

The Cosmic Distance Ladder

The Variable Universe – Lecture 12
Fall Semester 2022

Richard Anderson

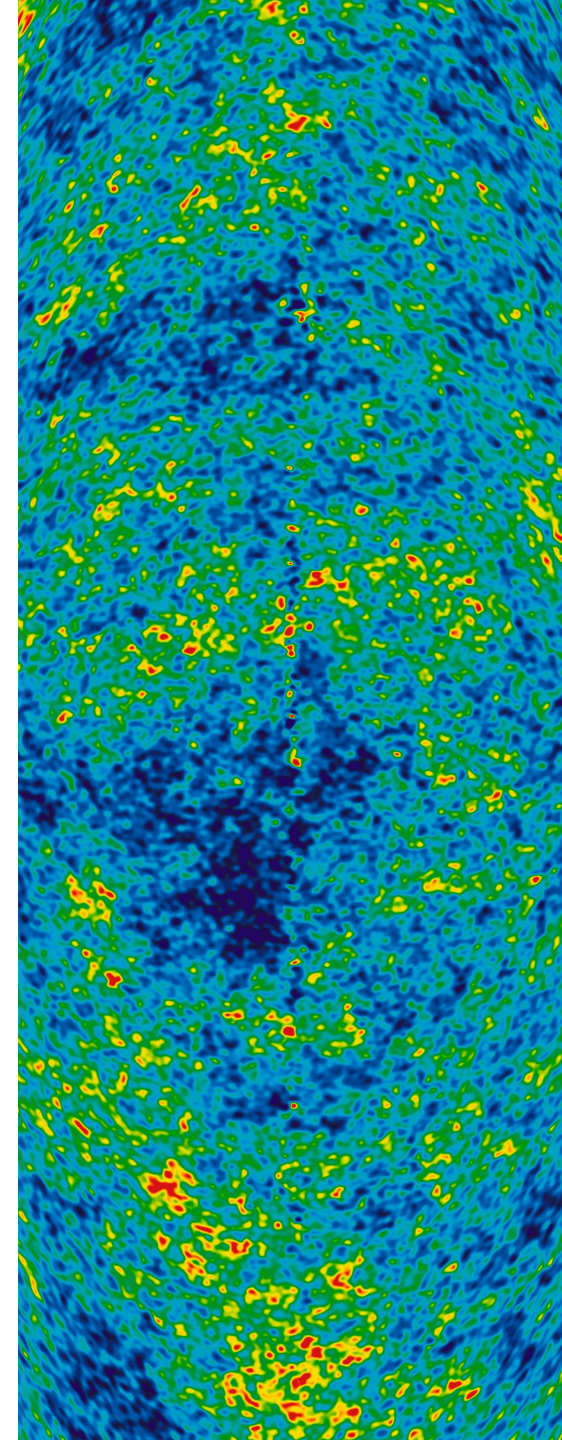
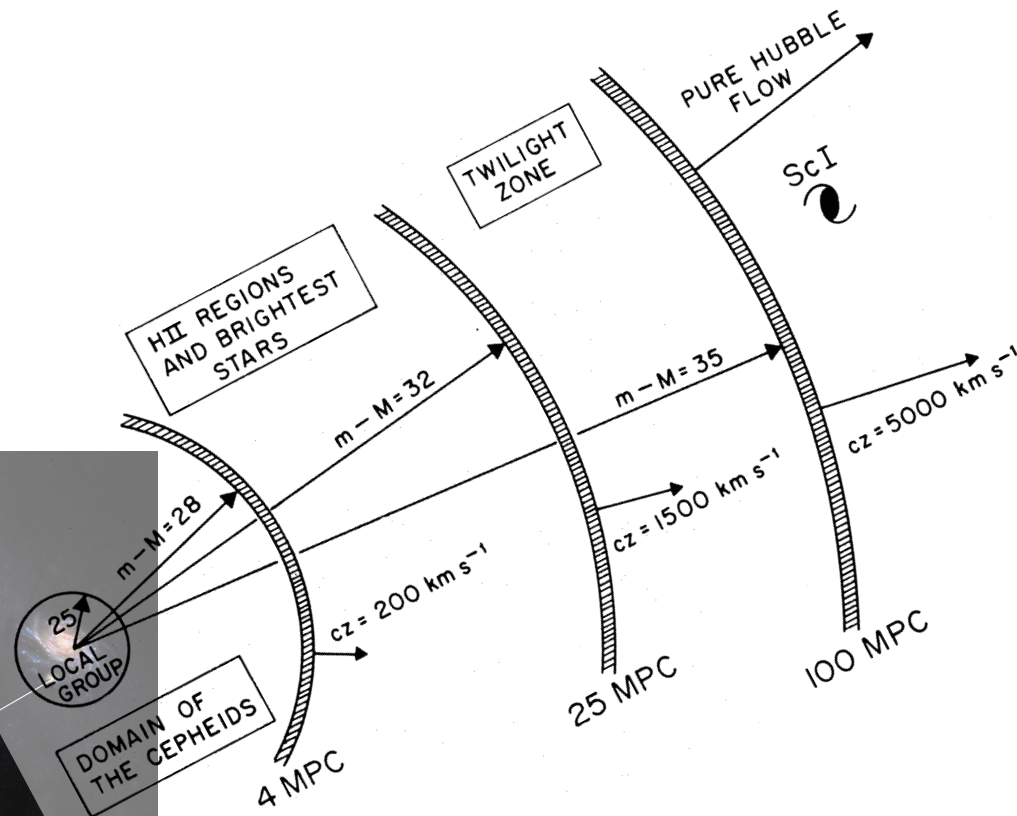
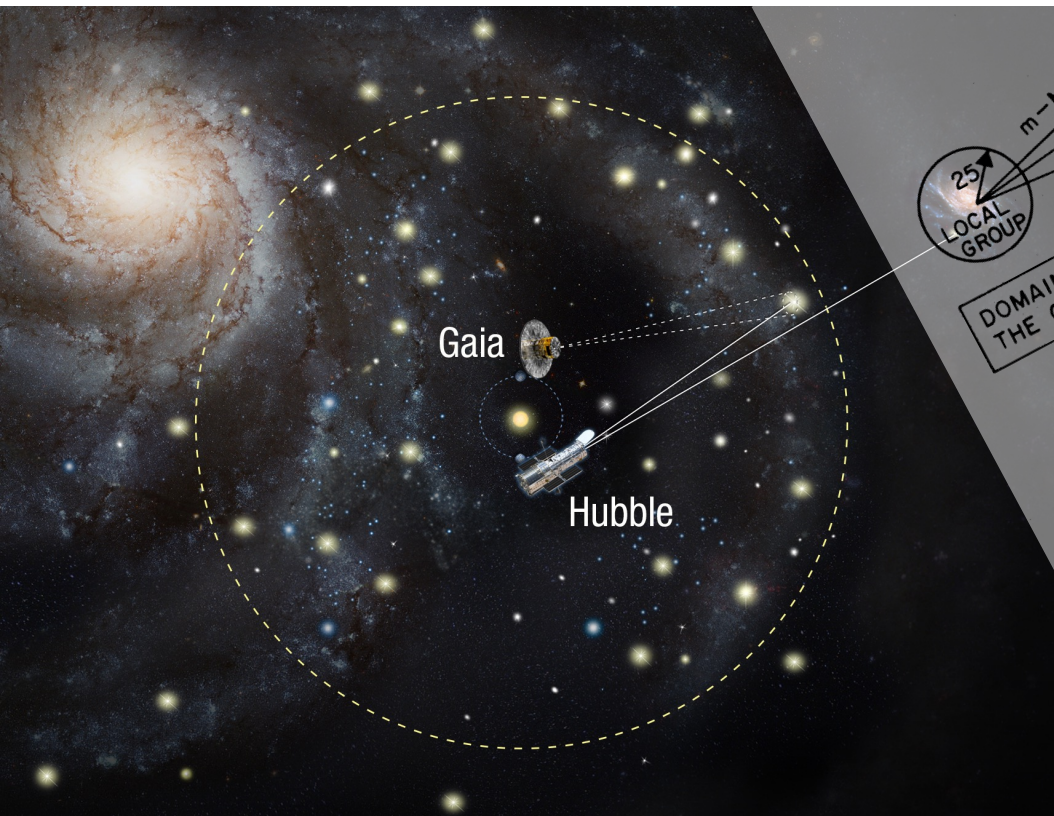
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Sauverny Observatory #265



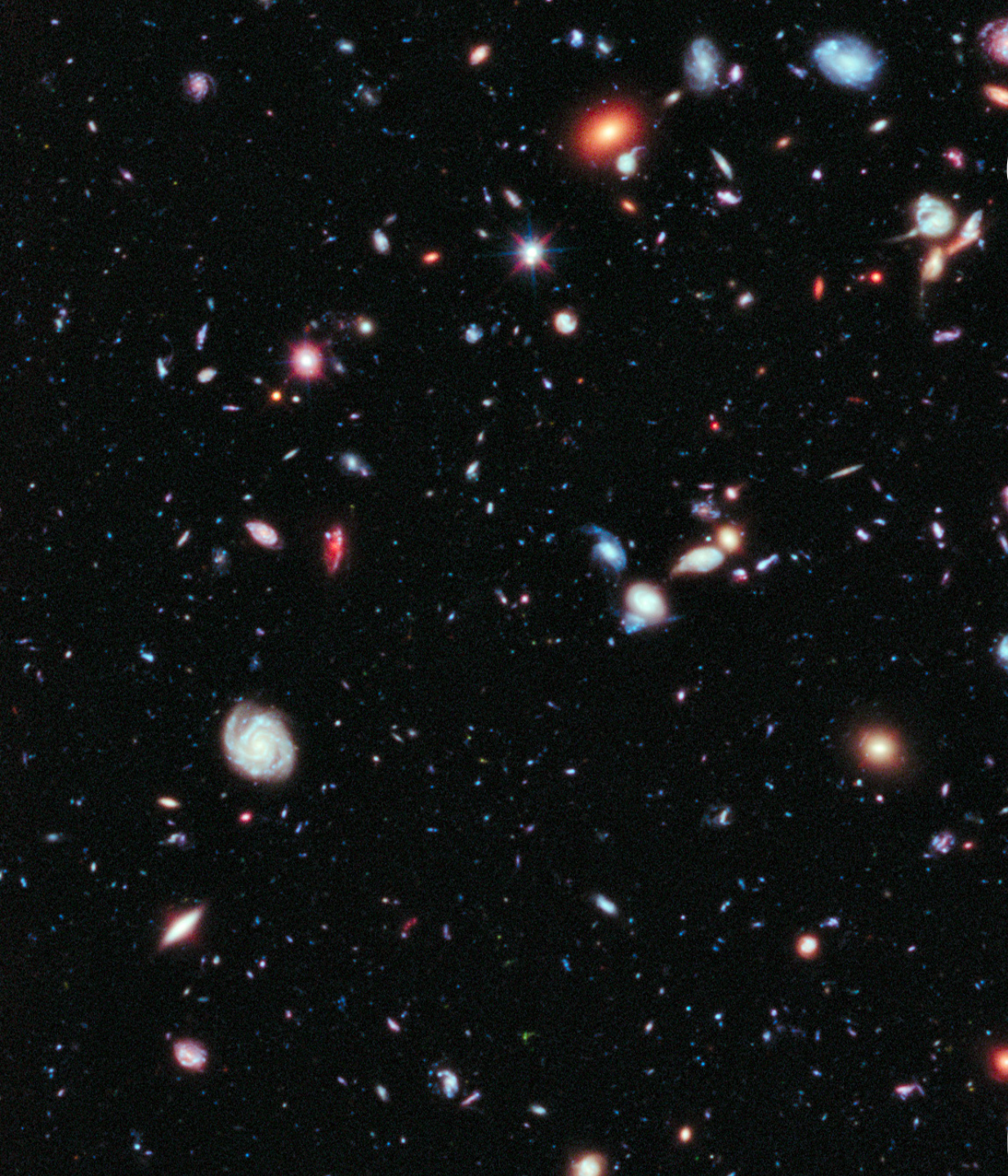
Overview



Distance is *the* fundamental problem of astronomy

Distance is required to interpret observations in an absolute sense





What's the basis for all (astronomical) distances?

The speed of light in vacuum: defines 1 m as distance traveled by light in vacuum in
 $t = 1 / 299792458$ s

31 Aug 2012!

RESOLUTION B2

on the re-definition of the astronomical unit of length.

*Proposed by the IAU Division I Working Group Numerical Standards
and supported by Division I*

The XXVIII General Assembly of International Astronomical Union,

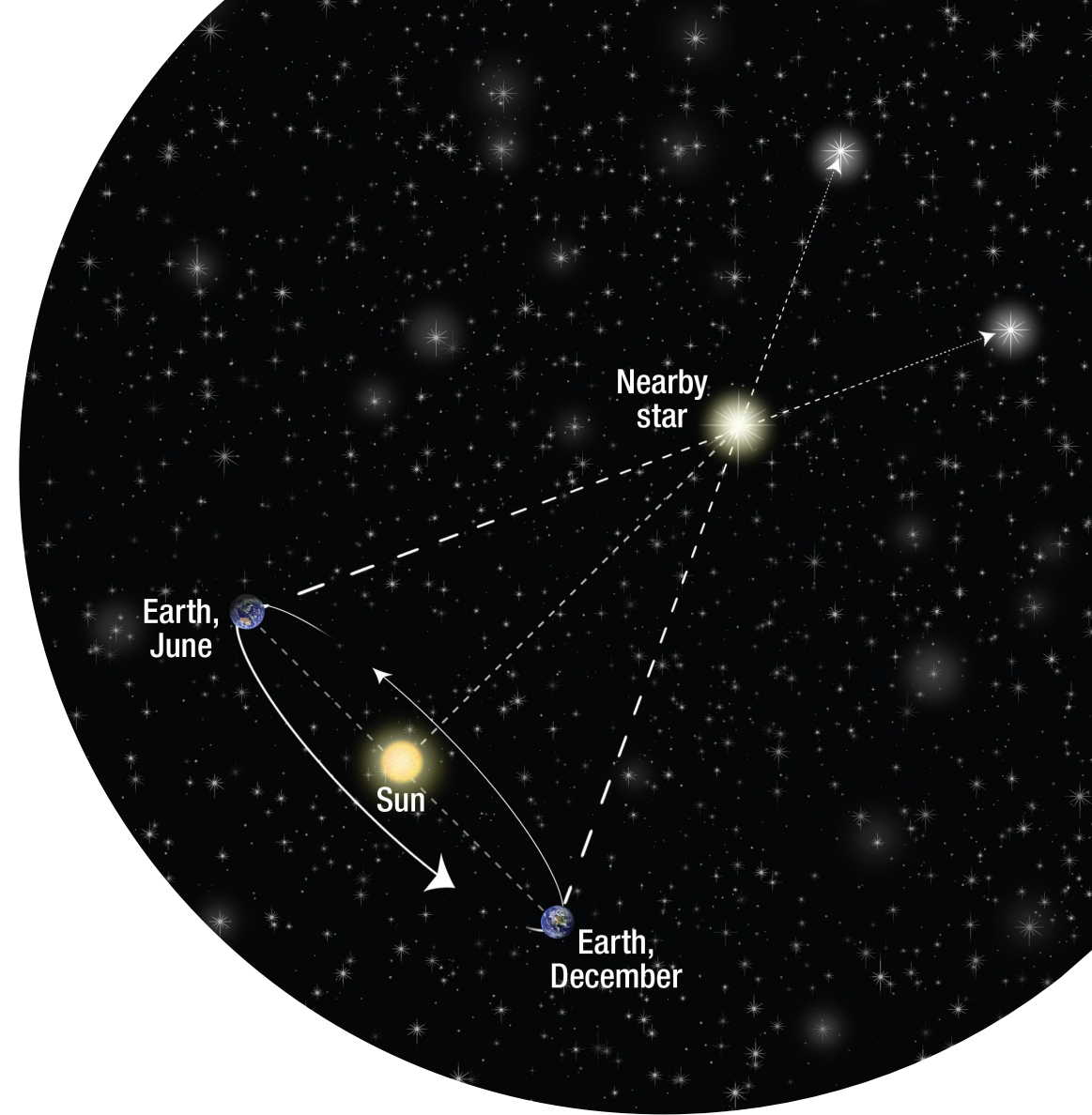
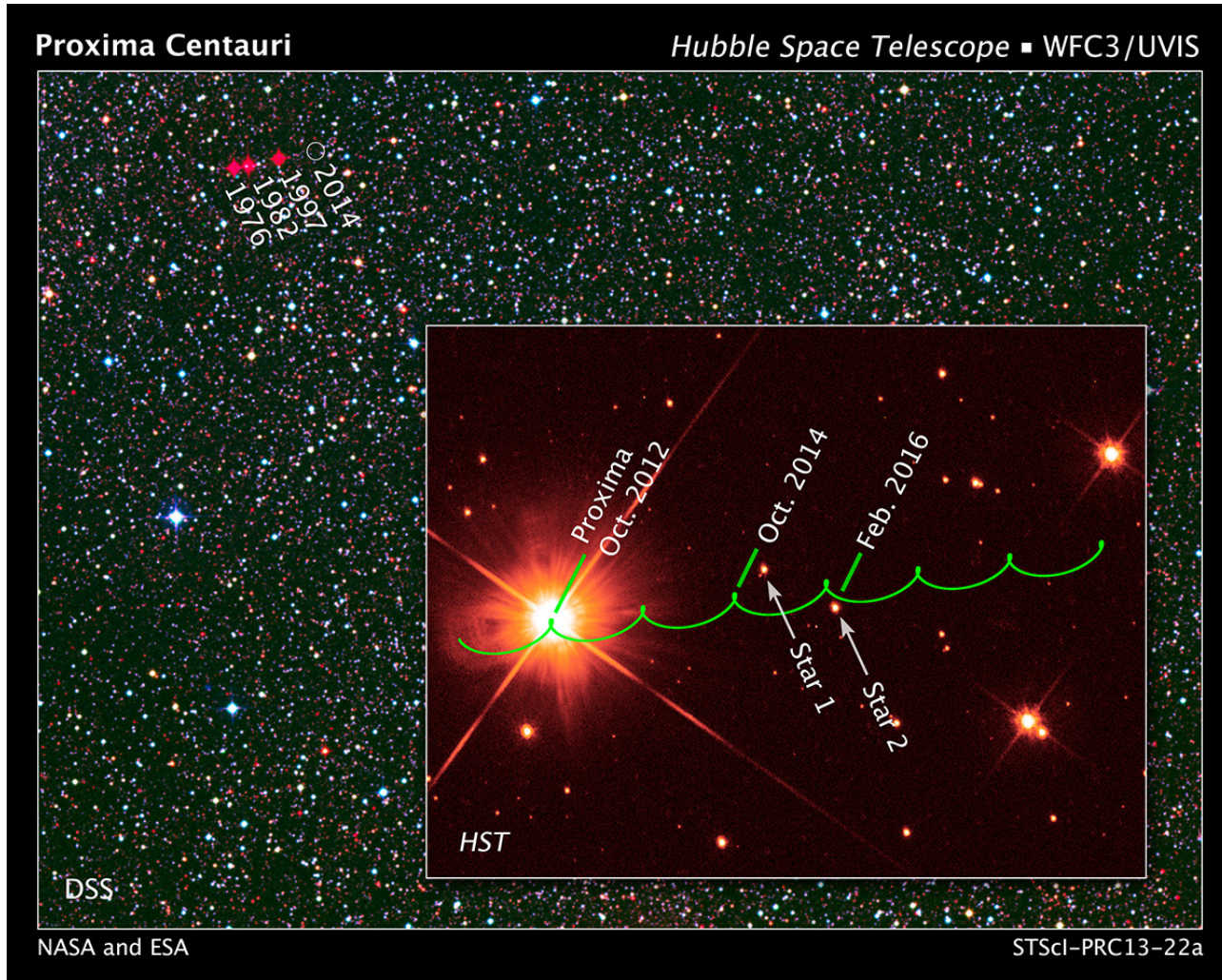
What is *the* fundamental
astronomical distance measure?

