AQFT - Exercise Set 5

Goldstone Theorem

Consider a QFT for a complex scalar field ψ invariant under the following $U(1)$ transformation

$$\psi(x) \to e^{i\alpha}\psi(x). \tag{1}$$

Let $J^\mu(x)$ be the current operator associated with the symmetry so that

$$Q = \int d^3x \, J^0(t, \mathbf{x}), \tag{2}$$

is the (time-independent) charge operator.

- Argue that
$$[J^0(t, \mathbf{x}), \psi(t, \mathbf{y})] = \psi(\mathbf{x}) \, \delta^{(3)}(\mathbf{x} - \mathbf{y}). \tag{3}$$

- Using the previous exercise, derive the Källén–Lehmann spectral representation of the two-point function
$$\langle 0| \, T(J^\mu(x)\psi(0)) \, |0\rangle \tag{4}$$

- Compute
$$\partial_\mu \langle 0| \, T(J^\mu(x)\psi(0)) \, |0\rangle \, . \tag{5}$$

- Assume now the symmetry to be spontaneously broken by the expectation value of the scalar field $\langle 0| \, \psi(0) \, |0\rangle = v \neq 0$. Prove that the spectral function of the previous correlator has support at zero invariant mass.