

Homework 8

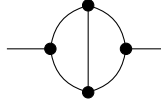
Two loop diagram for ϕ^3 in $d = 6 - \varepsilon$ dimensions

Let's consider a Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{g}{6!}\phi^3, \quad (1)$$

in $d = 6 - \varepsilon$ dimensions.

- Compute the divergent ($\varepsilon \rightarrow 0$) part of the following diagram contributing to the 2-pt function



- Show explicitly that non-local (proportional to $\log p^2$) terms cancel upon taking into account diagrams with the one-loop counter term $\delta_g^{(1)}$

$$\text{Diagram 1} + \text{Diagram 2} \quad (2)$$

Diagram 1: A horizontal line with a black dot on the left and a circle with an 'X' on the right. Two semi-circular arcs connect the dot and the circle. Below the circle is the label $\delta_g^{(1)}$.
 Diagram 2: A horizontal line with a circle with an 'X' on the left and a black dot on the right. Two semi-circular arcs connect the circle and the dot. Below the circle is the label $\delta_g^{(1)}$.

Optical theorem

From the unitarity of the S -Matrix one can derive the optical theorem (See for example Peskin Sec 7.3)

$$2\text{Im}\mathcal{M}(i \rightarrow f) = \sum_n \int d\Pi_n \mathcal{M}(i \rightarrow n) \mathcal{M}^*(f \rightarrow n), \quad (3)$$

where the sum is over all possible states and $d\Pi_n$ is the associated phase-space measure. Verify the equation to one loop order in the massive ϕ^4 theory when both i and f are two particle states.