The astronomical unit [au].
Now an SI quantity:
 $1 \text{ au} = 149,597,870,700 \text{ m}$

Direct distance measurement methods

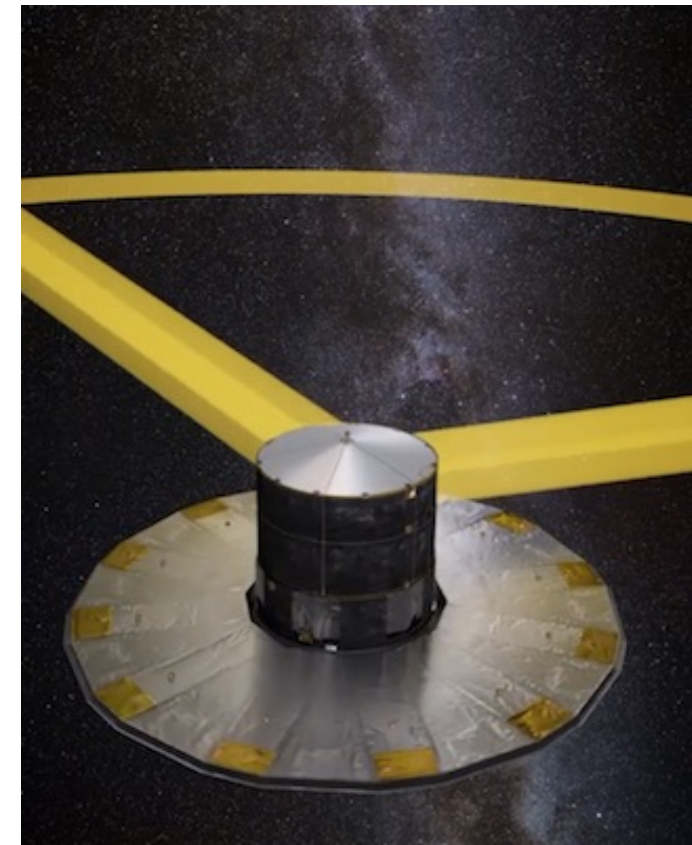
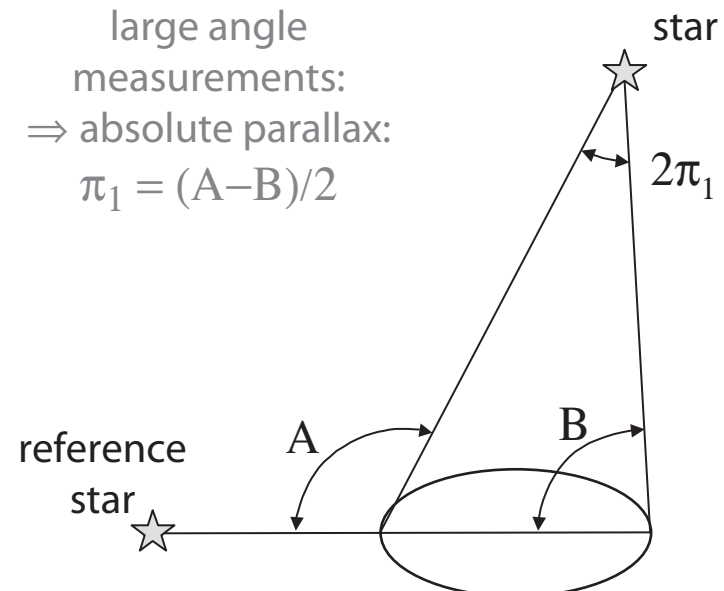
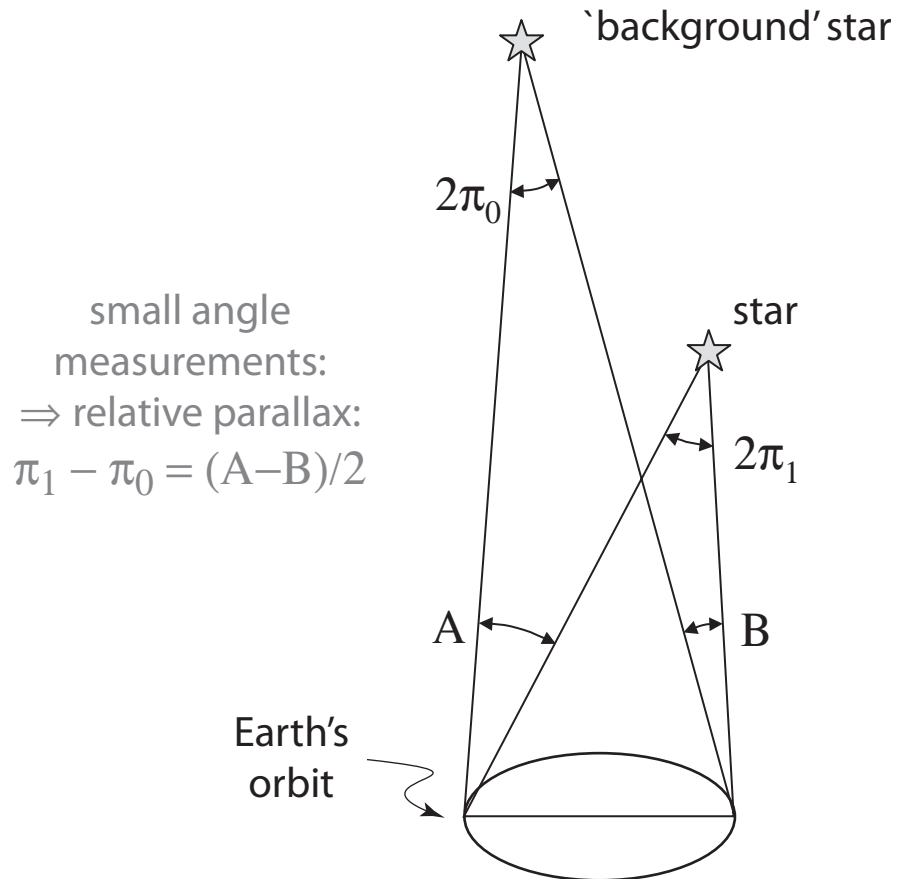
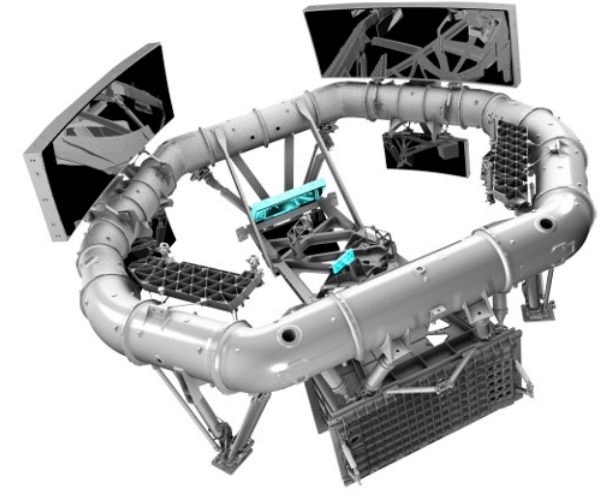
Which ones do you know?

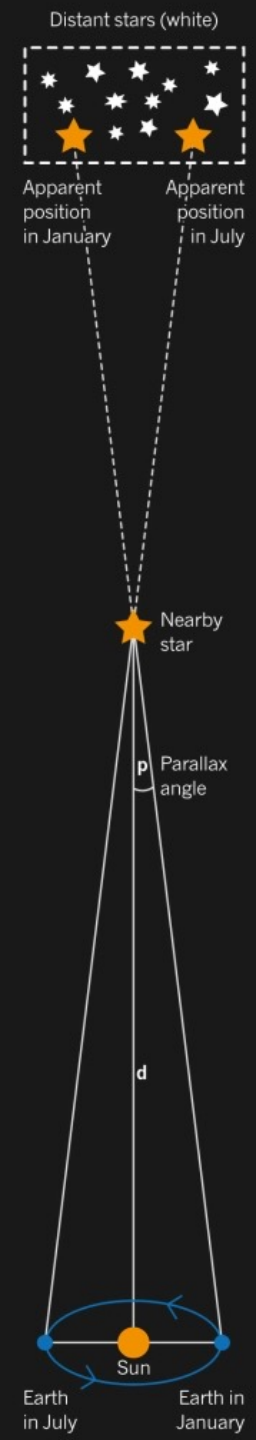
Parallax and Proper Motion



**HST: Narrow-angle astrometry!
(relative to background stars)**

Wide-angle astrometry





Parallax and Distance

Objects in mirror are **farther** than they appear!

$$d[\text{pc}] = \frac{1}{\varpi} \frac{[\text{au}]}{[\text{arcsec}]}$$

$$d(1\sigma) \in \left[\frac{1}{\varpi - \sigma_{\varpi}}, \frac{1}{\varpi + \sigma_{\varpi}} \right]$$

Gaia Data Release 2: Using Gaia parallaxes

X. Luri^[1], A.G.A. Brown^[4], L.M. Sarro^[6], F. Arenou^[2], C.A.L. Bailer-Jones^[3], A. Castro-Ginard^[1], J. de Bruijne^[5], T. Prusti^[5], C. Babusiaux^{[7][2]}, and H.E. Delgado^[6]

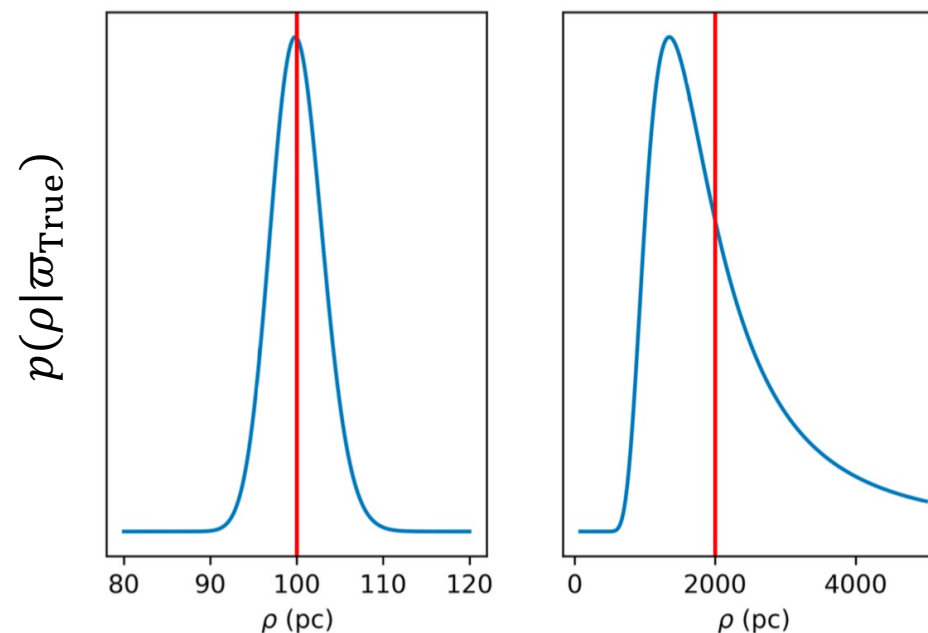
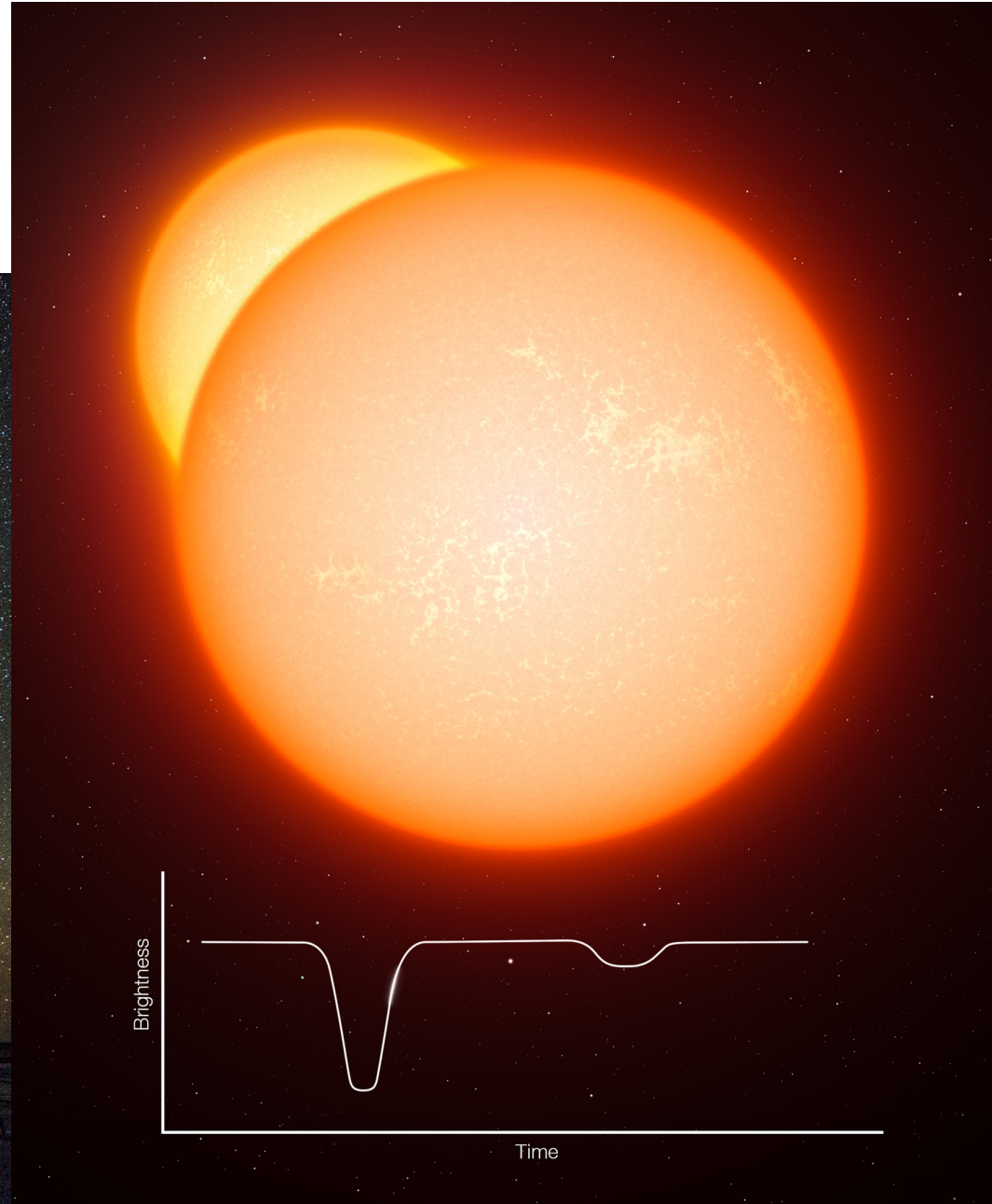


Fig. 3. PDF of $\rho = 1/\varpi$ in two extreme cases. The red vertical line indicates the true distance r . Left: Object at $r = 100$ pc with an uncertainty on the observed parallax of $\sigma_{\varpi} = 0.3$ mas. Right: Object at $r = 2000$ pc with an uncertainty on the observed parallax of $\sigma_{\varpi} = 0.3$ mas.

[Link to paper](#)

Detached Eclipsing Binary Distance to the Magellanic Clouds

Pietrzynski et al. ([2013](#), [2019](#)), Graczyk et al. ([2018](#))



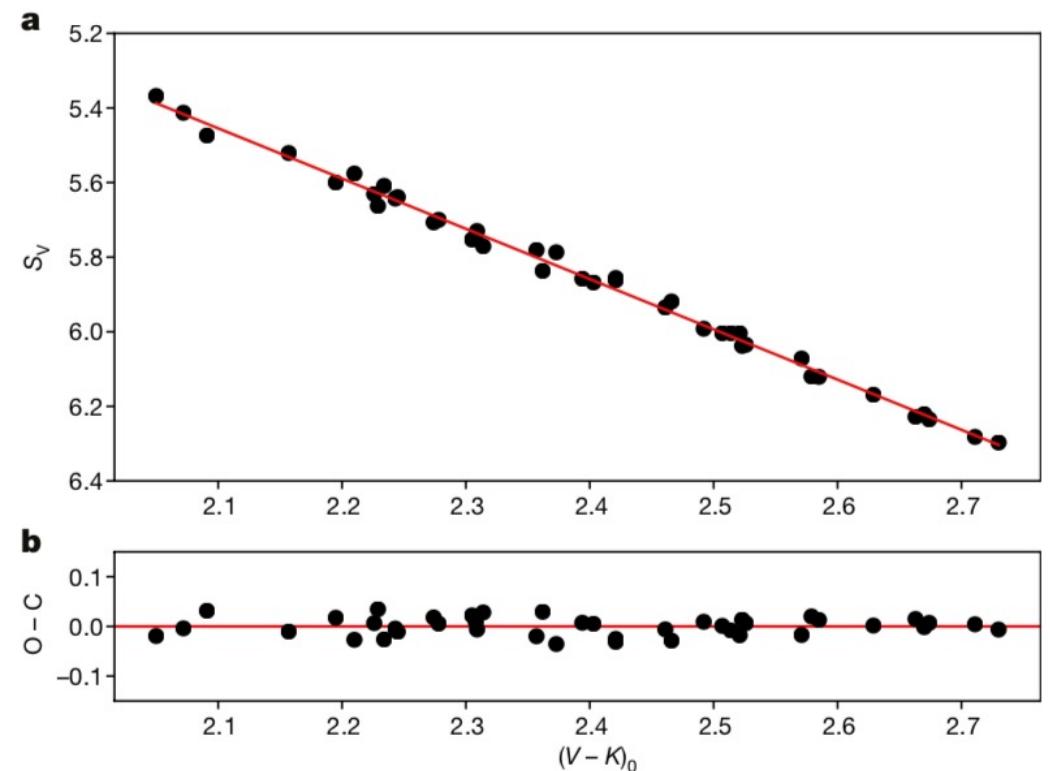
Detached eclipsing binary distances

- Distance = physical radius / angular diameter
- How do know angular diameter?
- Interferometry, surface brightness, ...

$$\phi = 10^{0.2(S_V - V_0)}$$

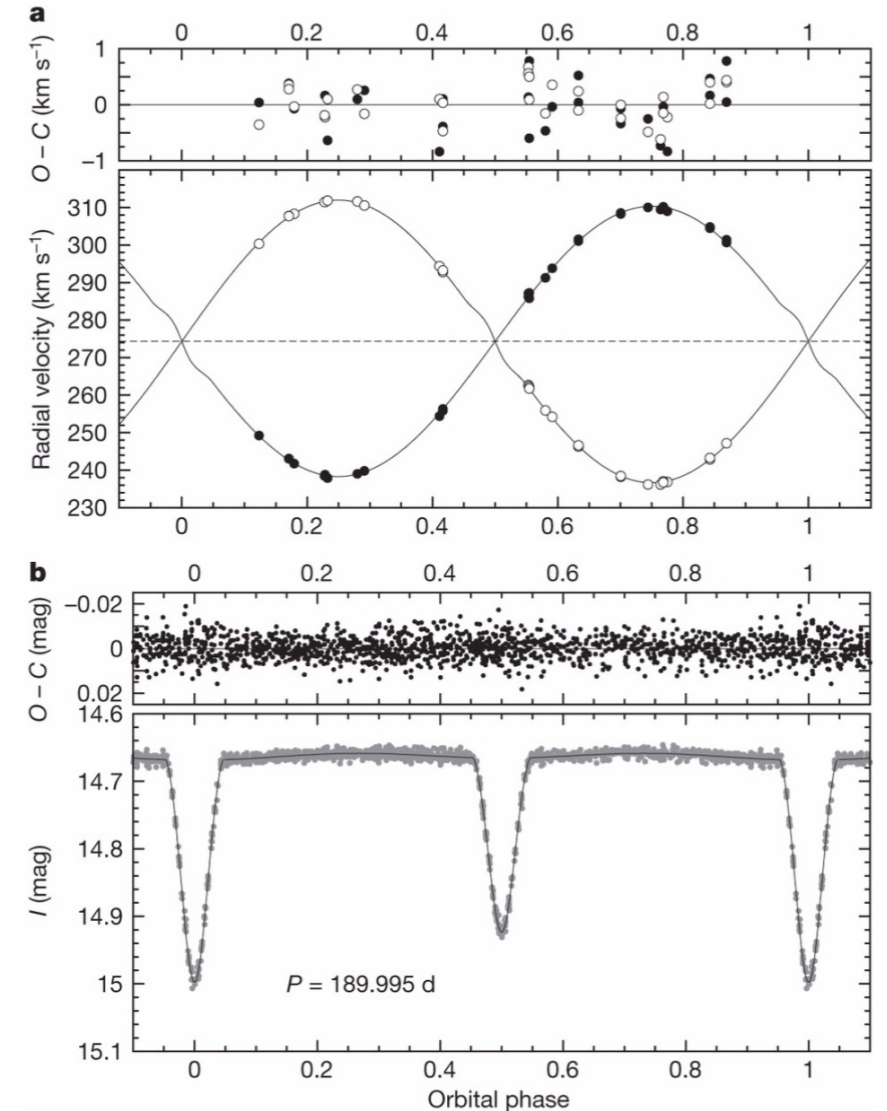
$$S_V = \alpha \cdot (V - K)_0 + \beta \text{ [mag]}$$

Surface brightness [mag/m²] is constant with distance (area compensates flux)!



Detached eclipsing binary distances

- Distance = physical radius / angular diameter
- How do know the physical radius?
- Eclipses: duration of transit and relative sizes
- Orbital velocities: eclipse path length



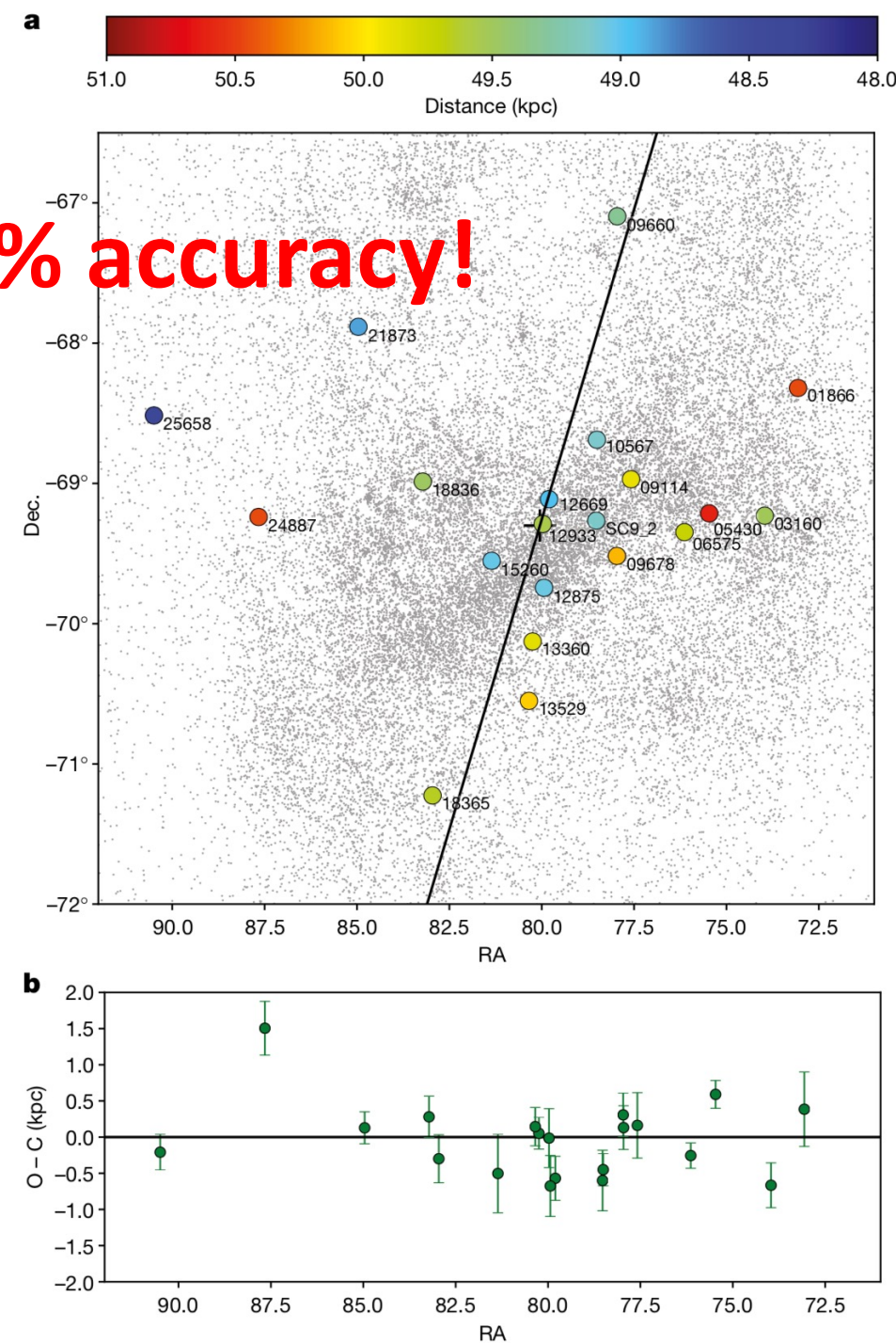
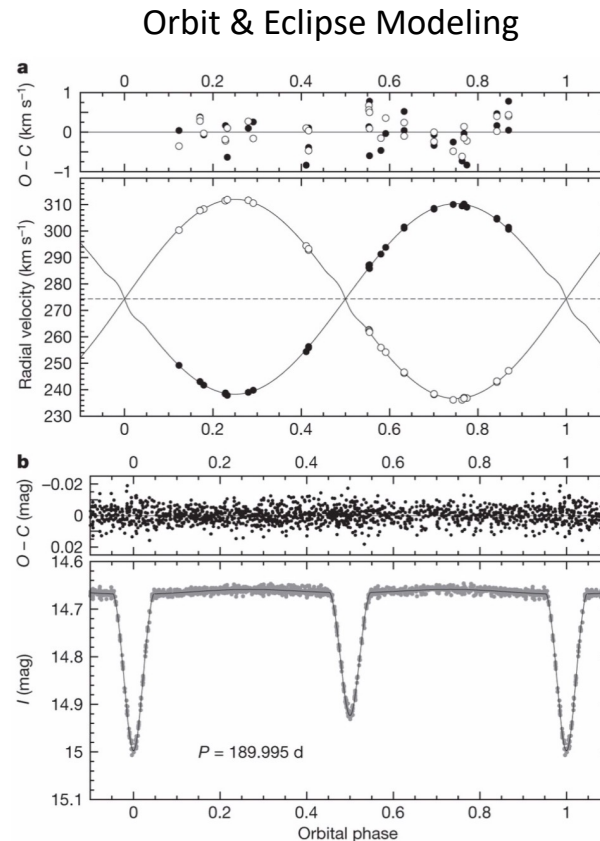
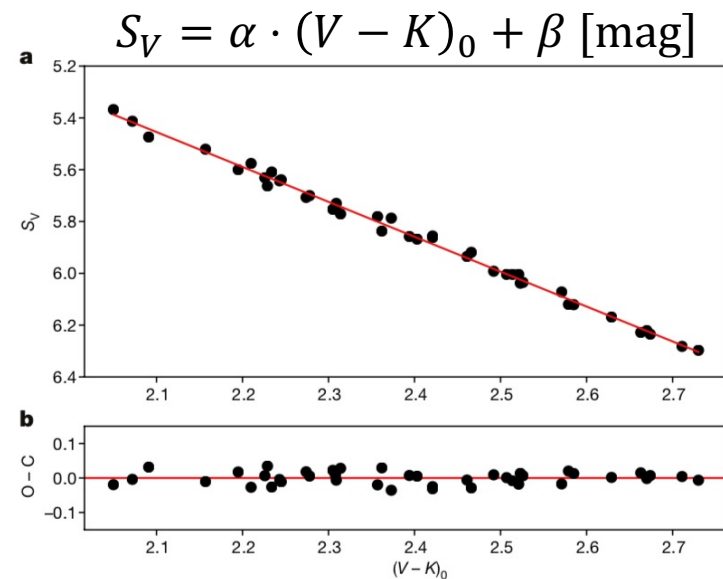
Detached Eclipsing Binary Distance of the Large Magellanic Cloud

Pietrzynski et al. ([2013](#), [2019](#)), Graczyk et al. ([2018](#))

1.09% accuracy!

