Homework 8

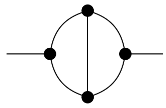
Two loop diagram for ϕ^3 in $d = 6 - \varepsilon$ dimensions

Let's consider a Lagrangian

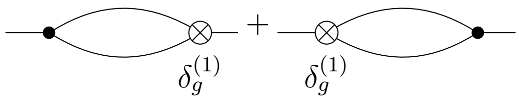
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{6!}\phi^3, \tag{1}$$

in $d = 6 - \varepsilon$ dimensions.

- Compute the divergent ($\varepsilon \to 0$) part of the following diagram contributing to the 2-pt function



- Show explicitly that non-local (proportional to $\log p^2$) terms cancel upon taking into account diagrams with the one-loop counter term $\delta_g^{(1)}$



(2)

Optical theorem

From the unitarity of the S-Matrix one can derive the optical theorem (See for example Peskin Sec 7.3)

$$2\mathrm{Im}\mathcal{M}(i \to f) = \sum_n \int d\Pi_n \mathcal{M}(i \to n)\mathcal{M}^*(f \to n), \tag{3}$$

where the sum is over all possible states and $d\Pi_n$ is the associated phase-space measure. Verify the equation to one loop order in the massive ϕ^4 theory when both i and f are two particle states.