AQFT - Exercise Set 7

Dimensional regularization

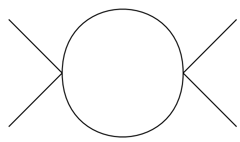
1. To familiarize yourself with the subject, read about dimensional regularization:

   [1] (Chapter 4) and

   [2] (in Chapter 7.5; you may also find helpful some parts in Chapter 10.2).

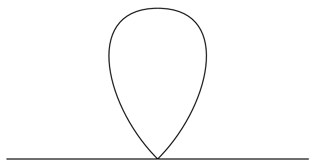
   In this homework we will be dealing only with scalar field theories.

2. Using dimensional regularization compute the diagram (1) in $d = 4 - \varepsilon$ dimensions



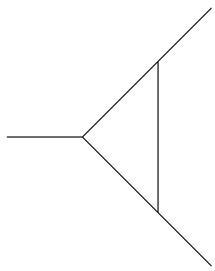
(1)

3. The same for the diagram (2)



(2)

4. Using dimensional regularization compute the divergent part of the integral corresponding to the following Feynman diagram in $d = 6 - \varepsilon$ dimensions



(3)

5. In dimensional regularization the integral

$$\int \frac{d^d k}{(k^2)^r} = 0 \tag{4}$$

   vanishes for arbitrary $r \in \mathbb{R}$. Using this property and the definition

$$I_2 = \int d^d k \frac{1}{k^2 (k+p)^2}, \tag{5}$$

   show that

- $$\int d^d k \frac{k_\mu}{k^2(k+p)^2} = -\frac{p_\mu}{2} I_2, \tag{6}$$
- $$\int d^d k \frac{k_\mu k_\nu}{k^2(k+p)^2} = \frac{d}{4(d-1)} \left(p_\mu p_\nu - \frac{1}{d}\delta_{\mu\nu} p^2 \right) I_2 \tag{7}$$

Pauli-Villars

There is yet another regularization, which is called Pauli-Villars. It corresponds to modifying the propagator by adding higher order derivatives to the Lagrangian

$$\phi \Box \phi \to \phi \left(\Box + a_0 \frac{m^2 \Box}{\Lambda^2} + a_1 \frac{\Box^2}{\Lambda^2} + a_2 \frac{\Box^3}{\Lambda^4} \right) \phi \underset{\Lambda\to\infty}{\to} \phi \Box \phi. \tag{8}$$

For this homework we focus on the following modification

$$\frac{1}{p^2+m^2} \to \frac{1}{p^2+m^2} + \frac{c_1}{p^2+M_1^2} + \frac{c_2}{p^2+M_2^2}, \tag{9}$$

with

$$c_1 = \frac{M_2^2 - m^2}{M_1^2 - M_2^2}, \quad c_2 = \frac{M_1^2 - m^2}{M_2^2 - M_1^2}, \quad M_{1,2} \propto \Lambda. \tag{10}$$

Convince yourself that this is equivalent to (8) and compute the integral corresponding to the diagram in (2).

Suggested reading

[1] J. C. Collins, *Renormalization: An Introduction to Renormalization, the Renormalization Group and the Operator-Product Expansion*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 1984.

[2] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995.