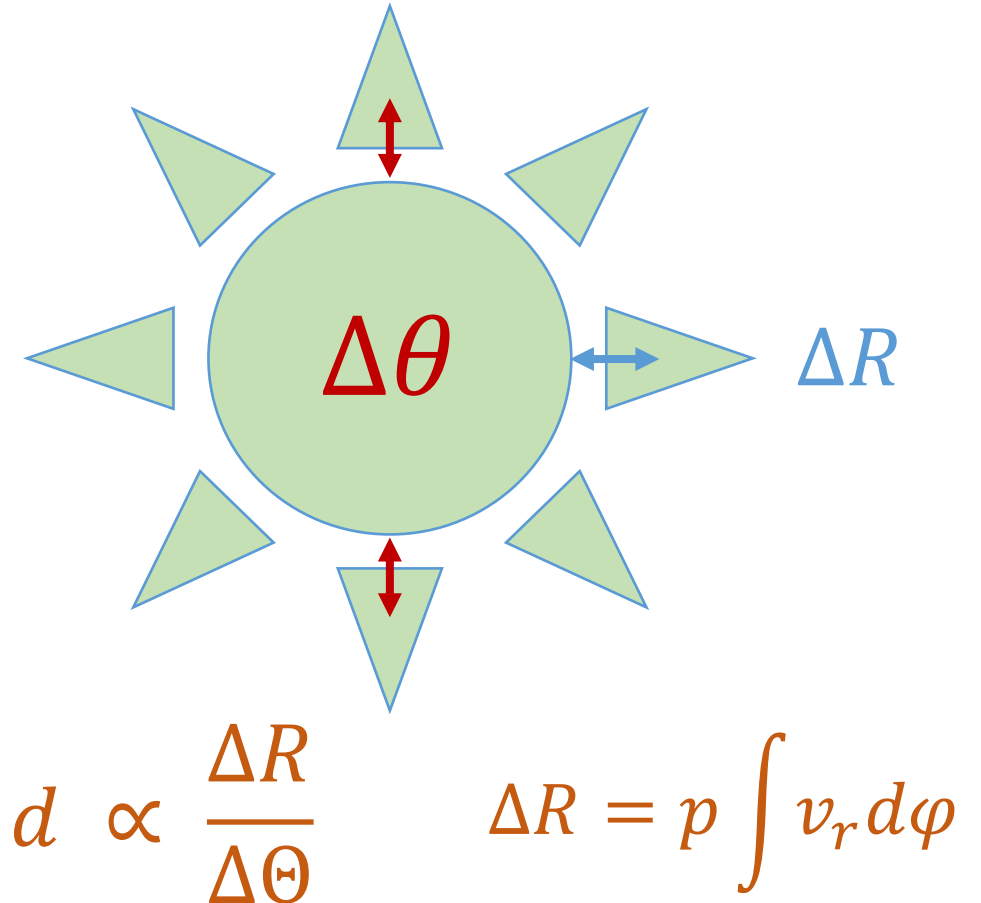
$$d = 49.59 \pm 0.09^{\text{stat}} \pm 0.53^{\text{syst}} \text{ kpc}$$

$$d \propto \frac{R}{\phi} \quad \xrightarrow{\phi = 10^{0.2(S_V - V_0)}}$$

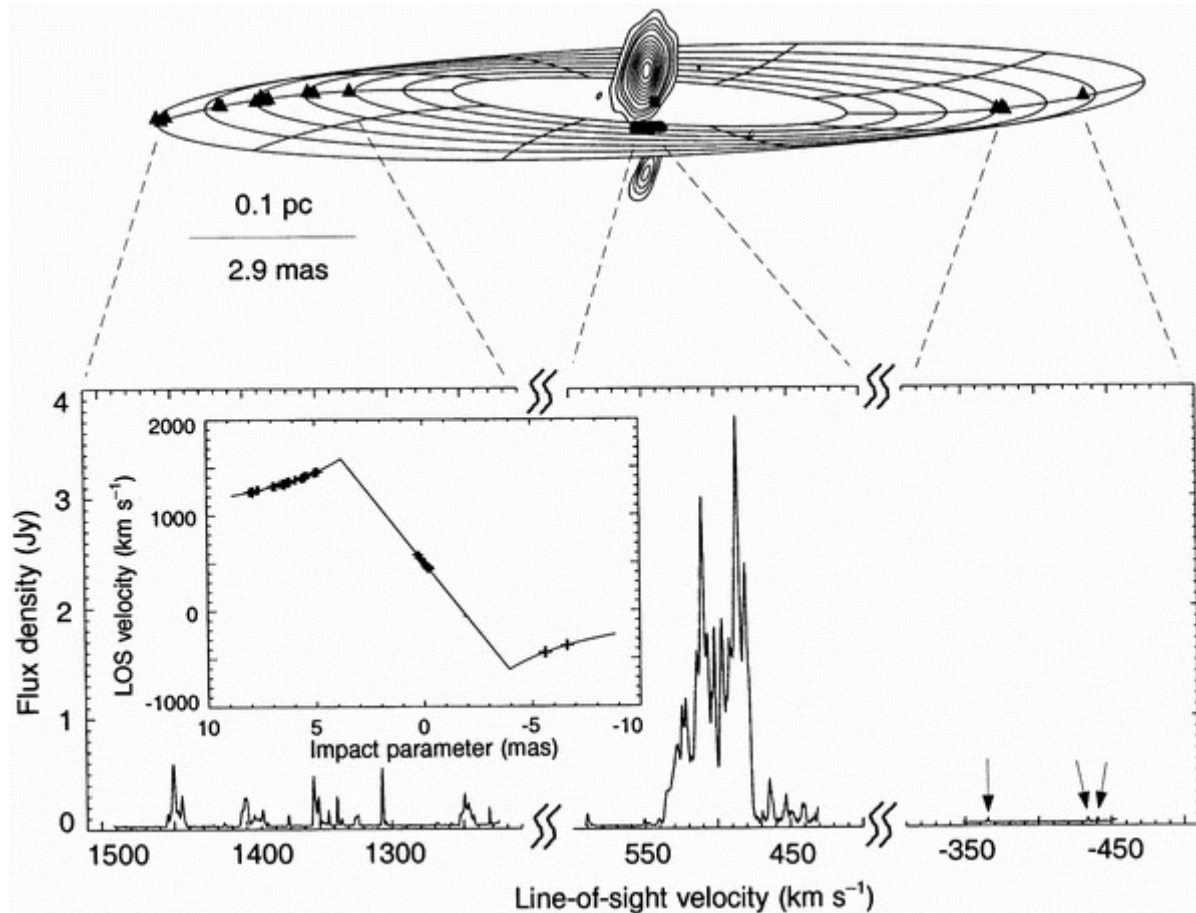


Baade-Wesselink-type Methods

- Angular distance determination
- Exploits the radial pulsation
- Compare *linear* to *angular* radius variations to determine distance
- Linear radius from line-of-sight velocities
- Angular radius from interferometry or surface brightness relations



Megamasers: geometric distances to < 200 Mpc



cd: The luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 5.4×10^{14} hertz ($\sim 550\text{nm}$) and that has a radiant intensity in that direction of $1/683$ watt per steradian.

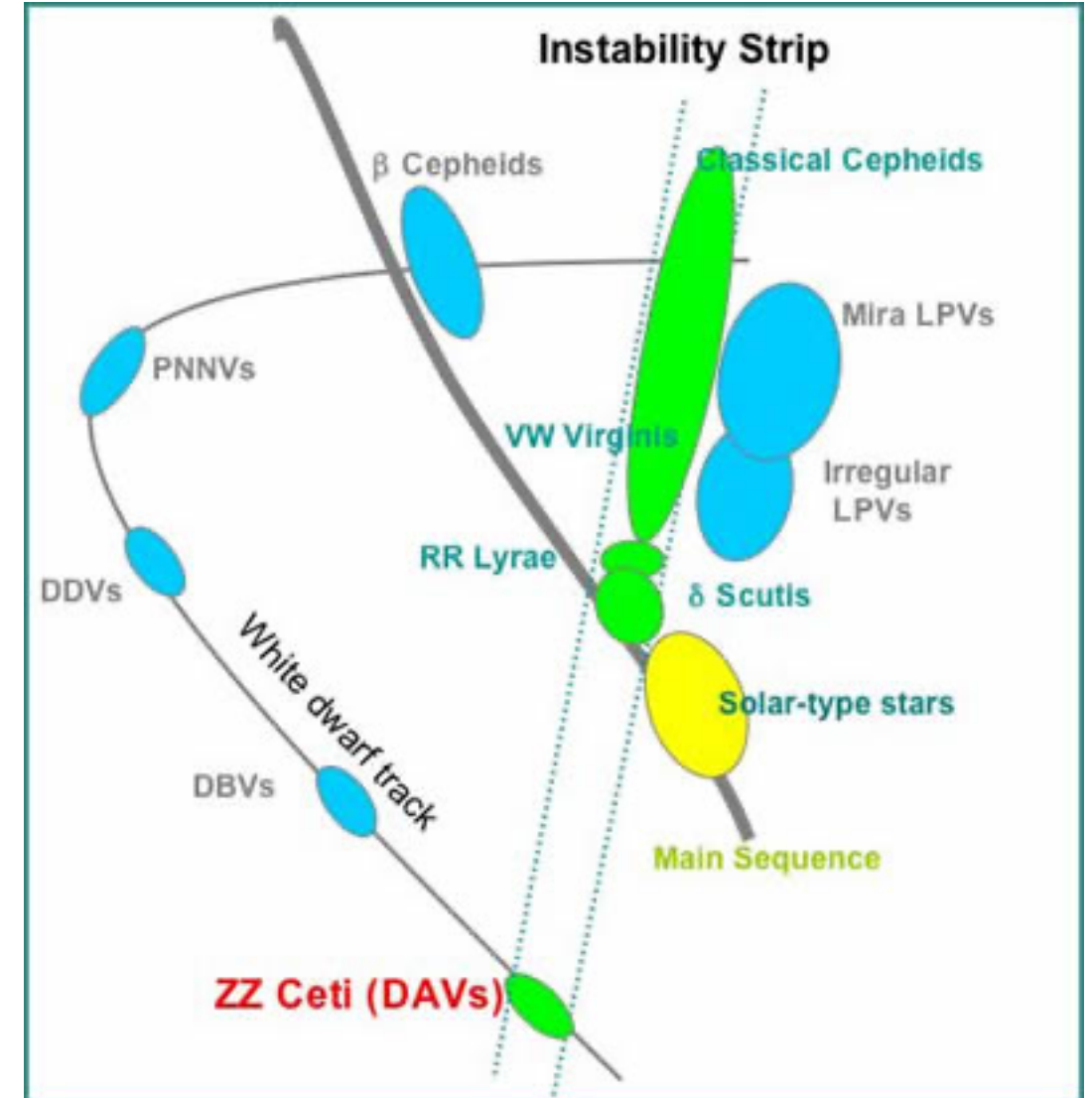
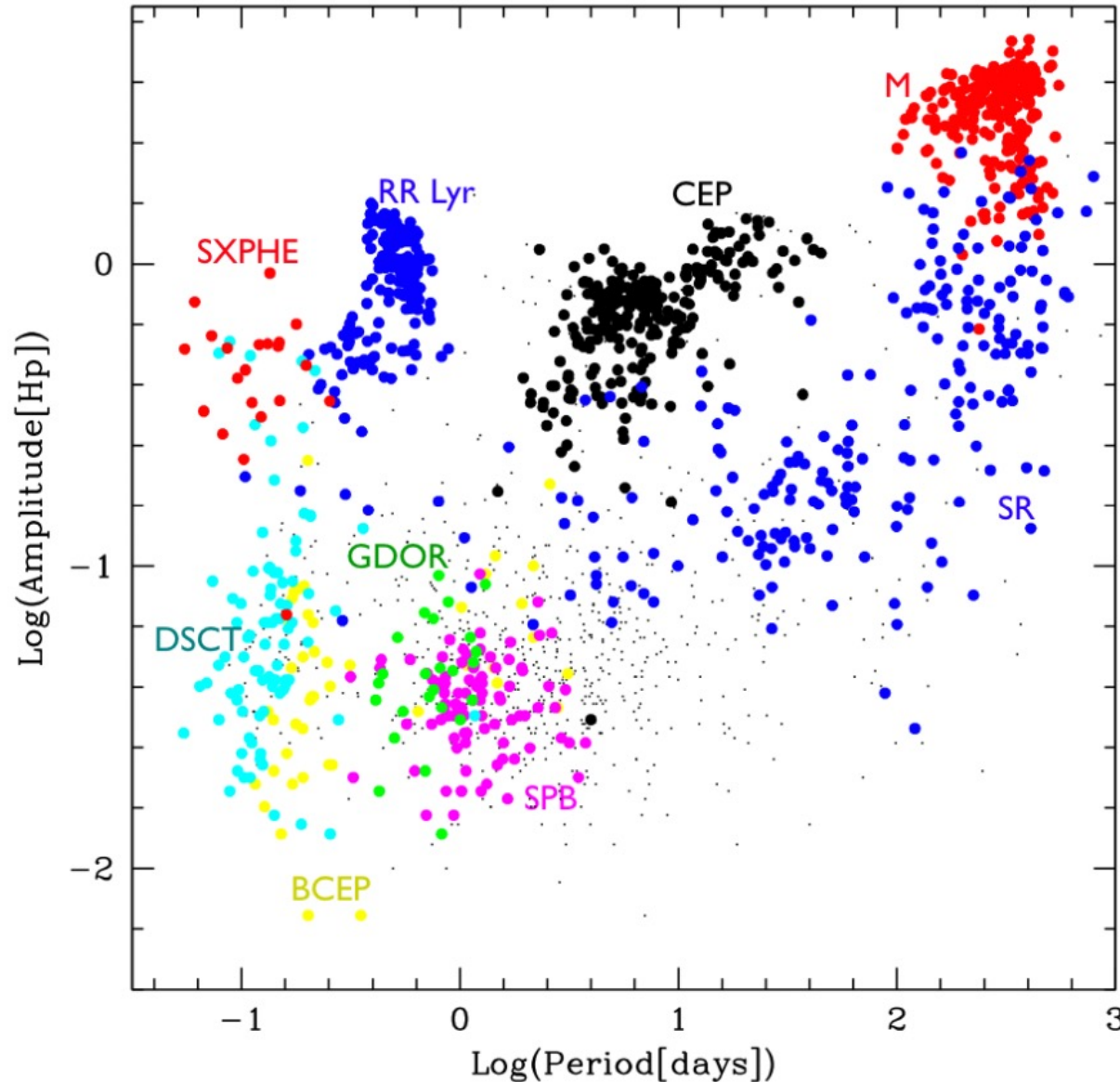


Stellar standard candles

What makes a good stellar standard candle?

What could be a good alternative to Cepheids?

Instability strips across the Hertzsprung Russell Diagram



The Leavitt Law

The Cepheid Period-Luminosity Relation

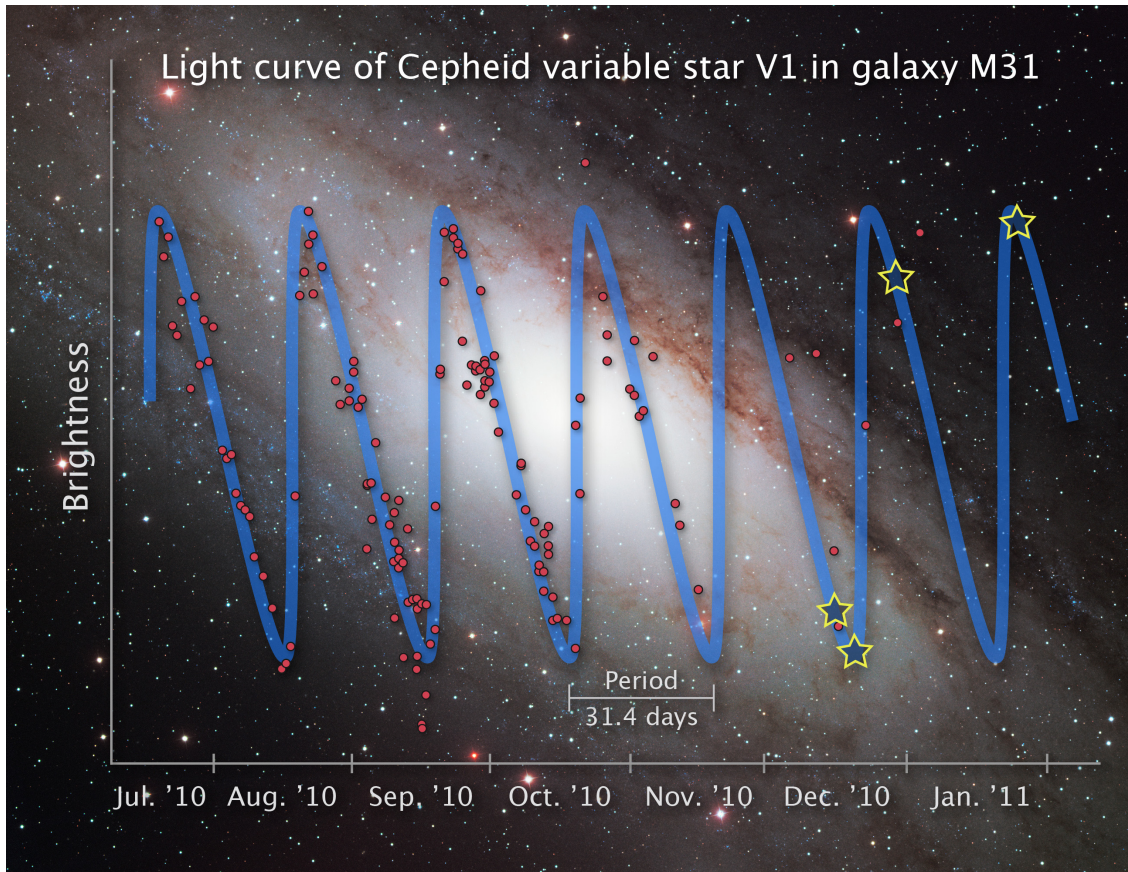
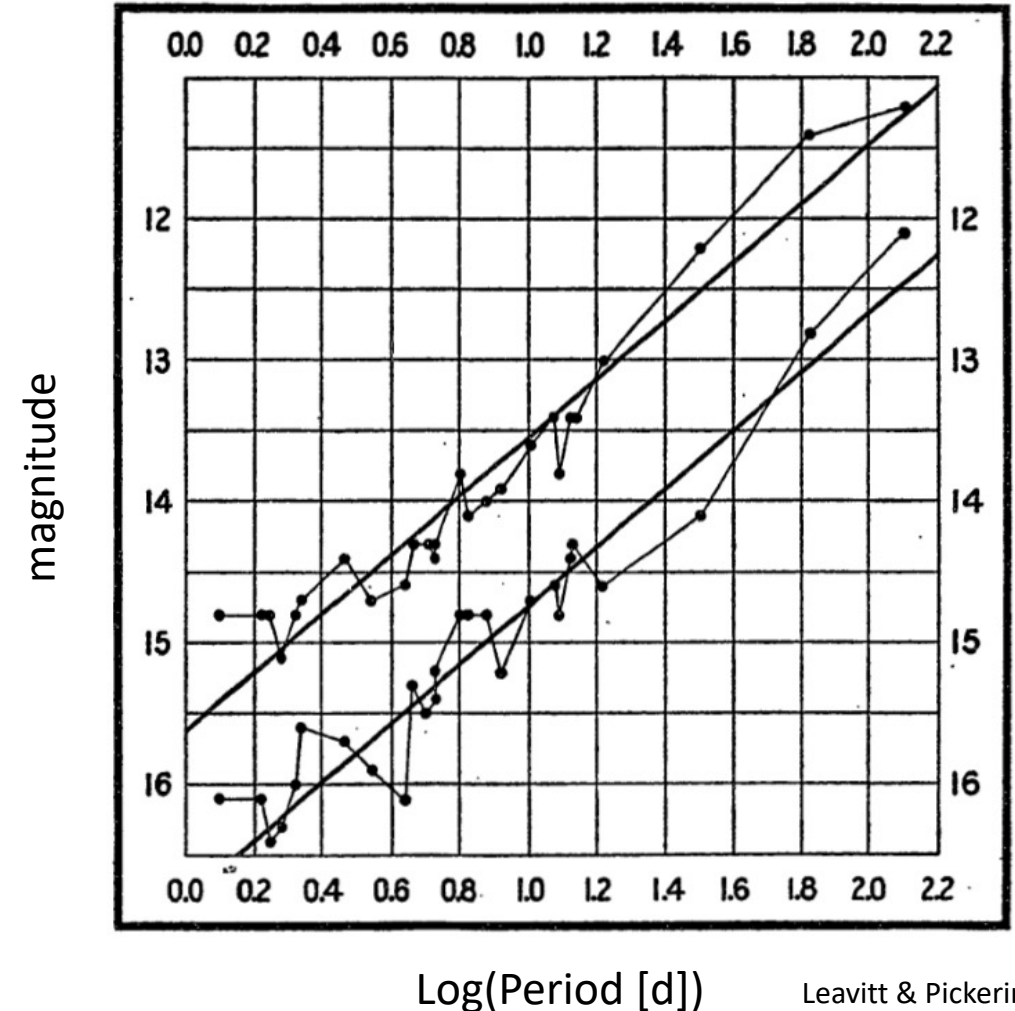


