Homework 9

1 Critical exponents

Consider the following $O(N)$ symmetric Lagrangian in $d = 4 - \varepsilon$ dimensions

$$\mathcal{L} = \frac{1}{2}(\partial\phi_i)^2 + \frac{\lambda}{4!}(\phi_i\phi_i)^2. \tag{1}$$

In order to simplify manipulations you may want to rewrite the Lagrangian

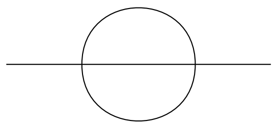
$$\mathcal{L} = \frac{1}{2}(\partial\phi_i)^2 + \frac{\lambda_{ijkl}}{4!}\phi_i\phi_j\phi_k\phi_l \tag{2}$$

with λ_{ijkl} totally symmetric

$$\lambda_{ijkl} = \frac{1}{3}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{3}$$

Using this model we try finding critical exponents. To this end,

- Computing the divergent part of the diagram



(4)

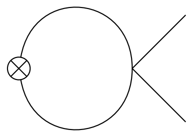
  show that the anomalous dimensions γ_ϕ of $\phi(x)$ is

$$\gamma_\phi(\lambda) = \frac{N+2}{36}\left(\frac{\lambda}{16\pi^2}\right)^2. \tag{5}$$

- One-loop contribution to the correlator

$$\langle \phi_k^2(x)\phi_i(y)\phi_j(z)\rangle \tag{6}$$

  is given by



(7)

  Computing the divergent part find the anomalous dimension γ_{ϕ^2} of the operator ϕ_i^2

$$\gamma_{\phi^2}(\lambda) = \frac{N+2}{3}\frac{\lambda}{16\pi^2}. \tag{8}$$

- Lastly, show that the one-loop beta function for $\lambda(\mu)$ is given by

$$\beta(\lambda) = \left(\frac{N+8}{3}\frac{\lambda}{16\pi^2} - \varepsilon\right)\lambda. \tag{9}$$

  Using that expression find the critical value for the coupling $\lambda = \lambda_*(\varepsilon)$: $\beta(\lambda_*) = 0$.

- Using Callan-Symanzik equation for two-point functions $\langle\phi\phi\rangle$ and $\langle\phi^2\phi^2\rangle$ find scaling dimensions of operators ϕ and ϕ^2 at critical point. Compare your result with the one obtained in [1] using conformal bootstrap method.

2 Current non-renormalization

For this section we are going back to Yukawa theory considered in the previous homework

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + i\bar{\psi}\partial\!\!\!/\psi - g\phi\bar{\psi}\psi - \frac{\lambda}{4!}\phi^4. \tag{10}$$

It follows from Ward identities that the conserved current

$$j^\mu = \bar{\psi}\gamma^\mu\psi \tag{11}$$

is not renormalized, meaning that its anomalous dimension is zero. Show that by explicit computations at one loop.

References

[1] F. Kos, D. Poland, D. Simmons-Duffin and A. Vichi, *Precision Islands in the Ising and $O(N)$ Models*, *JHEP* **08** (2016) 036 [1603.04436].