HW9

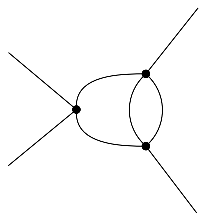
Two loop renormalization of ϕ^4

In class we found the structure of one-loop counter terms δ_λ and δ_{m^2} for a theory given by

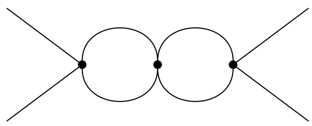
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\delta_{m^2}}{2}\phi^2 + \frac{\delta_\lambda}{4!}\phi^4 \tag{1}$$

For what follows we focus on $m^2 = 0$ for simplicity. In this case the advantage of dimensional regularization is manifest: all diagrams with tadpoles can be neglected, for they are zero.

1. Draw all (connected one particle irreducible) diagrams contributing to 4-pt function at two loops. Here is a couple of them (but be aware of other channels)

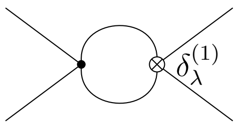


(2)



(3)

2. Show that including diagrams with one-loop counter terms, like the one below,

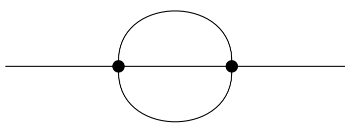


(4)

is crucial for cancelling non-local divergences.

3. Find the corresponding two loop counter term $\delta_\lambda^{(2)}$, whose inclusion renders 4-pt function finite.

4. There is one new structure appearing at two loops that we haven't encountered before. Compute two-loop contribution to 2-pt function coming from



(5)

Show that the corresponding divergence can be eliminated with the following counter term

$$\mathcal{L} \supset \frac{1}{2}\delta_Z(\partial\phi)^2 \tag{6}$$

This counter term can be viewed as a rescaling of the field

$$\phi = \sqrt{Z}\phi_R \equiv \sqrt{1+\delta_Z}\phi_R, \tag{7}$$

and (for historical reasons) is called the wave function renormalization