Illustration Credit: [NASA](#), [ESA](#) and Z. Levay ([STScI](#)). Science Credit: [NASA](#), [ESA](#), the [Hubble Heritage](#) Team ([STScI/AURA](#)) and the American Association of Variable Star Observers

$$M = m - 5 \log d + 5$$

$$M = \alpha \cdot \log(P) + \beta$$

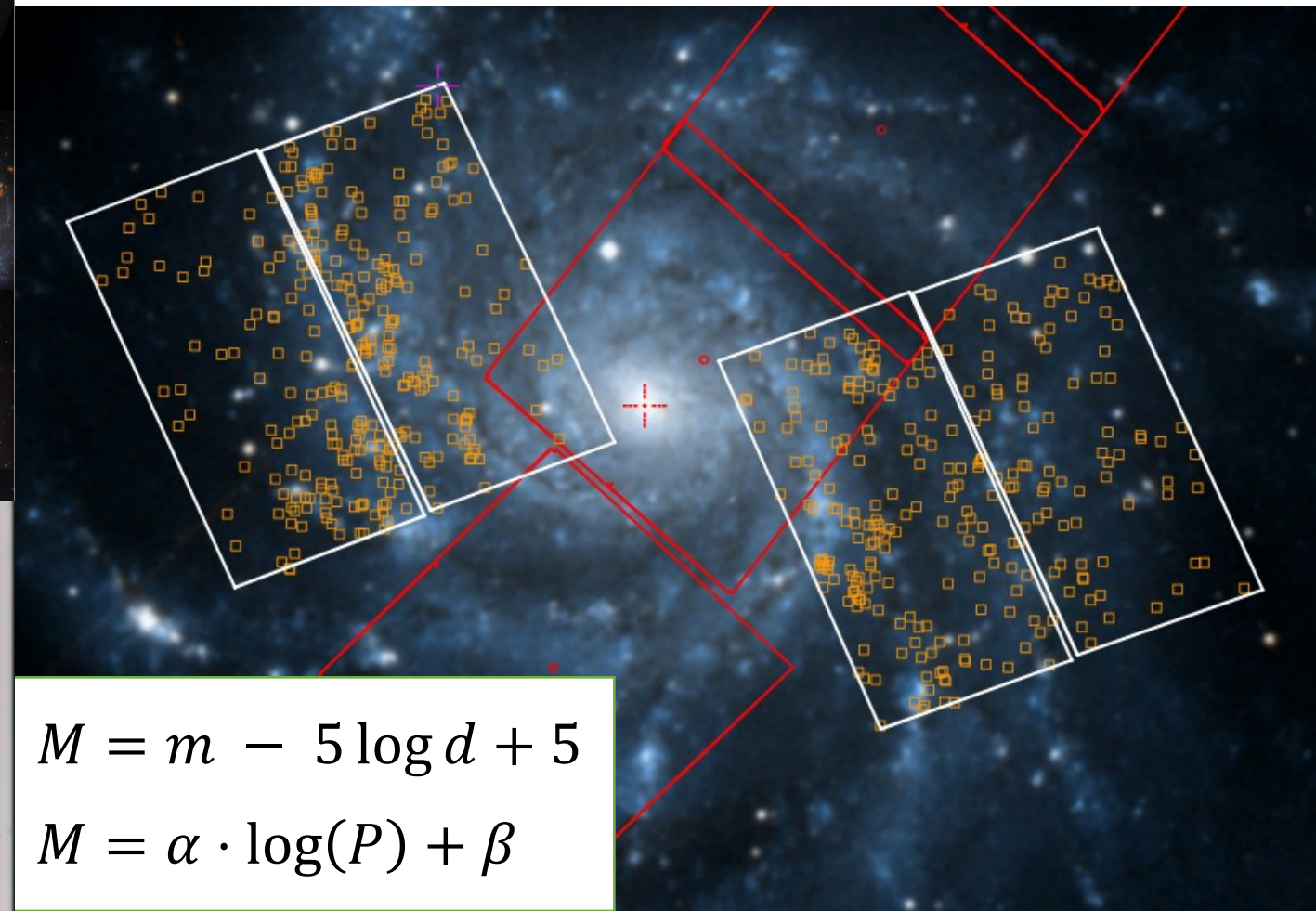
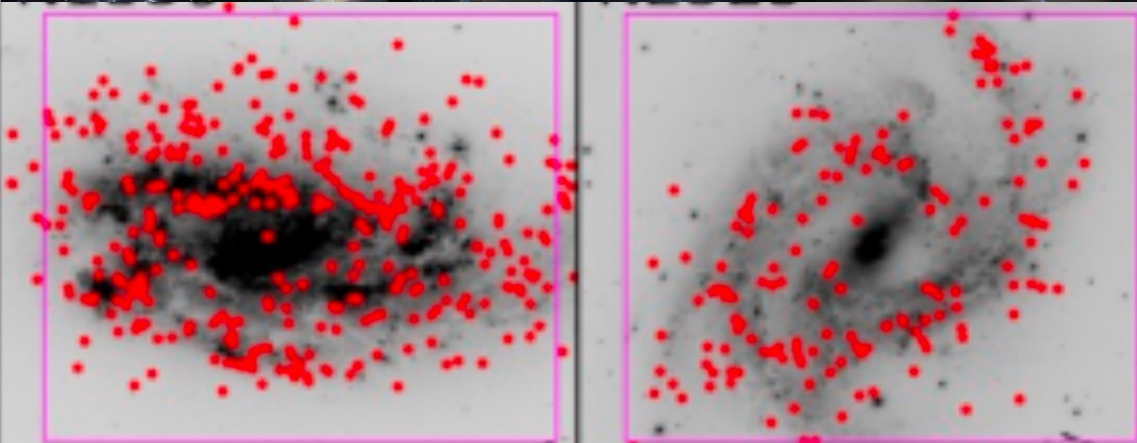


Many pulsating stars obey Leavitt laws!

- Classical Cepheids
- Type II Cepheids[low mass, rare, less clean]: BL Her, W Vir, RV Tau
- RR Lyrae stars
- Anomalous Cepheids
- Mira stars & other long-period or semi-regular variables
- OGLE Small Amplitude Red Giants – OSARGs
- delta Scuti stars
- Even some eclipsing binaries obey period-luminosity relations



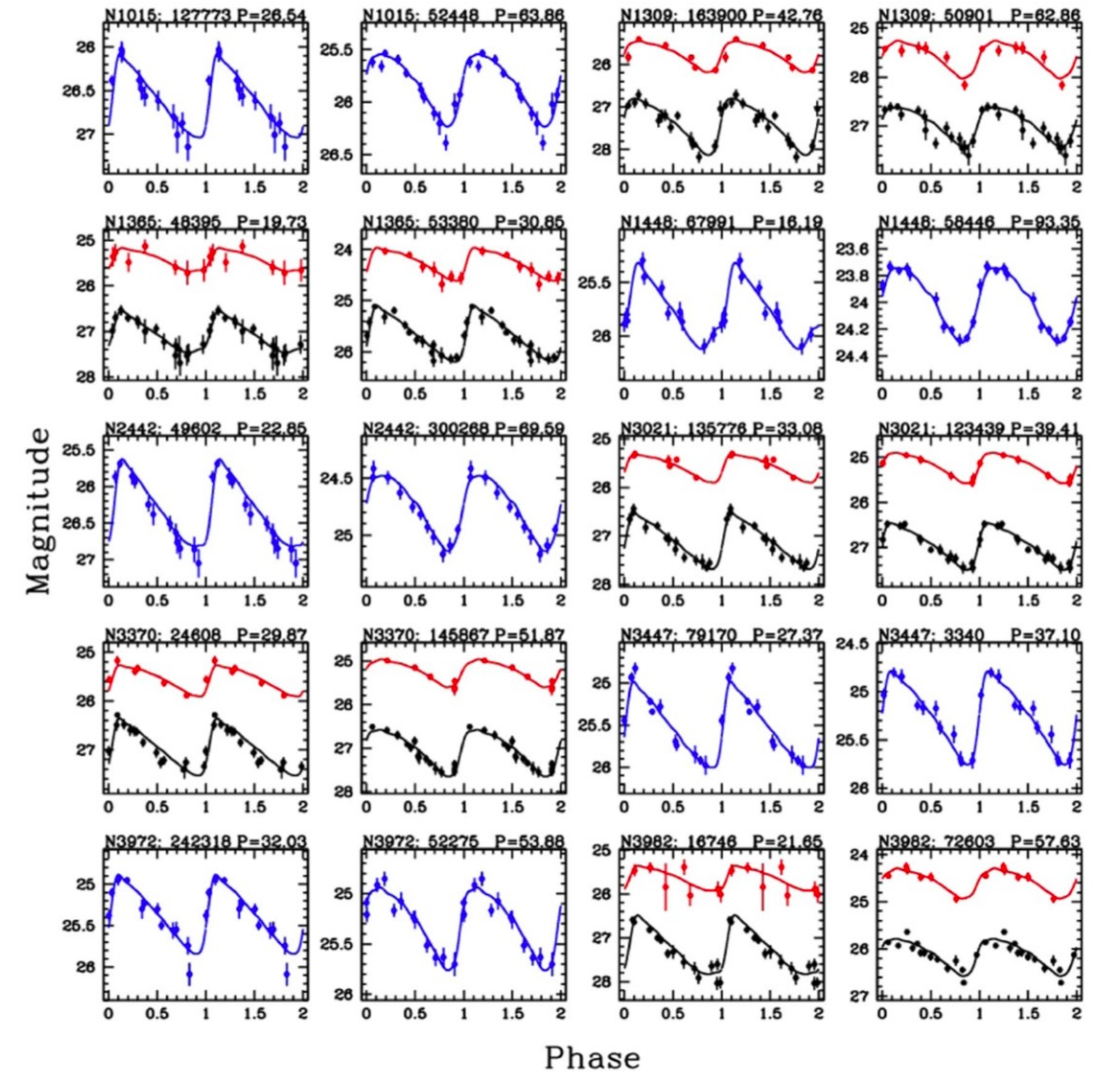
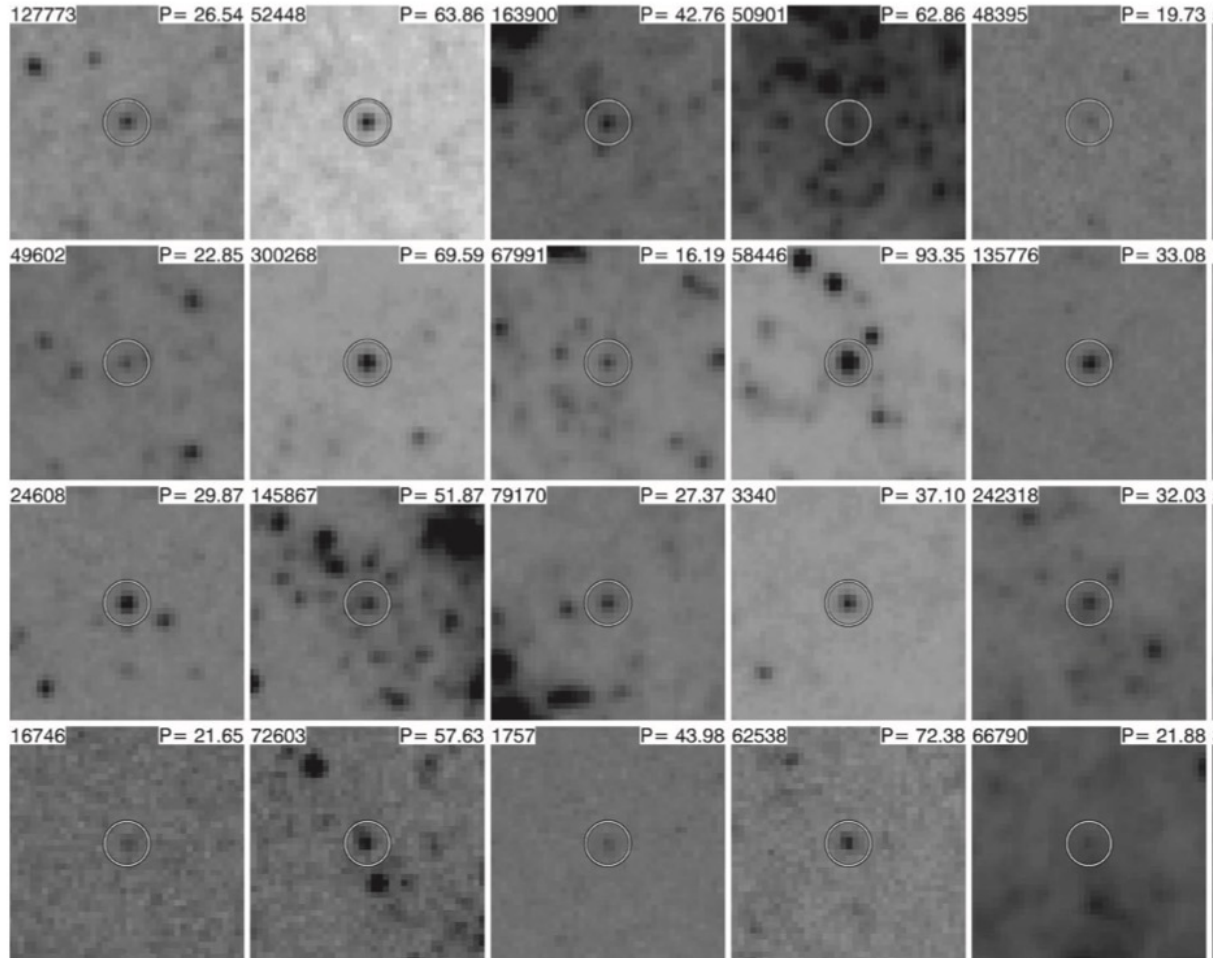
Cepheid distances to other galaxies (< 70 Mpc)



$$M = m - 5 \log d + 5$$

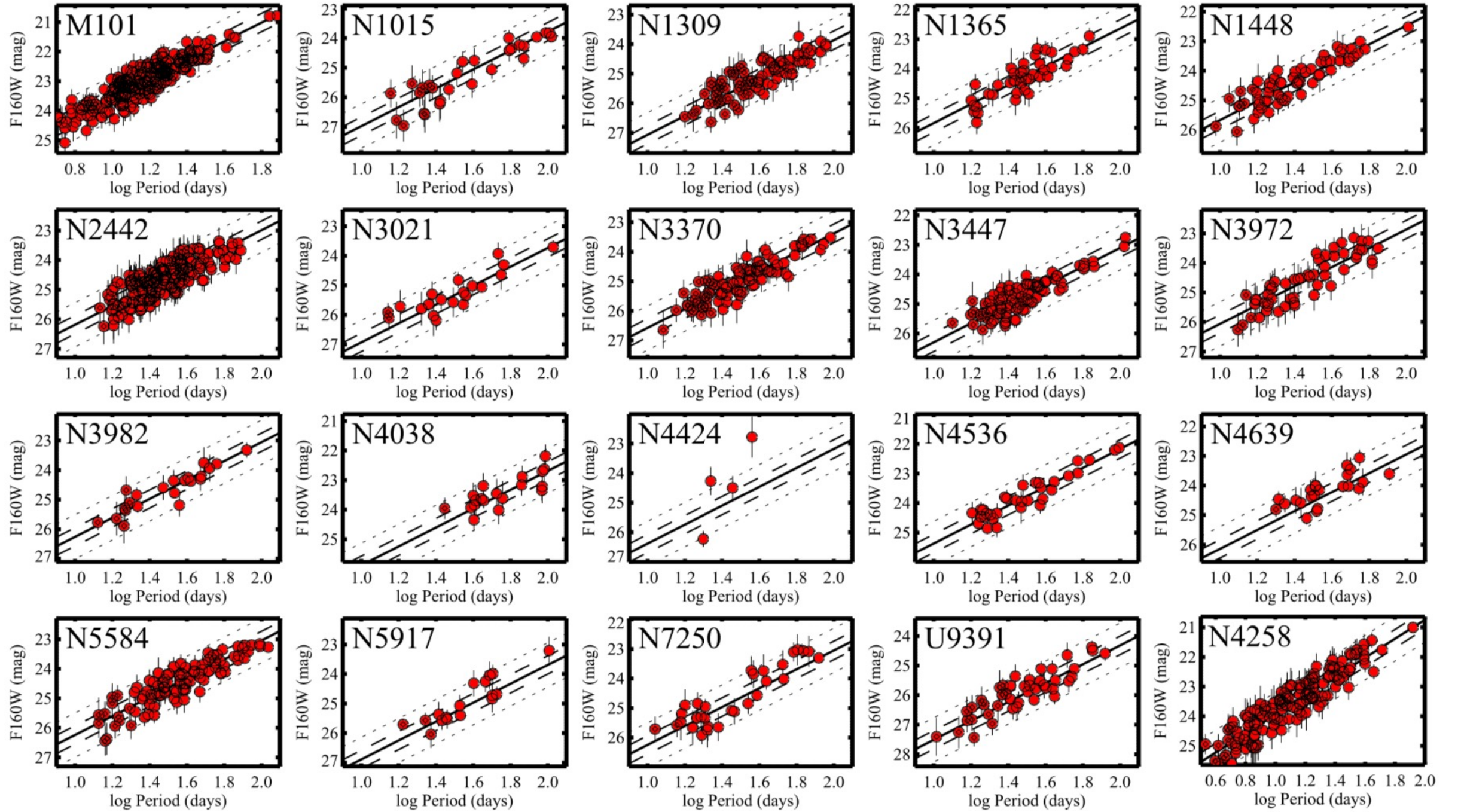
$$M = \alpha \cdot \log(P) + \beta$$

Cepheid photometry and periods



Reddening-free “Wesenheit” magnitudes

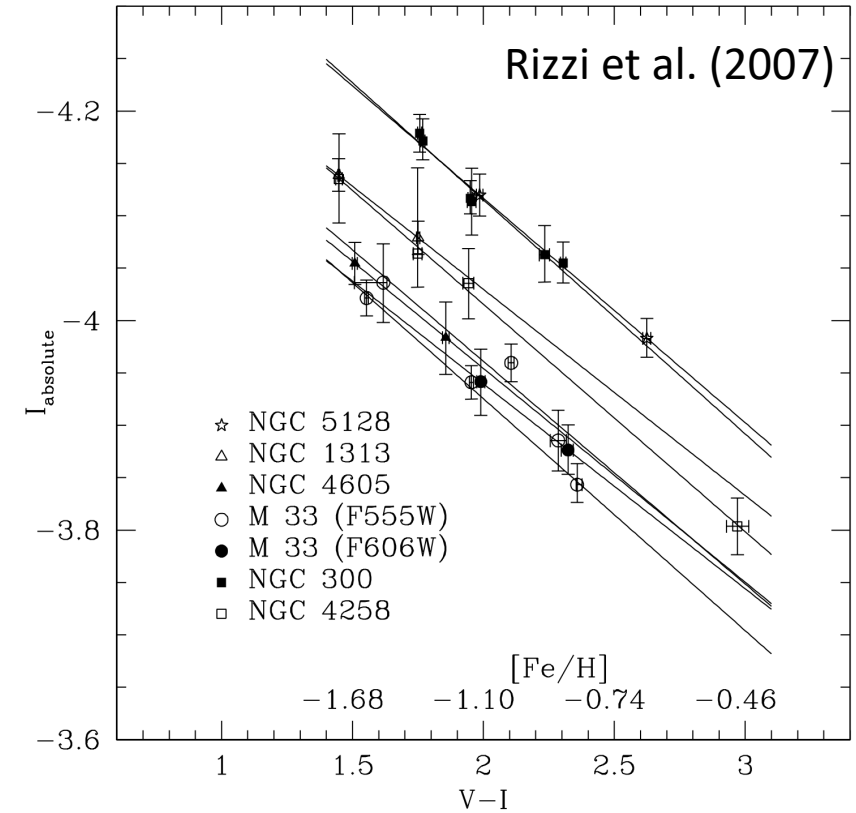
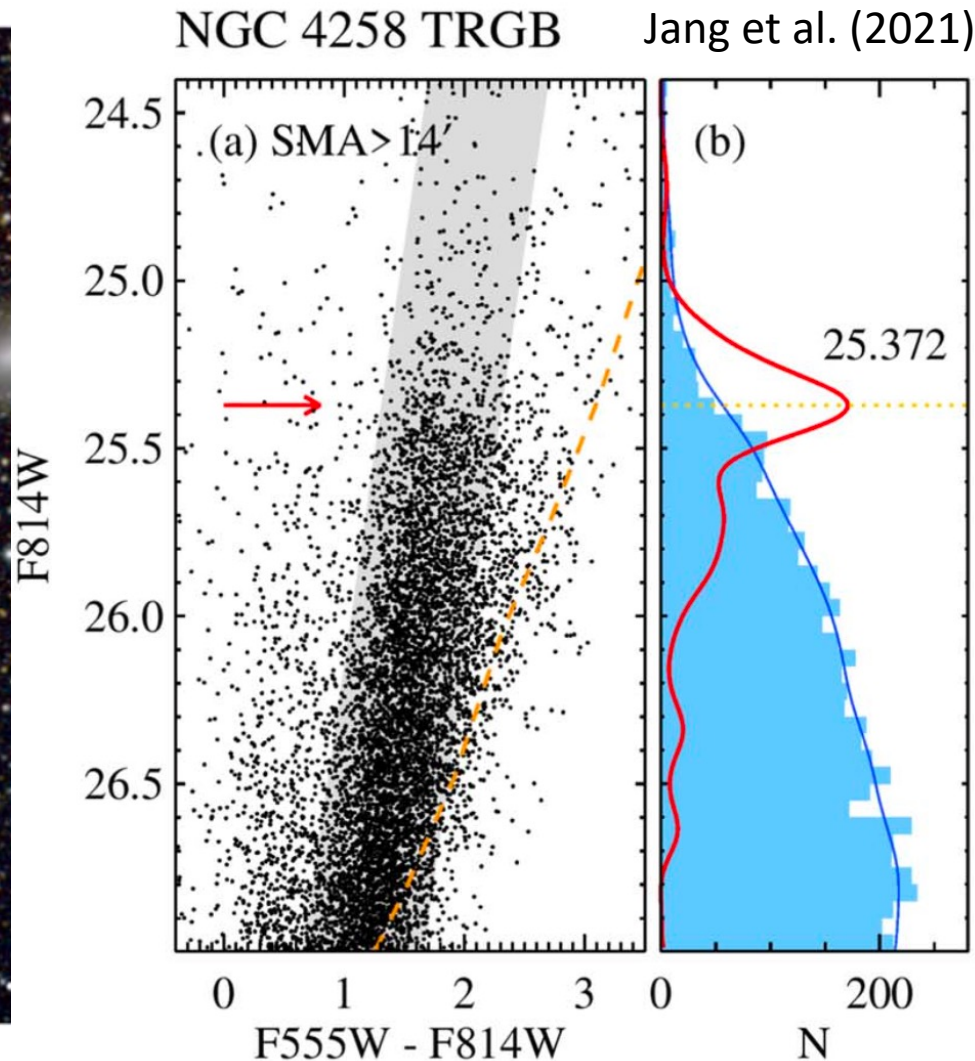
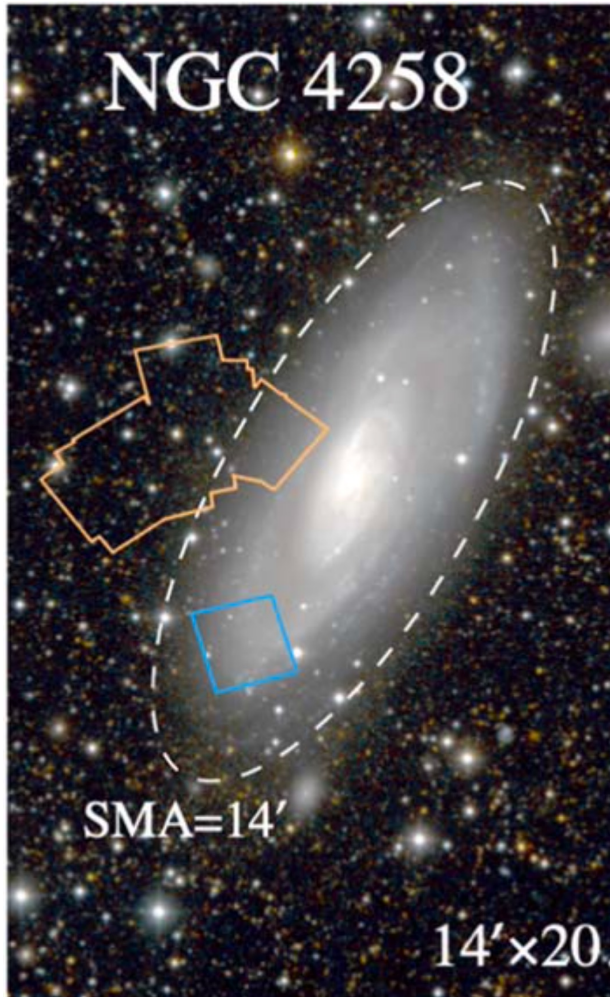
- Wesenheit function defines a reddening-free magnitude
- $m_{H,VI}^W = m_H - R_{H,VI}^W(m_V - m_I) \equiv m_H^0 - R_{H,VI}^W(m_V^0 - m_I^0)$
- Assuming that
- $R_{H,VI}^W = \frac{A_H}{A_V - A_I}$ is correctly described by a reddening law normalized to a value of $R_V = A_V/E(B - V)$
- $R_{H,VI}^W \approx 0.39$ for H = F160W, V = F555W, I = F814W
- In Wesenheit formalism: $\mu = m^0 - M^0 = m^W - M^W$
- This is key to current accuracy of distance ladder



