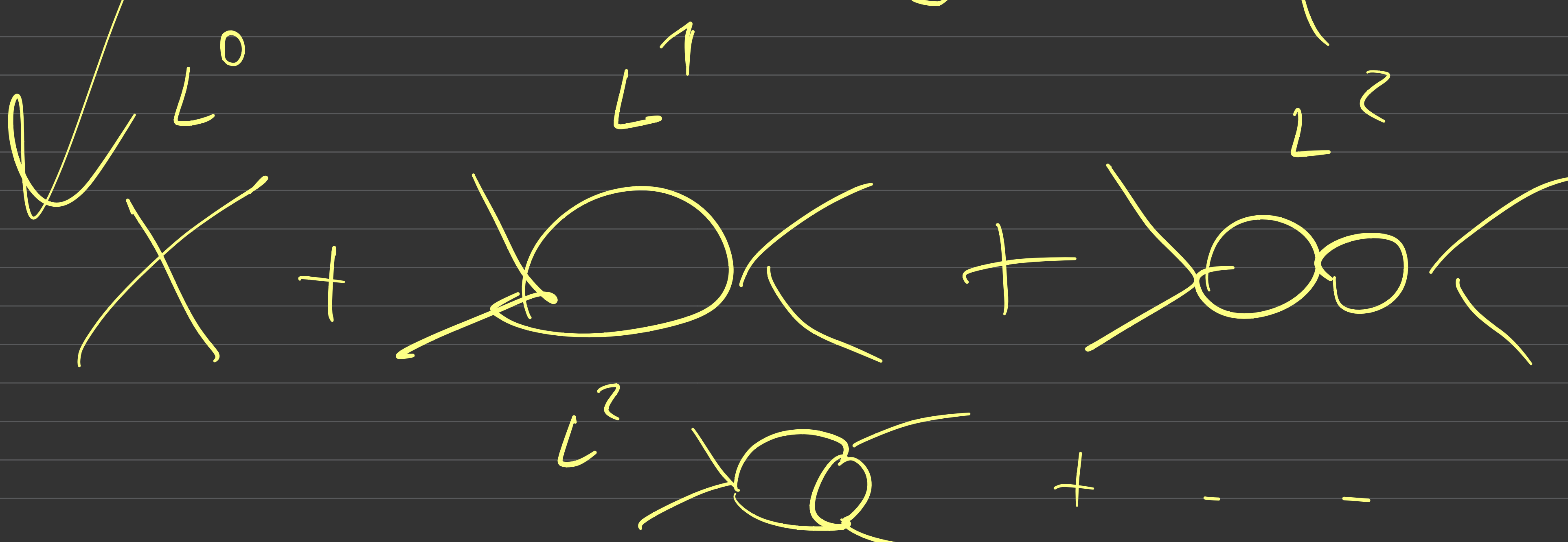
$$\cdot\ \beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3) > 0$$

$$\mu\frac{d}{d\mu}\lambda(\mu) = \frac{3\lambda^2(\mu)}{16\pi^2} \implies \boxed{\lambda(\mu') = \frac{\lambda(\mu)}{1 - \frac{3}{16\pi^2}\lambda(\mu)\ln\frac{\mu'}{\mu}}}$$

$$\bullet \quad \sigma(E) \approx \frac{1}{E^2} \cdot \lambda(E)^2 \left(1 + c\,\lambda(E) + \dots \right)$$

$$\approx \frac{1}{E^2}\, \lambda(E)^2 \approx \frac{1}{E^2} \left[\frac{\lambda(\mu)}{1 - \frac{3\lambda(\mu)}{16\pi^2} \ln E/\mu} \right]^2$$

$$\approx \frac{1}{E^2}\, \lambda(\mu)^2 \left(\sum \left(\frac{3\lambda}{16\pi^2}\right)^n L^n \right)$$



$$L \equiv \ln \frac{E}{\mu}$$

$$\frac{d\lambda}{d\ln\mu} = c_1\lambda^2 + c_2\lambda^3 + \cdots + c_\ell\lambda^{\ell+1} + \cdots$$

$$\frac{d\ln\lambda}{d\ln\mu} = \lambda\left(c_1 + c_2\lambda + \cdots + c_\ell\lambda^{\ell-1} + \cdots\right)$$

$$\begin{array}{ccc}
\downarrow & \downarrow & \downarrow \\
(\lambda L)^n & \lambda(\lambda L)^n & \lambda^{\ell-1}(\lambda L)^n \\
\lambda^n L^n & \lambda^{n+1} L^n & \lambda^{\ell+n-1} L^n
\end{array}$$

RG asymptotics of $\lambda\phi^4$ in 4D

$$\lambda(\mu') = \frac{\lambda(\mu)}{1 - \frac{3}{16\pi^2}\lambda(\mu)\ln\frac{\mu'}{\mu}}$$

- IR $\quad \ln \mu'/\mu \longrightarrow -\infty \quad \Longrightarrow \quad \lambda(\mu') \simeq \dfrac{16\pi^2}{3\ln\frac{\mu}{\mu'}} \longrightarrow 0$

- UV $\quad \dfrac{3\lambda(\mu)}{16\pi^2}\ln\dfrac{\Lambda_*}{\mu} = 1 \quad \Longrightarrow \quad \lambda(\Lambda_*) = \infty \quad$ Landau Pole

$$\ln\frac{\Lambda_*}{\mu} = \frac{16\pi^2}{3\lambda(\mu)} \longrightarrow \Lambda_* = \mu\, e^{\frac{16\pi^2}{3\lambda(\mu)}}$$