The Tip of the Red Giant Branch

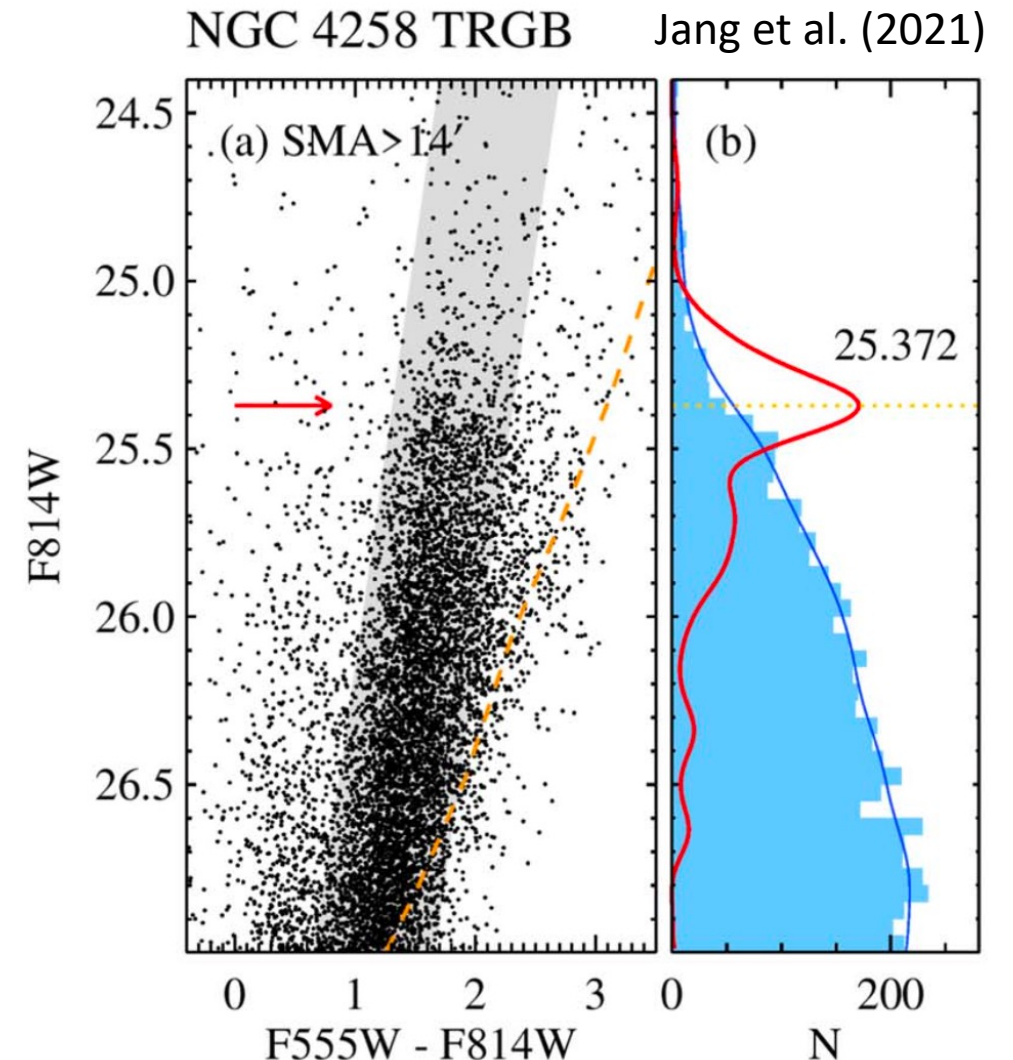
The most precise alternative to Cepheids

The Tip of the Red Giant Branch (TRGB)



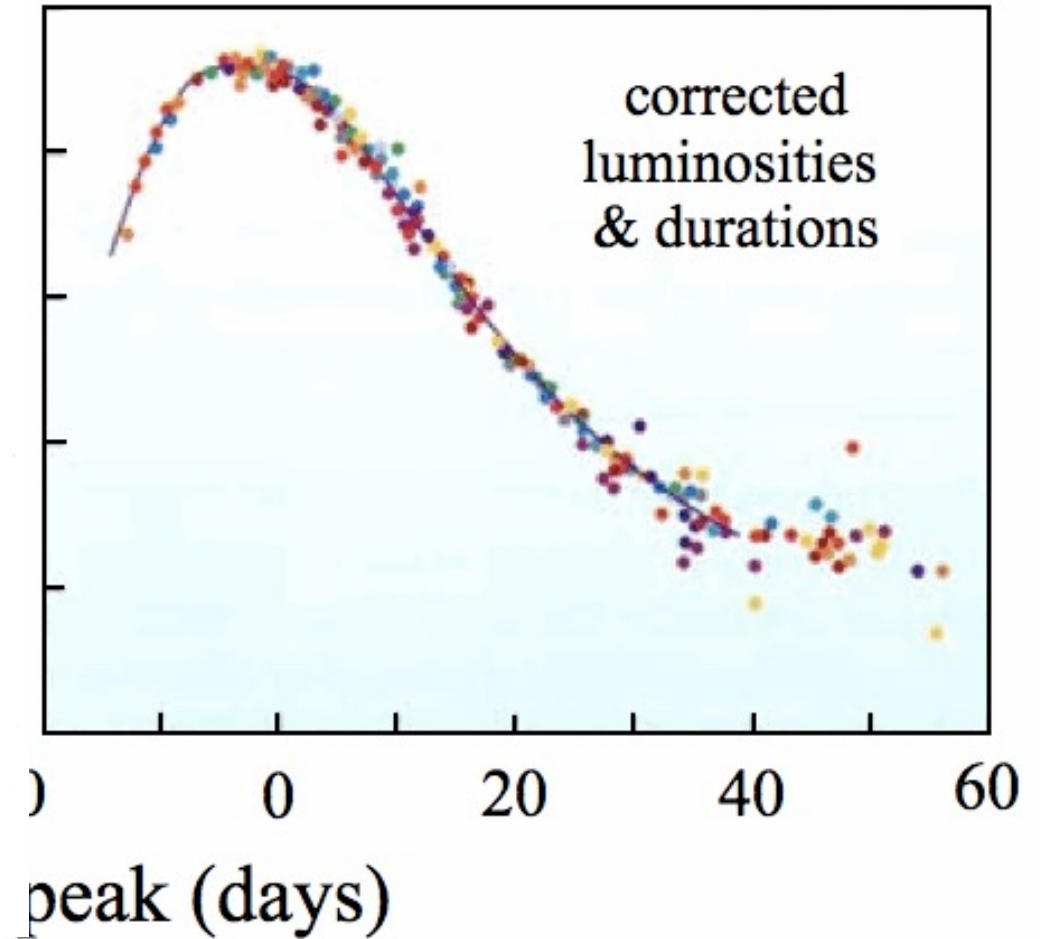
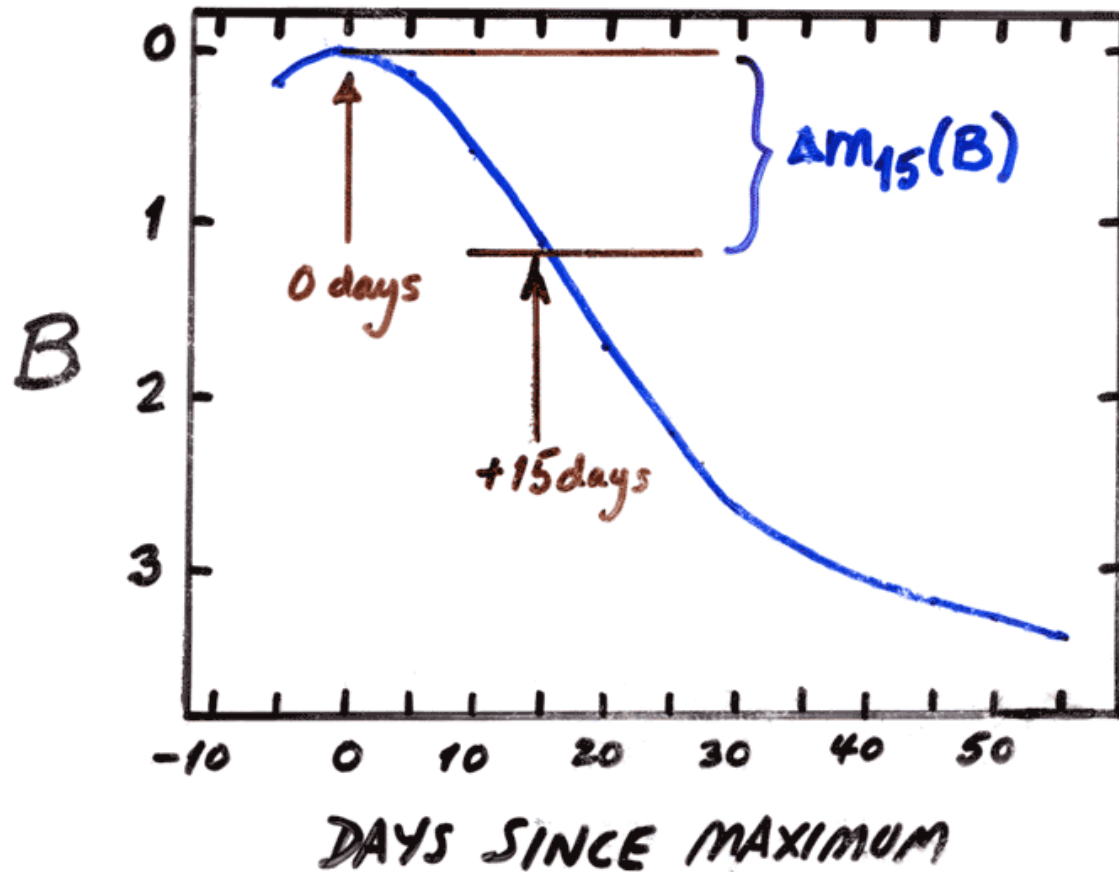
The Tip of the Red Giant Branch (TRGB)

- Most precise alternative to Cepheids for SN calibration
- RGB position in HRD depends on metallicity
- Tip depends on wavelength and metallicity
- Calibrations with or without metallicity differ by a few percent
- Main benefits:
 - No time series needed
 - IR brightness
 - Halos are dust-free & low in crowding



Supernova distances and
The Hubble diagram:
Distance-redshift relation

SN Ia Standardization



Light curve fits

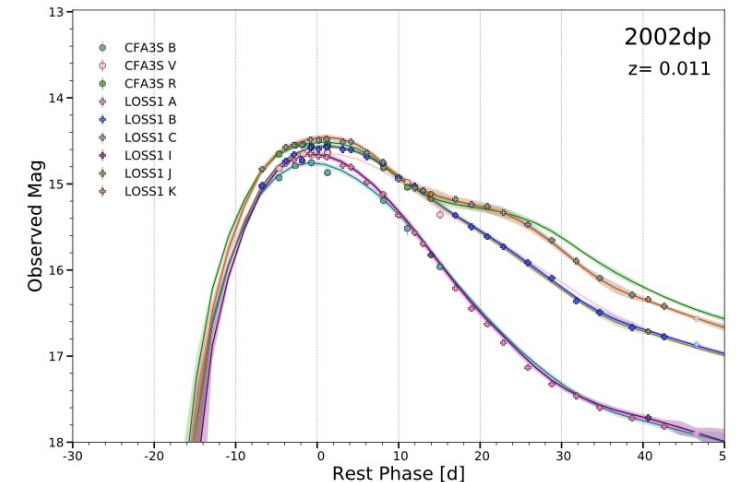
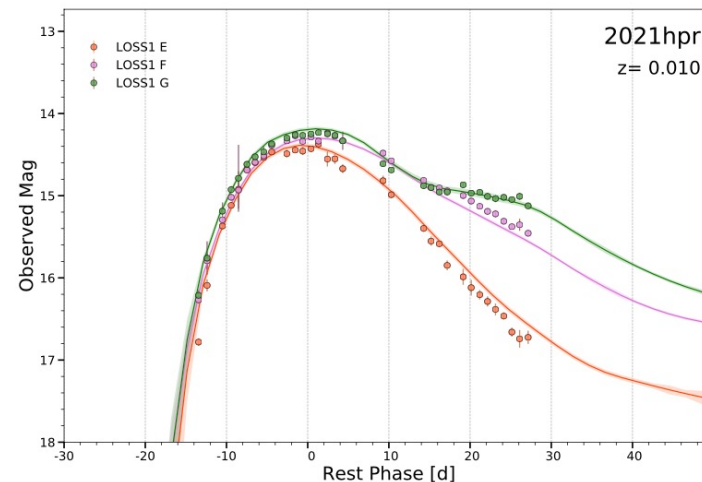
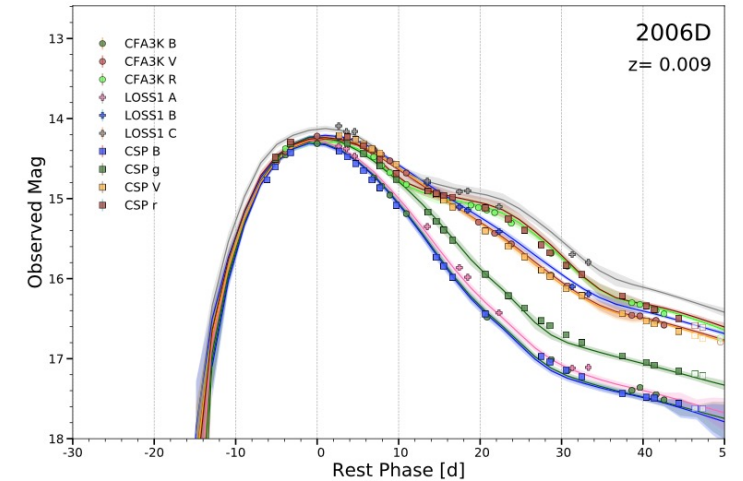
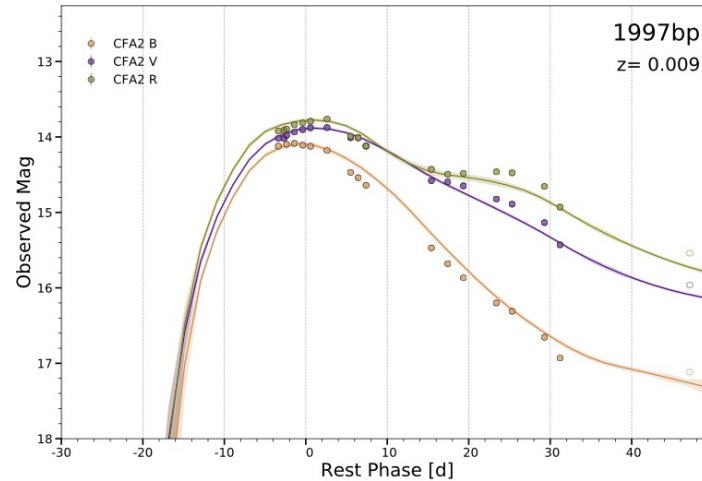
[Scolnic et al. \(2021\)](#)

SALT2 light curve fitter

c : color
 α : 0.148
 β : 3.112
 x_1 : stretch
 x_0 : amplitude
 m_B : $-2.5 \log_{10}(x_0)$
 $\delta_{\mu-host}$: host mass-step
 due to dust

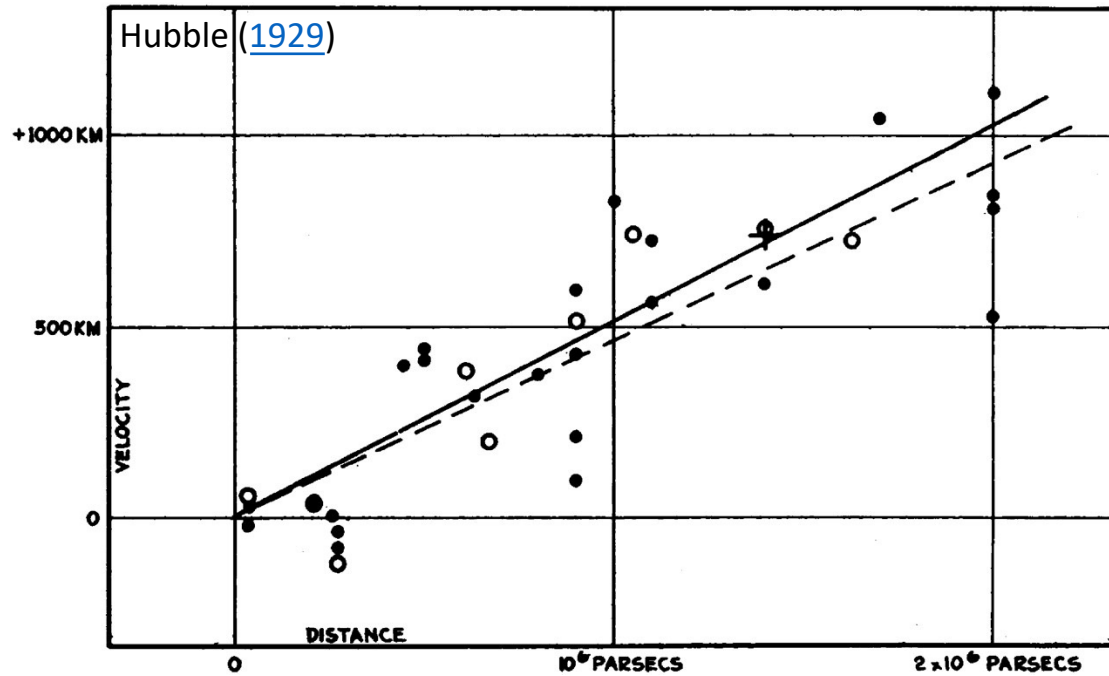
[Brout & Scolnic \(2021\)](#)

$$\mu = m_B + \alpha x_1 + \beta c - M - \delta_{\mu-bias}$$

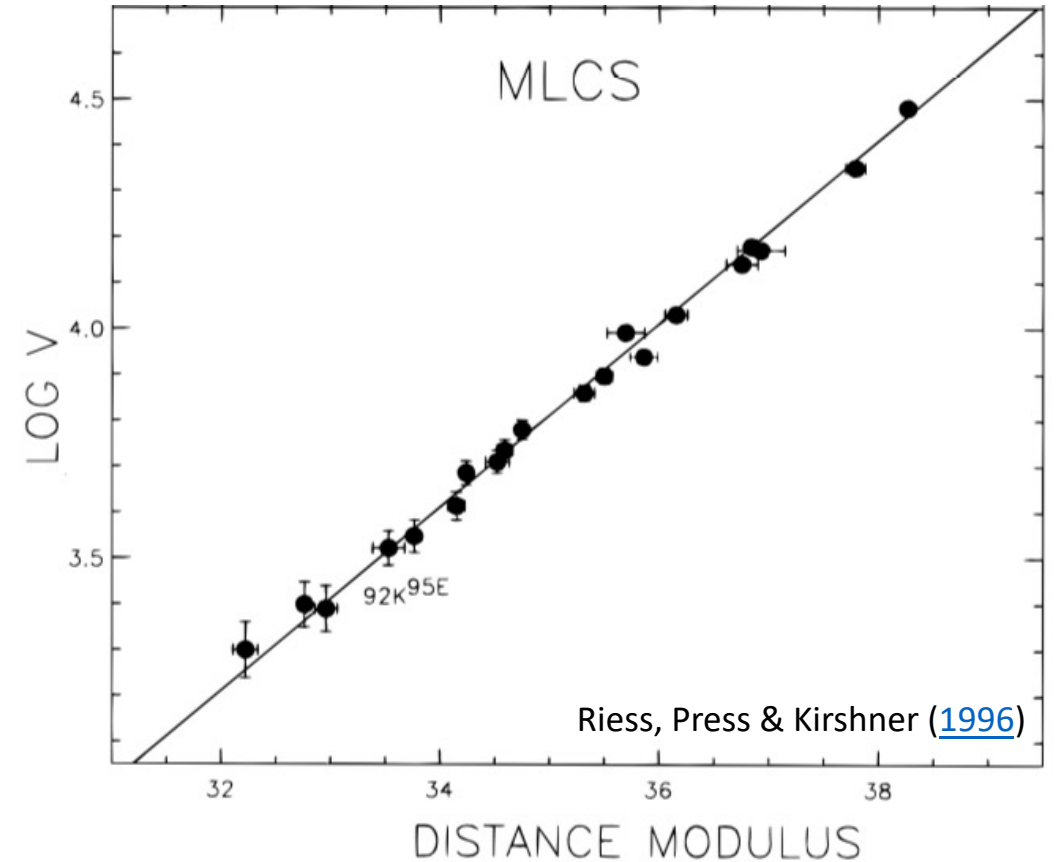


The Hubble-Lemaître Law

Lemaître ([1927](#)), Hubble ([1929](#)), IAU vote [2018](#)



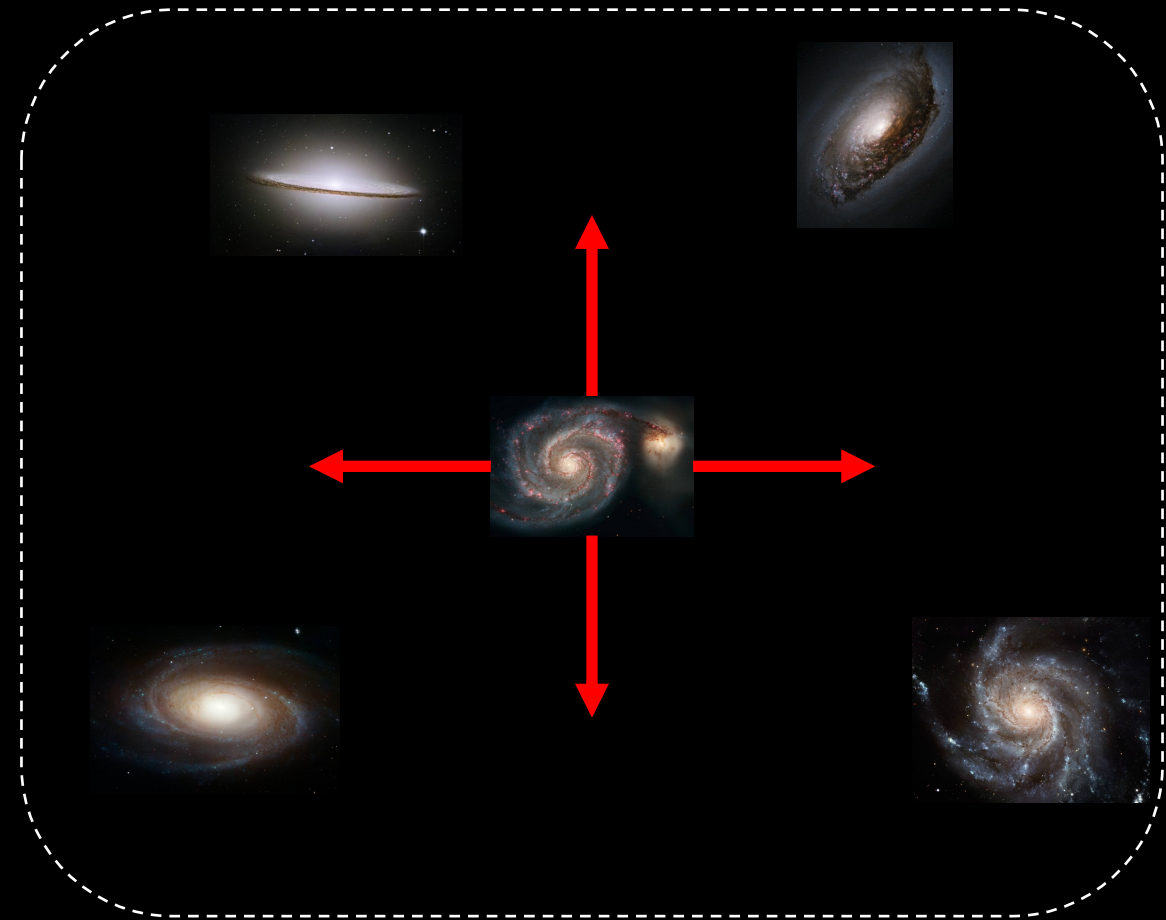
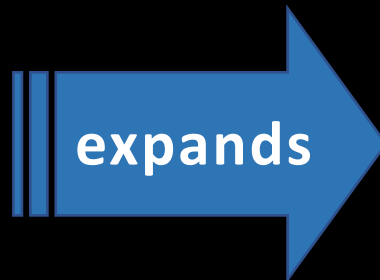
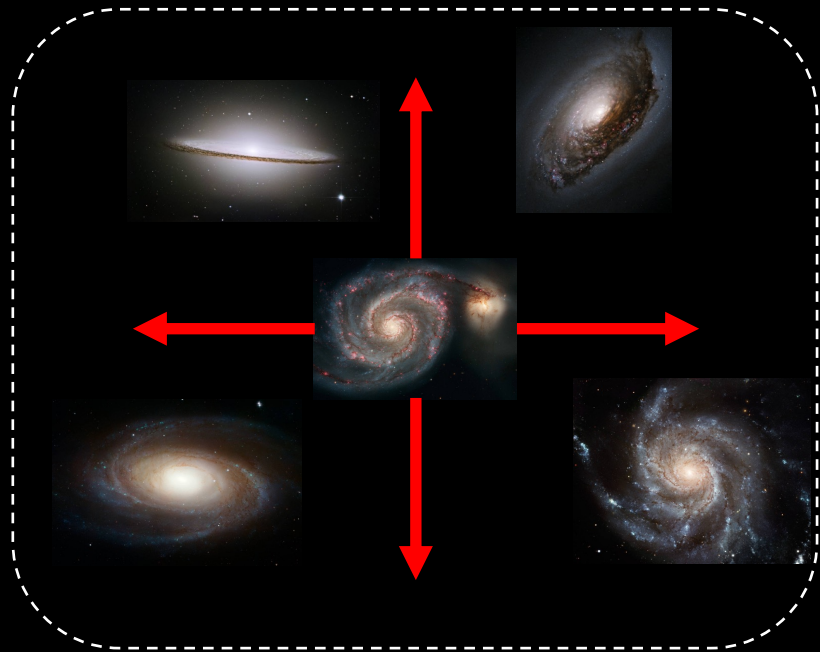
$$v = H_0 D$$



H_0 measures expansion rate of today's Universe
Hubble-Lemaître law:

$$H_0 = v/D$$

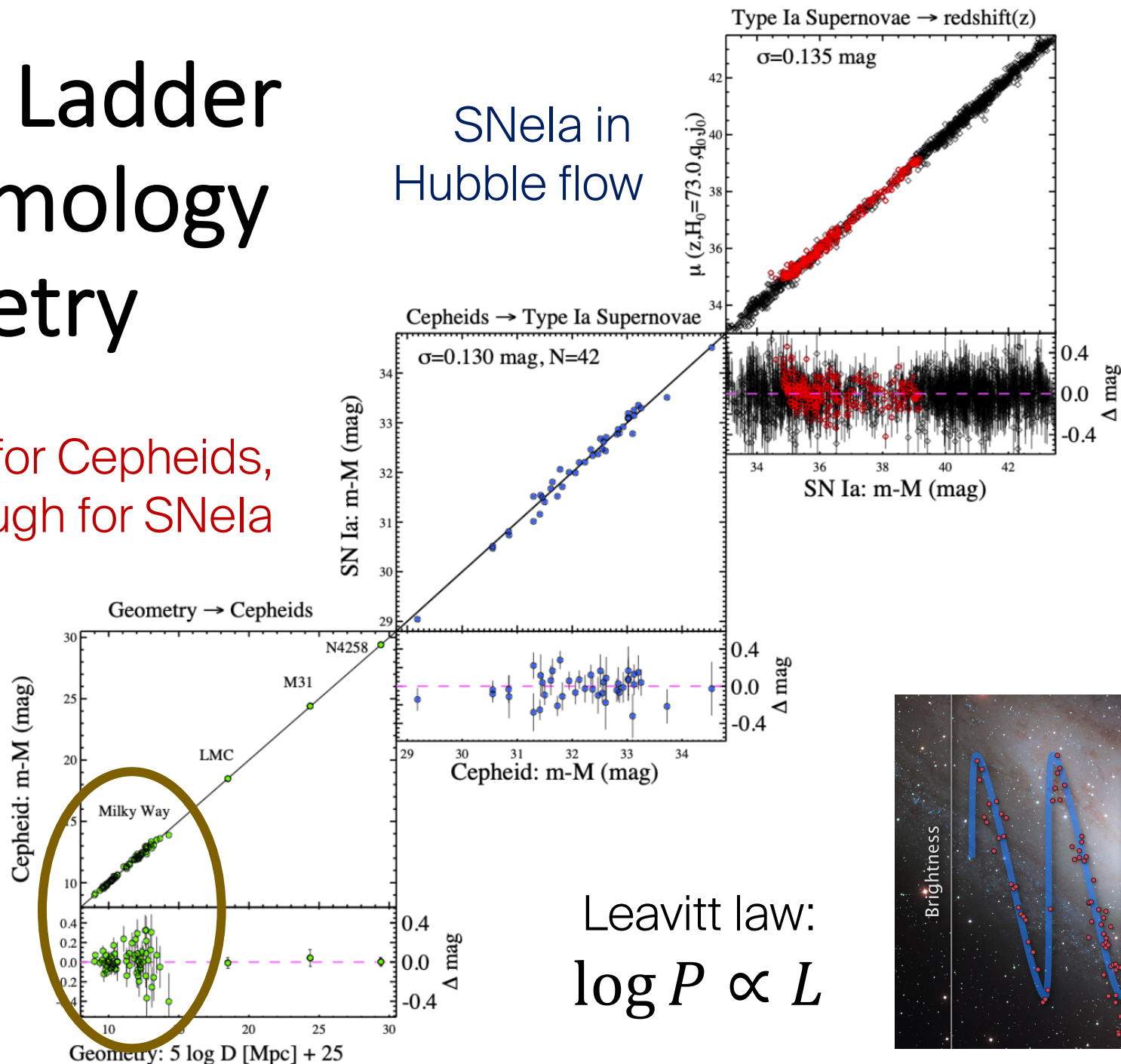
$$D_L = \frac{cz}{H_0} \left\{ 1 + \frac{1}{2}(1 - q_0)z - \frac{1}{6} \left[1 - q_0 - 3q_0^2 + j_0 \pm \frac{c^2}{H_0^2 R^2} \right] z^2 + O(z^3) \right\}$$



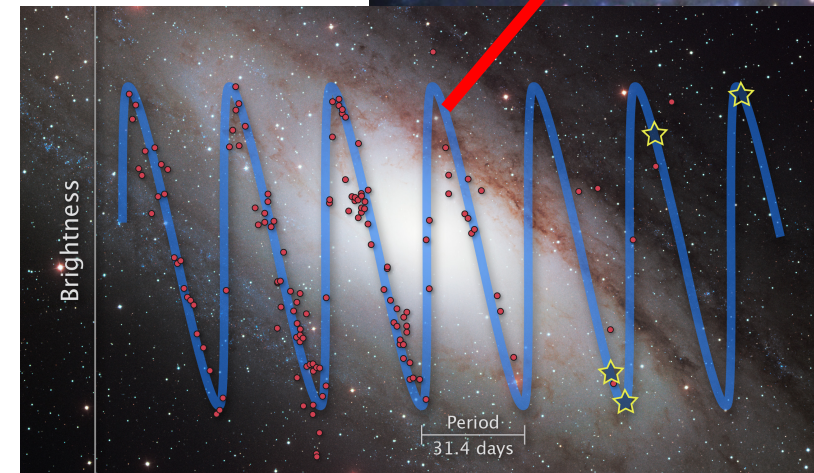
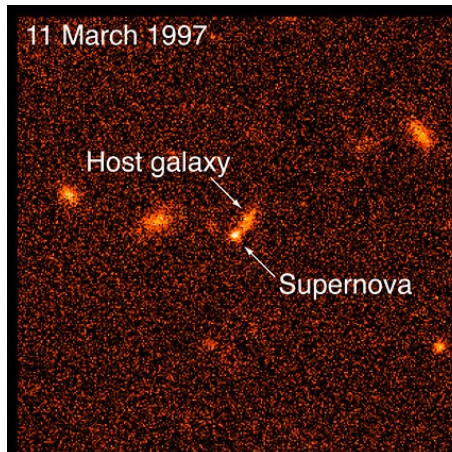
Distance Ladder links cosmology to geometry

Close enough for Cepheids,
far enough for SNeIa

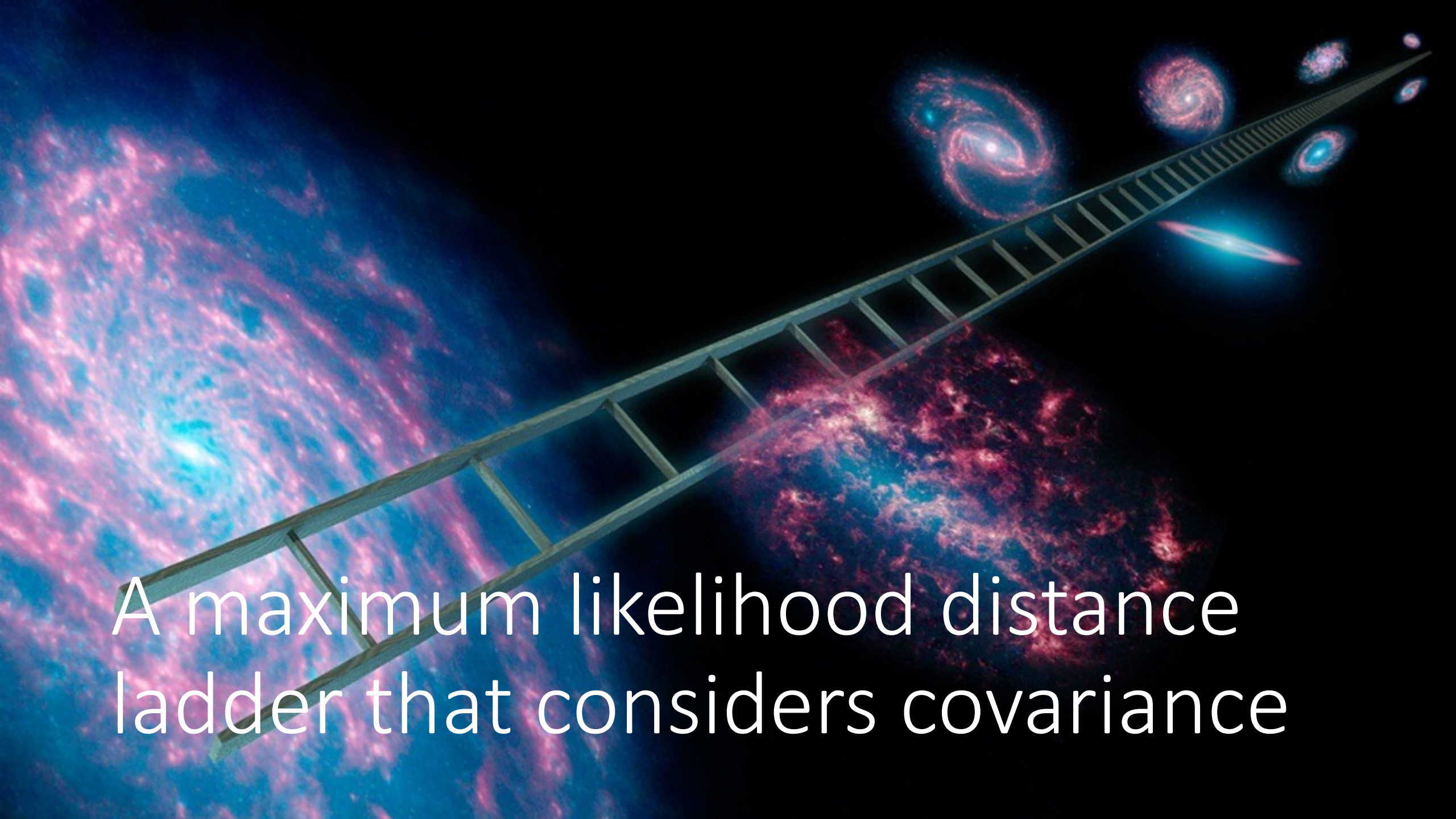
Gaia Parallaxes



SNela in
Hubble flow



Leavitt law:
 $\log P \propto L$



A maximum likelihood distance
ladder that considers covariance

The mathematical formalism

$$\log H_0 = 0.2M_B^0 + a_B + 5$$

Distance-redshift relation:
$$a_B = \log cz \left\{ 1 + \frac{1}{2}[1 - q_0]z - \frac{1}{6}[1 - q_0 - 3q_0^2 + j_0]z^2 + O(z^3) \right\} - 0.2m_B^0$$

Standardized SNIa B-band magnitude
$$M_B^0 = m_{B,i}^0 - \mu_{0,i}$$

$$\chi^2 = (y - Lq)^T C^{-1} (y - Lq)$$

$$q_{opt} = (L^T C^{-1} L)^{-1} L^T C^{-1} y^T$$

Generalized least squares fit parameters

$$Cq = (L^T C^{-1} L)^{-1}$$

Covariance matrix contains errors & covariance

The mathematical formalism $\log H_0 = 0.2M_B^0 + a_B + 5$

Cepheids

$$\overset{\text{Mean mag}}{\boxed{m_{H,i,j}^W}} = \overset{\text{Distance}}{\boxed{\mu_{0,i}}} + \overset{\text{Fiducial 10d}}{\boxed{M_{H,1}^W}} + \overset{\text{Leavitt law}}{\boxed{b_W (\log P_{i,j} - 1)}} + \overset{\text{Metallicity effect}}{\boxed{Z_W [\text{O}/\text{H}]_{i,j}}}$$

$$M_{H,j}^W = m_{H,N4258,j}^W - \mu_{0,N4258} + \Delta\mu_{N4258}$$

SN Ia

$$\overset{\text{Maximum light standardized mag}}{\boxed{m_B^0}} = \overset{\text{Distance}}{\boxed{\mu_{0,i}}} + \overset{\text{Fiducial}}{\boxed{M_B^0}}$$

$$a_B = \log cz \left\{ 1 + \frac{1}{2} [1 - q_0] z - \frac{1}{6} [1 - q_0 - 3q_0^2 + j_0] z^2 + O(z^3) \right\} - \boxed{0.2m_B^0}$$

$$\chi^2 = (y - Lq)^T C^{-1} (y - Lq)$$

Observables

$$y = \begin{pmatrix} m_{H,1}^W \\ \vdots \\ m_{H,nh}^W \\ \hline m_{H,N4258}^W - \mu_{0,N4258} \\ m_{H,M31}^W \\ m_{H,LMC}^W - \mu_{0,LMC} \\ \hline m_{B,1}^0 \\ \vdots \\ m_{B,ncc}^0 \\ \hline M_{H,1,HST}^W \\ M_{H,1,Gaia}^W \\ 0 \\ 0 \\ 0 \\ \hline m_{B,1}^0 - 5 \log cz_1\{\} - 25 \\ \vdots \\ m_{B,nhf} - 5 \log cz_{nhf}\{\} - 25 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & .. & 0 & 0 & 1 & 0 & 0 & \log P_{N,1} & 0 & [O/H]_{N,1} & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & 0 \\ 0 & .. & 1 & 0 & 1 & 0 & 0 & \log P_{N,nh} & 0 & [O/H]_{N,nh} & 0 & 0 \\ \hline 0 & .. & 0 & 1 & 1 & 0 & 0 & \log P_{N4258} & 0 & [O/H]_{N4258} & 0 & 0 \\ 0 & .. & 0 & 0 & 1 & 0 & 1 & \log P_{M31} & 0 & [O/H]_{M31} & 0 & 0 \\ 0 & .. & 0 & 0 & 1 & 1 & 0 & \log P_{LMC} & 0 & [O/H]_{LMC} & 1 & 0 \\ \hline 1 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & 0 \\ 0 & .. & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & .. & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .. & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & .. & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .. & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$$

} Cepheids in SN Ia hosts
 } Cepheids in anchors or non-SN Ia hosts
 } SNe Ia in Cepheid hosts
 } External constraints
 } SNe Ia in Hubble flow

Fit parameters

$$q = \begin{pmatrix} \mu_{0,1} \\ \vdots \\ \mu_{0,nh} \\ \Delta\mu_{N4258} \\ M_{H,1}^W \\ \Delta\mu_{LMC} \\ \mu_{M31} \\ b \\ M_B^0 \\ Z_W \\ \Delta z_p \\ 5 \log H_0 \end{pmatrix}$$

$$m_{H,i,j}^W = \mu_{0,i} + M_{H,1}^W + b_W (\log P_{i,j} - 1) + Z_W [O/H]_{i,j}$$

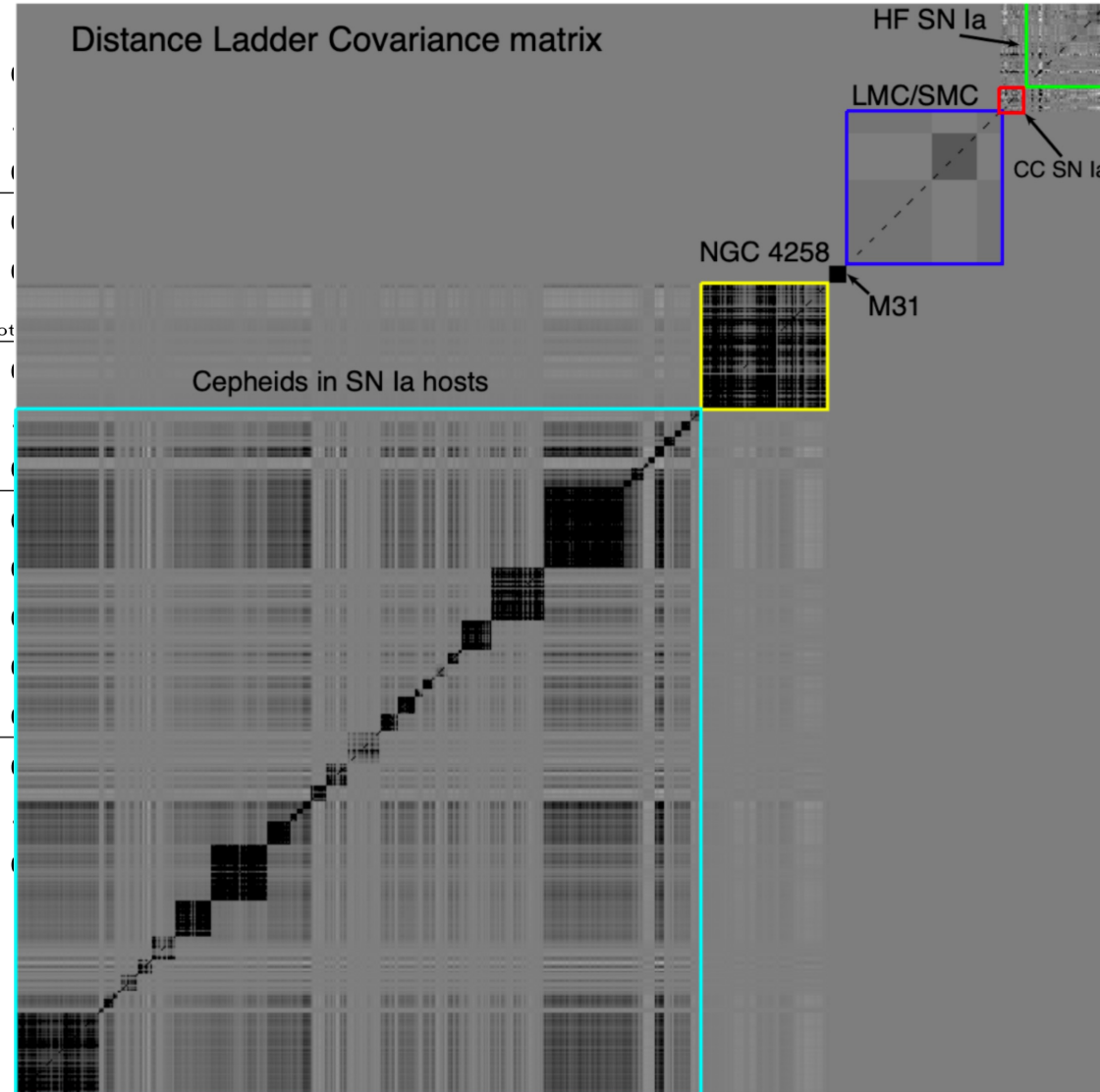
$$\chi^2 = (y - Lq)^T C^{-1} (y - Lq)$$

$$C = \begin{pmatrix} \sigma_{\text{tot},1}^2 & .. & Z_{\text{cov}} & Z_{\text{cov}} & 0 & 0 & 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ Z_{\text{cov}} & .. & \sigma_{\text{tot,nh}}^2 & Z_{\text{cov}} & 0 & 0 & 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ \hline Z_{\text{cov}} & .. & Z_{\text{cov}} & \sigma_{\text{tot,N4258}}^2 & 0 & 0 & 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & \sigma_{\text{tot,M31}}^2 & 0 & 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & 0 & \sigma_{\text{tot,LMC}}^2 & 0 & .. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ \hline 0 & .. & 0 & 0 & 0 & 0 & \sigma_{m_B,1}^2 & .. & \text{SN}_{\text{cov}} & 0 & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \text{SN}_{\text{cov}} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \sigma_{m_B,\text{ncc}}^2 & 0 & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \text{SN}_{\text{cov}} \\ \hline 0 & .. & 0 & 0 & 0 & 0 & 0 & .. & 0 & \sigma_{HST}^2 & 0 & 0 & 0 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & .. & 0 & 0 & \sigma_{Gaia}^2 & 0 & 0 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & .. & 0 & 0 & 0 & \sigma_{\text{grnd,zp}}^2 & 0 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & .. & 0 & 0 & 0 & 0 & \sigma_{\mu,\text{N4258}}^2 & 0 & 0 & .. & 0 \\ 0 & .. & 0 & 0 & 0 & 0 & 0 & .. & 0 & 0 & 0 & 0 & 0 & \sigma_{\mu,\text{LMC}}^2 & 0 & .. & 0 \\ \hline 0 & .. & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \text{SN}_{\text{cov}} & 0 & 0 & 0 & 0 & 0 & \sigma_{m_B,z,1}^2 & .. & \text{SN}_{\text{cov}} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & .. & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \text{SN}_{\text{cov}} & 0 & 0 & 0 & 0 & 0 & \text{SN}_{\text{cov}} & .. & \sigma_{m_B,z,\text{nhf}}^2 \end{pmatrix}$$

$$\sigma_{\text{tot},j}^2 = \begin{pmatrix} \sigma_{\text{tot},1,1}^2 & .. & C_{1,1,n,bkgd} \\ .. & .. & .. \\ C_{1,n,1,bkgd} & .. & \sigma_{\text{tot},1,n}^2 \end{pmatrix}$$

$$\chi^2 = (y - Lq)^T C^{-1} (y - Lq)$$

$$C = \begin{pmatrix} \sigma_{\text{tot},1}^2 & \dots & Z_{\text{cov}} & Z_{\text{cov}} & 0 \\ \dots & \dots & \dots & \dots & \dots \\ Z_{\text{cov}} & \dots & \sigma_{\text{tot,nh}}^2 & Z_{\text{cov}} & 0 \\ Z_{\text{cov}} & \dots & Z_{\text{cov}} & \sigma_{\text{tot,N4258}}^2 & 0 \\ 0 & \dots & 0 & 0 & \sigma_{\text{tot,M31}}^2 \\ 0 & \dots & 0 & 0 & 0 & \sigma_{\text{tot}}^2 \\ \hline 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & 0 \\ \hline 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 \\ \hline 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} \sigma_{\text{tot},1,1}^2 & \dots & C_{1,1,n,bkgd} \\ \dots & \dots & \dots \\ C_{1,n,1,bkgd} & \dots & \sigma_{\text{tot},1,n}^2 \end{pmatrix}$$

5-parameter Distance Ladder with many variants & systematics checks

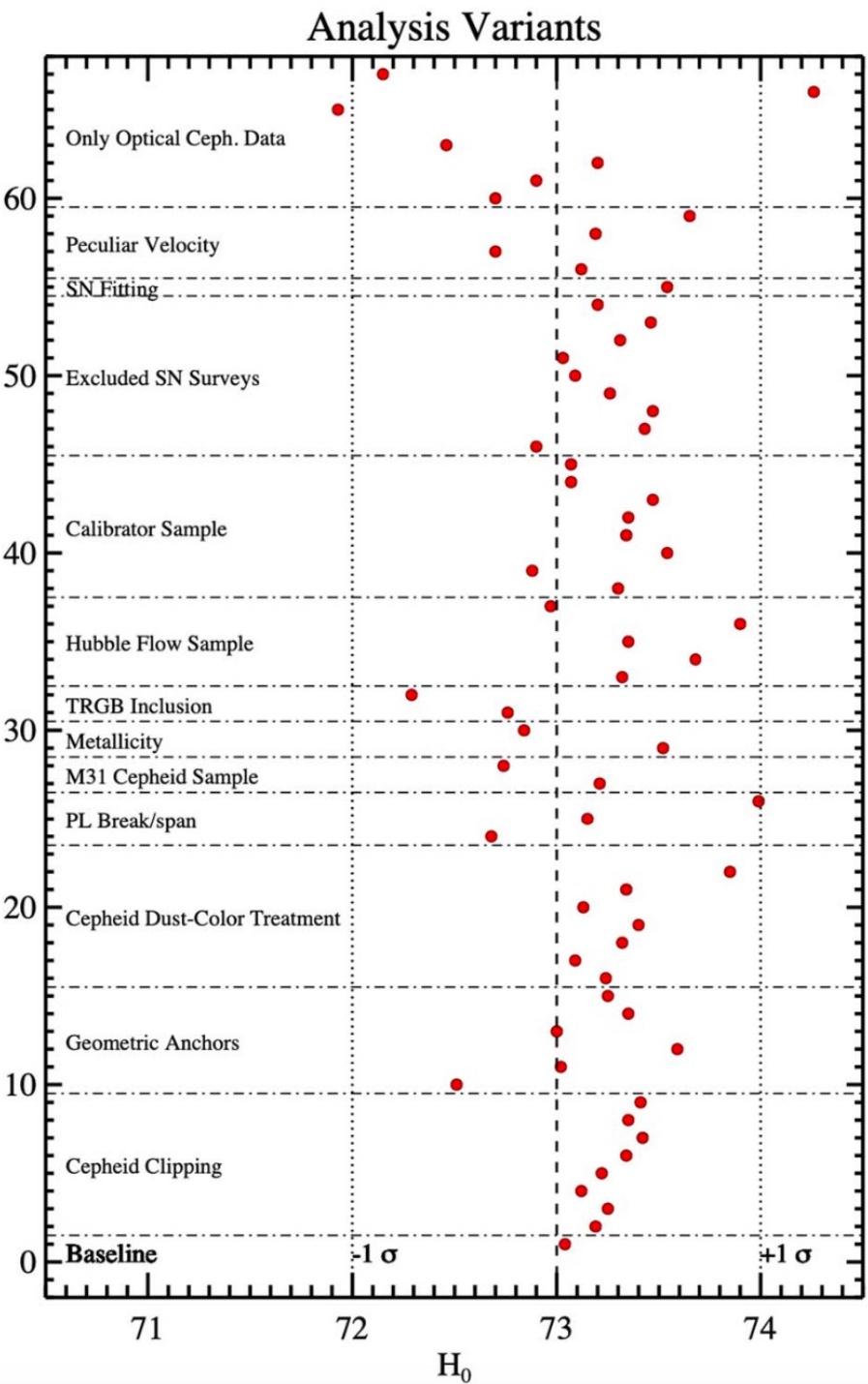
THE ASTROPHYSICAL JOURNAL LETTERS, 934:L7 (52pp), 2022 July 20 Riess et al.

Table 5
Fits for H_0

Fit	Variant	χ^2_{dof}	N	H_0	b	γ	M_W^0	M_B^0	a_b
1	Baseline	1.03	3445	73.04 1.01	-3.299 0.015	-0.217 0.046	-5.894	-19.253	0.714158
Cepheid-clipping Variants: Section 6.1									
2	global $\sigma_{\text{clip}} = 3.3$	1.03	3446	73.19 1.01	-3.298 0.015	-0.216 0.046	-5.891	-19.249	0.714174
3	individual $P-L$ $\sigma_{\text{clip}} = 3.3$	0.99	3370	73.25 1.02	-3.296 0.015	-0.201 0.045	-5.893	-19.248	0.714315
4	tight:one-by-one MAD $\sigma_{\text{clip}} = 3.0$	0.99	3429	73.12 1.01	-3.299 0.015	-0.220 0.045	-5.893	-19.251	0.714175
5	tight:global $\sigma_{\text{clip}} = 3.0$	0.99	3432	73.22 1.01	-3.298 0.015	-0.215 0.045	-5.891	-19.248	0.714194
6	loose:global $\sigma_{\text{clip}} = 5.0$	1.16	3475	73.34 1.01	-3.294 0.016	-0.221 0.049	-5.888	-19.244	0.714183
7	loose:one-by-one MAD $\sigma_{\text{clip}} = 5.0$	1.15	3474	73.42 1.01	-3.295 0.016	-0.222 0.048	-5.888	-19.242	0.714178
8	loose:individual $P-L$ $\sigma_{\text{clip}} = 5.0$	1.05	3397	73.35 1.01	-3.296 0.015	-0.202 0.046	-5.892	-19.244	0.714257
9	none	1.23	3481	73.41 1.01	-3.290 0.016	-0.206 0.050	-5.885	-19.242	0.714248

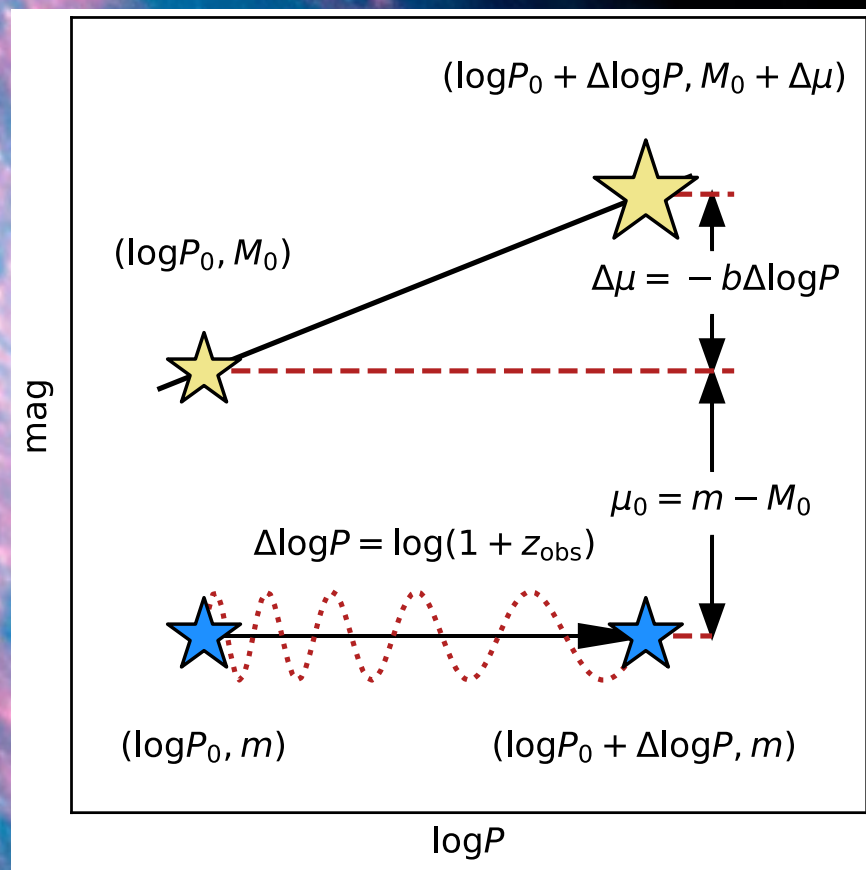
$$\log H_0 = 0.2M_B^0 + a_B + 5$$

$$H_0 = 73.04 \pm 1.04 \text{ km/s/Mpc}$$



Cepheid Redshift-Leavitt bias

RIA, A&A **631**, A165 (2019)



$z_{\text{obs}} \approx 0$



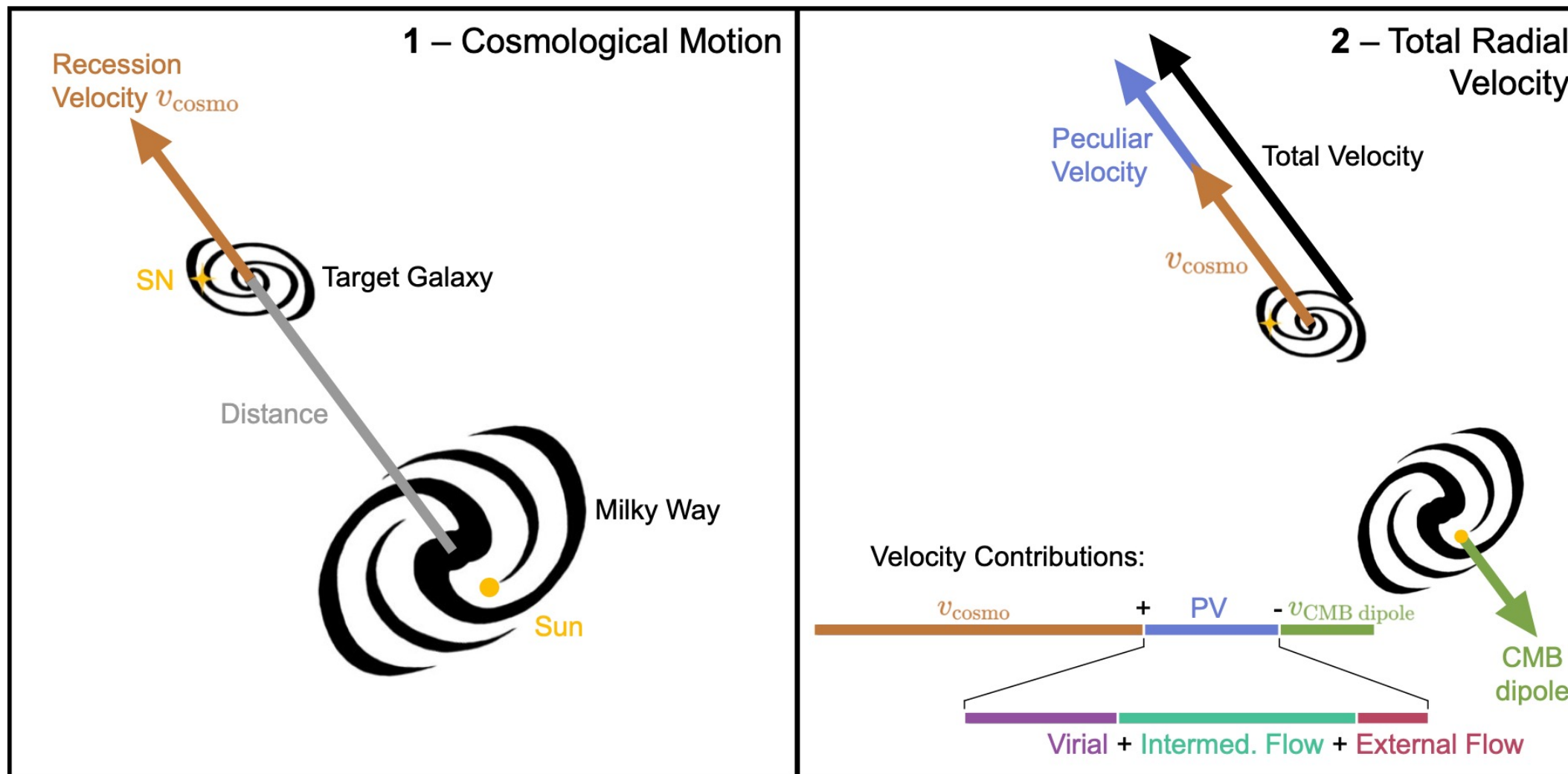
$$M = \alpha + \beta \log P$$

Everything is moving!

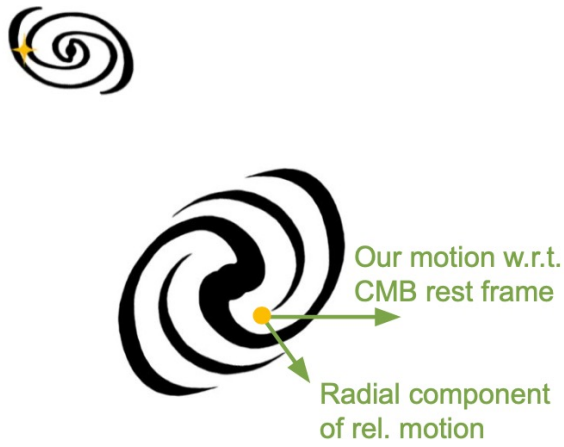
$$z_{obs} = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} \approx \frac{D_L H_0}{c} \text{ measures the combined motion}$$

cosmological Peculiar motion target, observer Peculiar motion target, observer

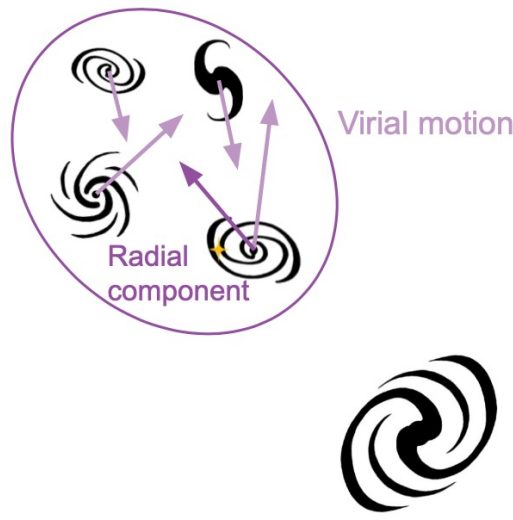
$$(1 + z_{obs}) = (1 + \bar{z})(1 + z_{pec}^{gal})(1 + z_{pec}) \left(1 + z_{\phi}^{gal}\right) (1 + z_{\phi})$$



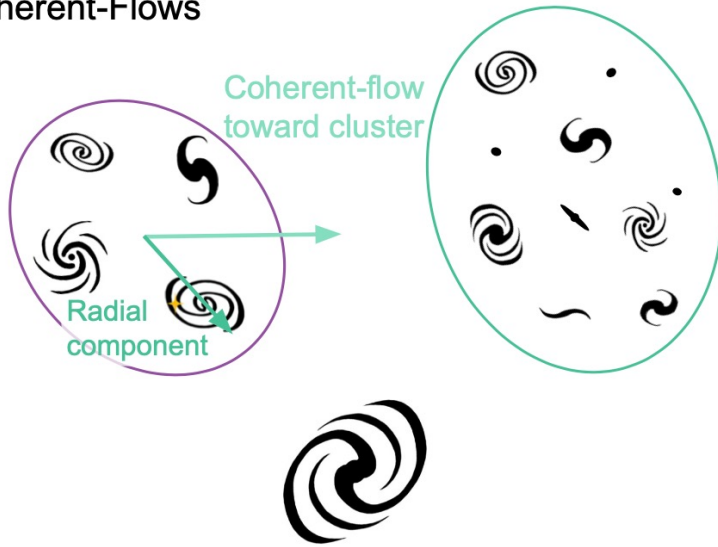
A – Our Motion Relative to CMB Rest Frame



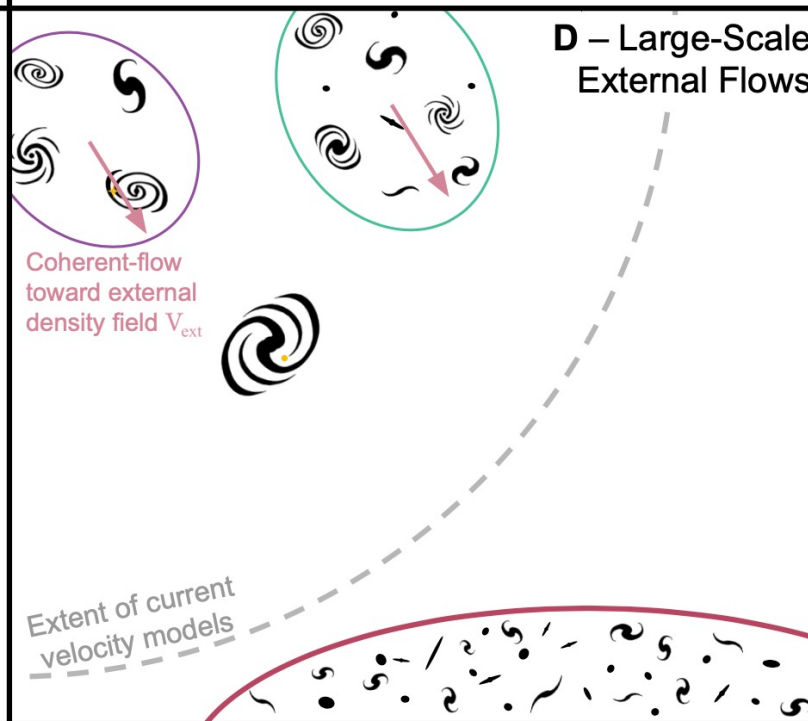
B – Small-Scale Motion



C – Intermediate-Scale Coherent-Flows

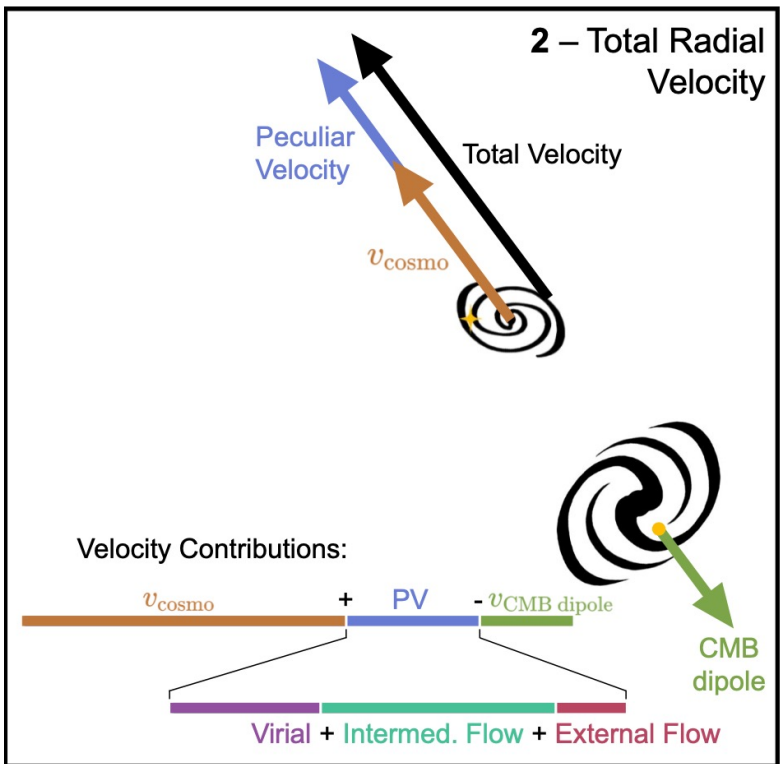


D – Large-Scale External Flows

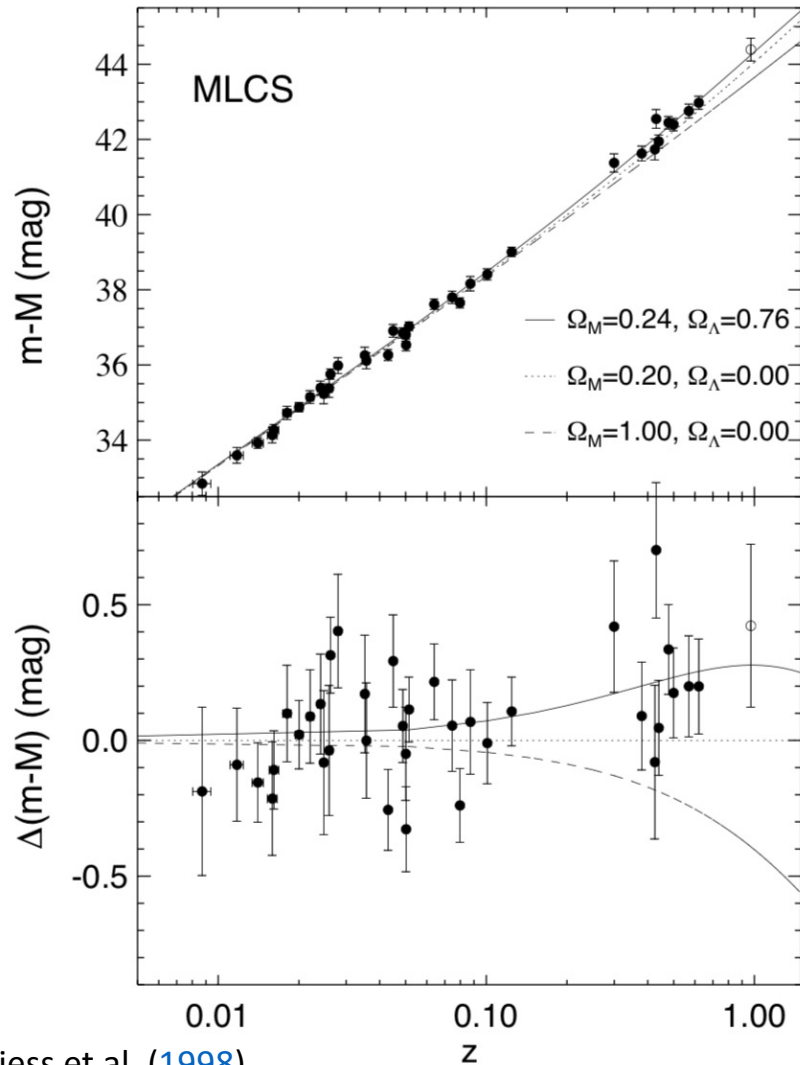


$$z_{\text{cosmo}} = \left(\frac{(1 + z_{\text{observed}})}{(1 + z_{\text{ecliptic}})(1 + z_{\text{CMB dipole}})(1 + z_{\text{pec}})} \right) - 1$$

$$z_{\text{pec}} = \left((1 + z_{\text{virial}})(1 + z_{\text{coh.}})(1 + z_{\text{ext. coh.}}) \right) - 1$$



Acceleration



Riess et al. ([1998](#))

$$D_L = \frac{cz}{H_0} \left\{ 1 + \frac{1}{2}(1 - q_0)z - \frac{1}{6} \left[1 - q_0 - 3q_0^2 + j_0 \pm \frac{c^2}{H_0^2 R^2} \right] z^2 + O(z^3) \right\}$$

$$r(t) = a(t) \cdot r_0$$

$$H_0 = \frac{\dot{a}}{a}$$

$$q_0 = -\frac{\ddot{a}}{a} H_0^{-2}$$

$$j_0 = \frac{\dddot{a}}{a} H_0^{-3}$$

$$q_0 = \frac{\Omega_M}{2} + (1 + 3w) \frac{\Omega_{DE}}{2} < 0$$

The Nobel Prize in Physics 2011



Saul
Perlmutter



Brian P.
Schmidt

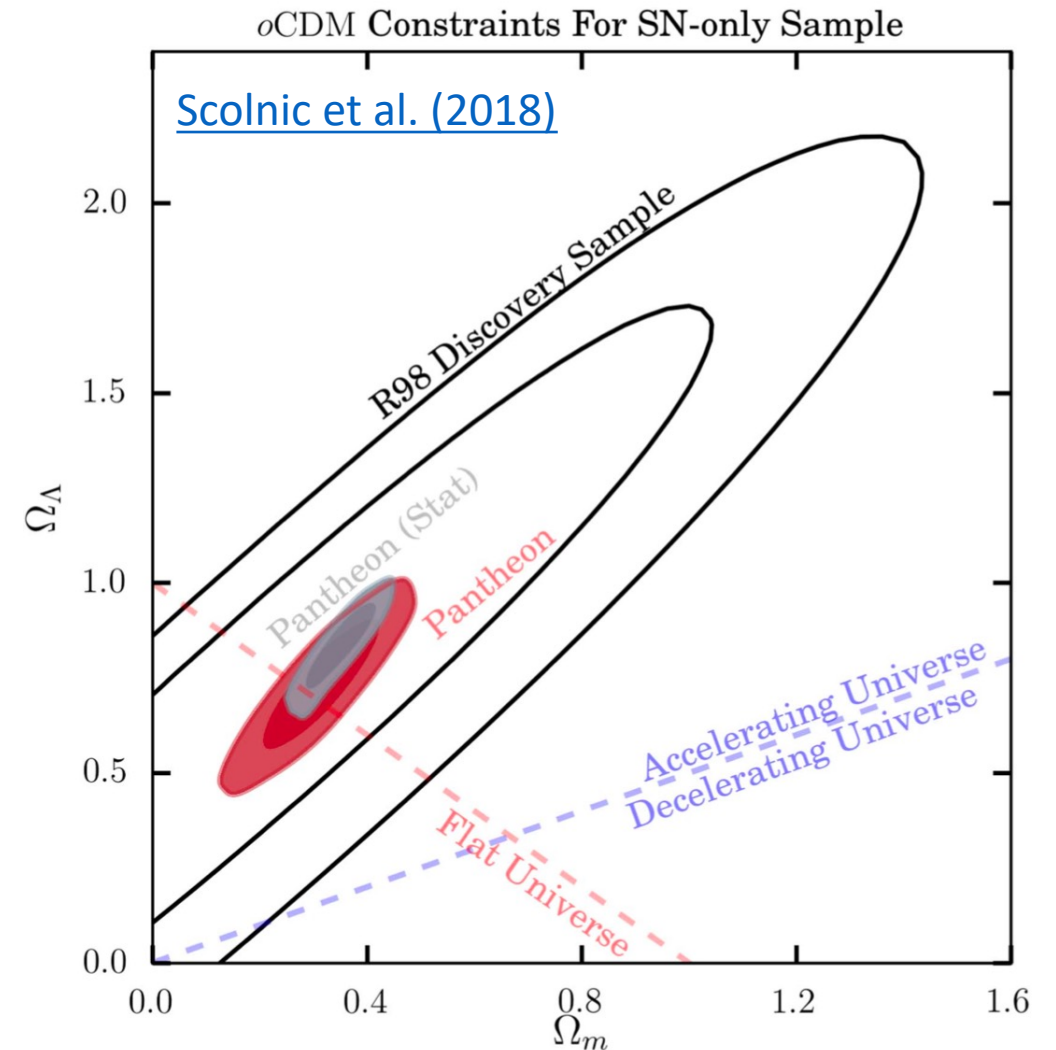
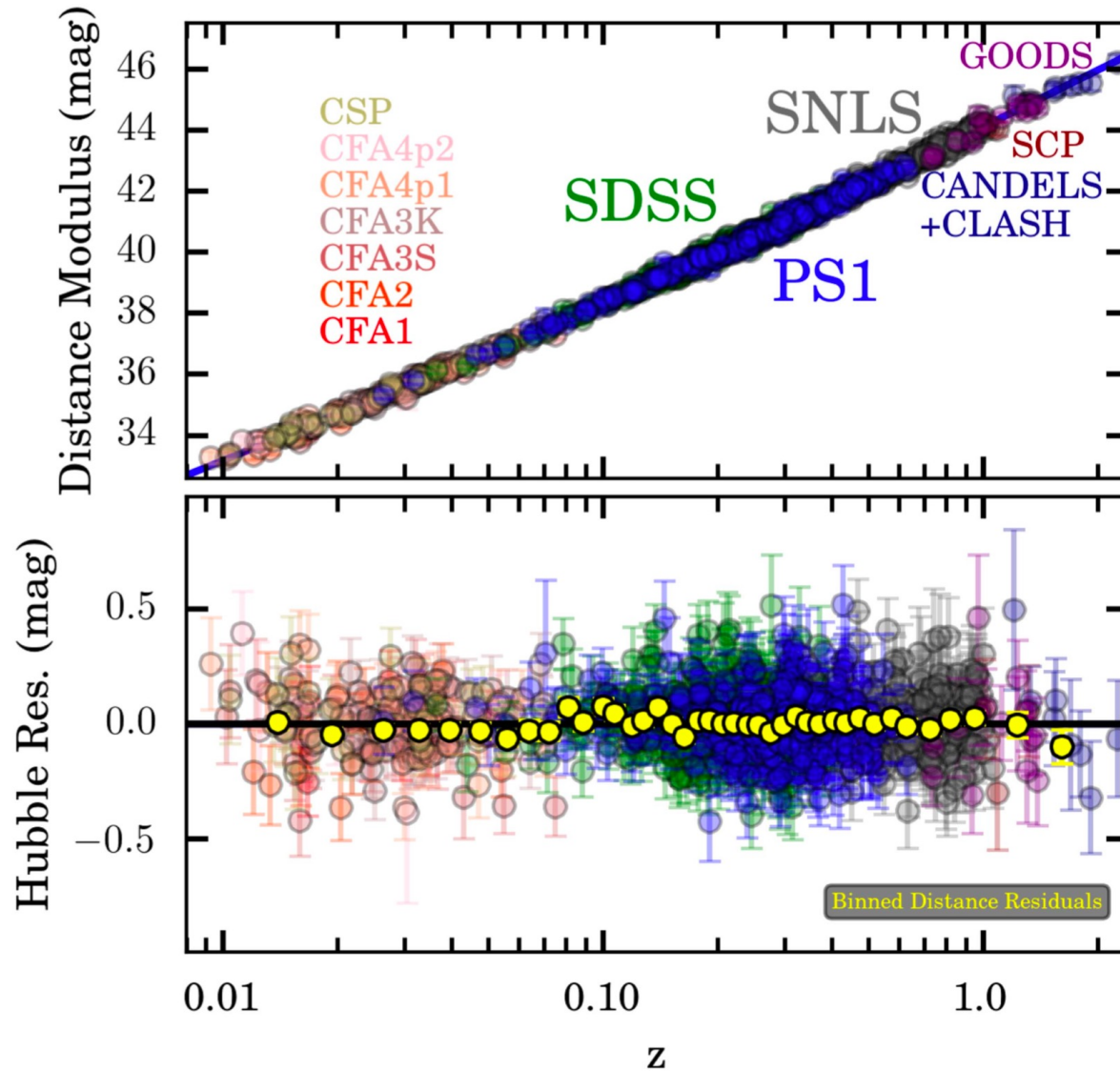


Adam G.
Riess

Credit: The Nobel Foundation, Photos by U. Montan

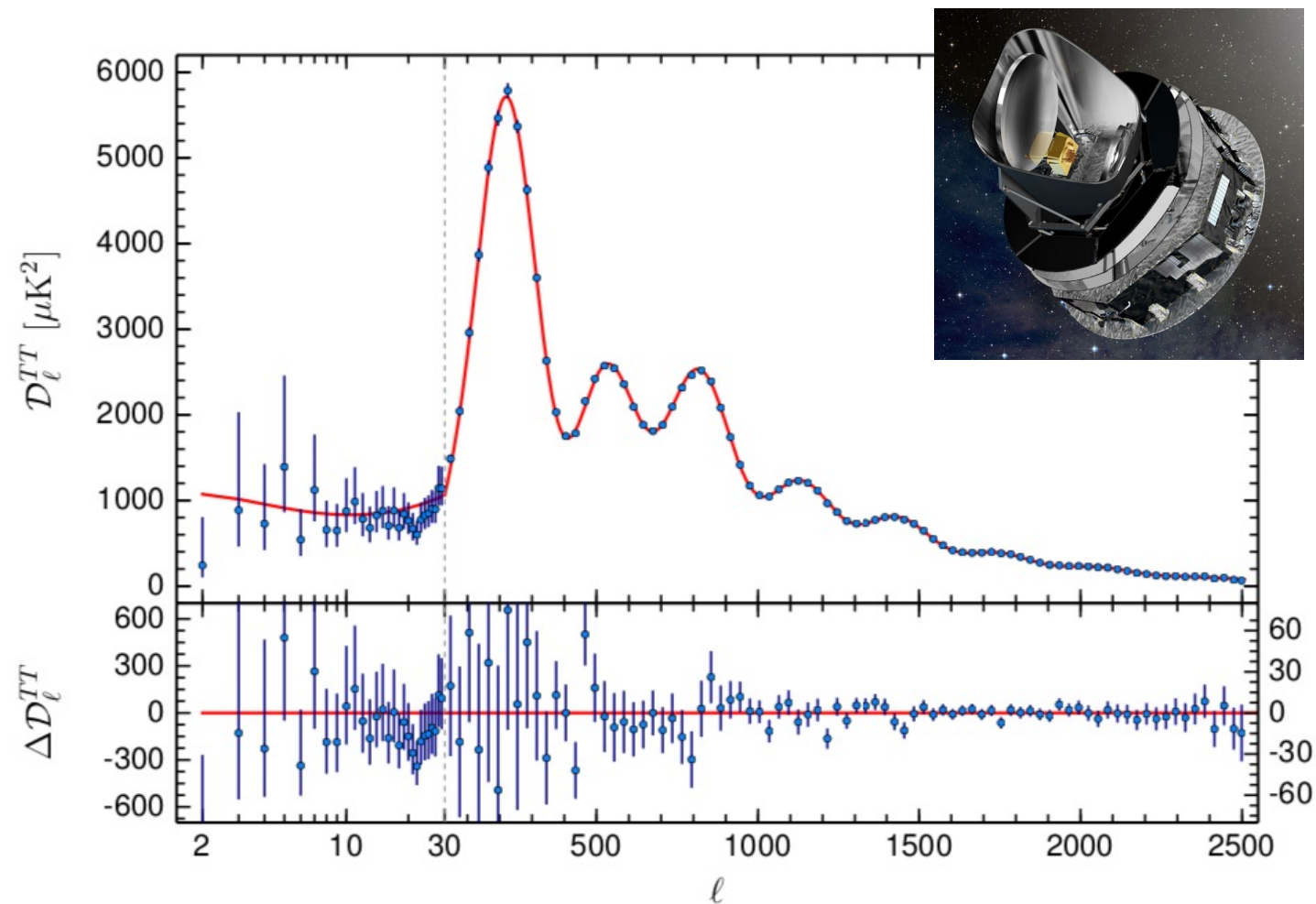
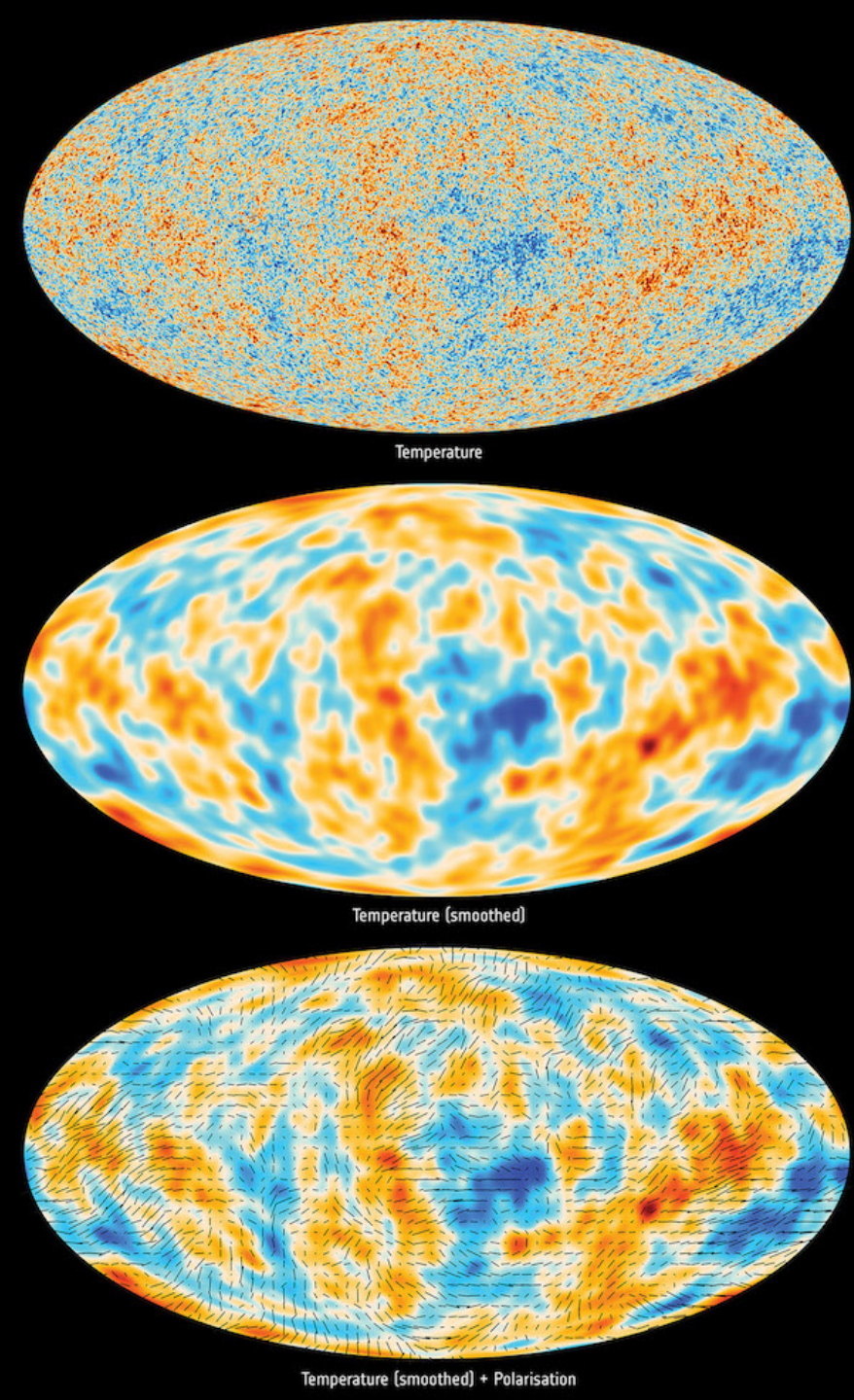
Precision SN Cosmology

$$q_0 = \frac{\Omega_M}{2} + (1 + 3w) \frac{\Omega_{DE}}{2} < 0$$



The Hubble tension: hunting for
cracks in the Λ CDM cosmology

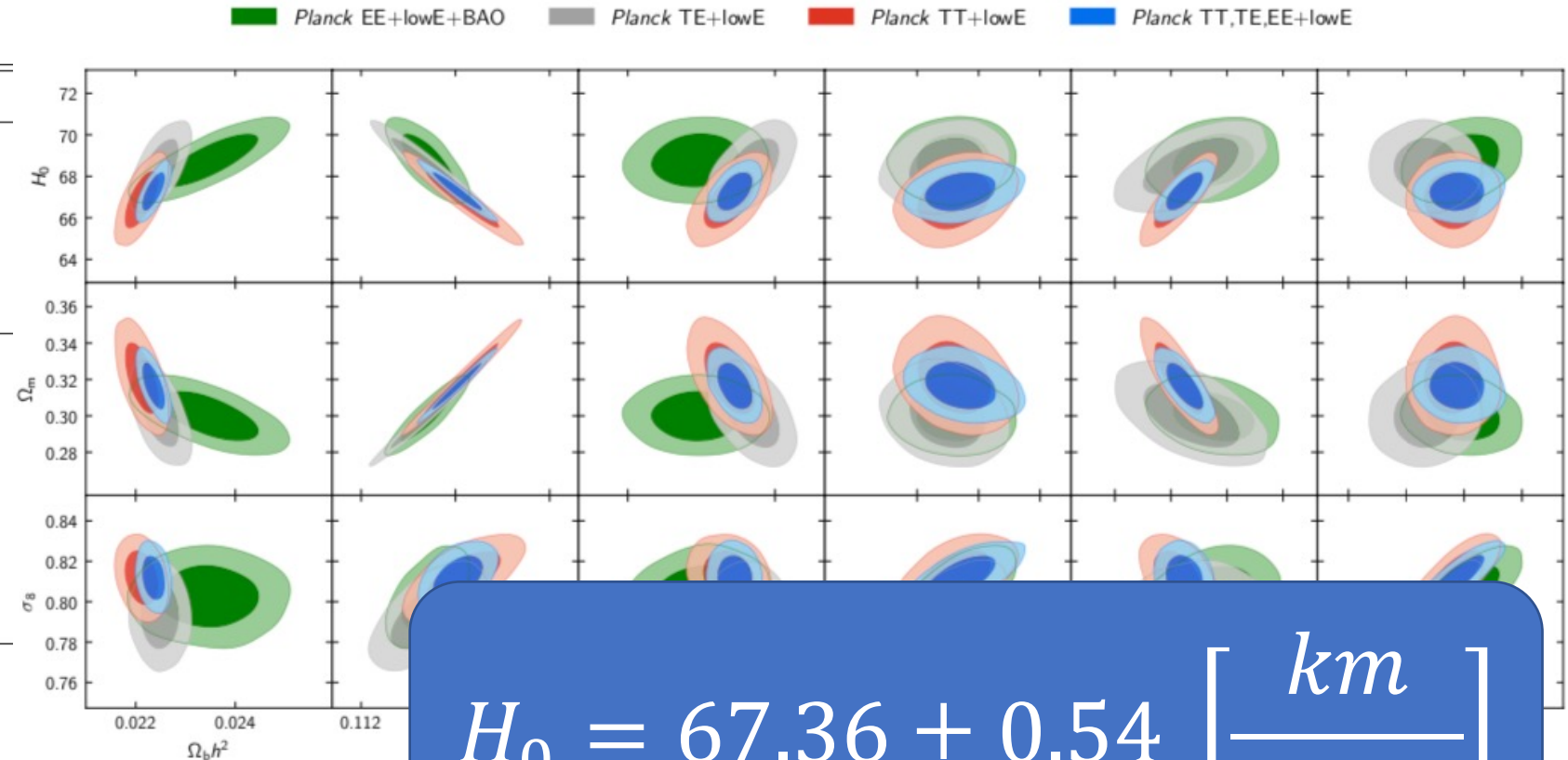
The Cosmic Microwave Background



Planck Collaboration ([2015](#))

Cosmological Parameters from the *Planck* CMB

Parameter	Plik best fit	Plik [1]
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012
$100\theta_{\text{MC}}$	1.040909	1.04092 ± 0.00031
τ	0.0543	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014
n_s	0.96605	0.9649 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011
H_0 [km s ⁻¹ Mpc ⁻¹] ...	67.32	67.36 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073
Age [Gyr]	13.7971	13.797 ± 0.023
σ_8	0.8120	0.8111 ± 0.0060
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$..	0.8331	0.832 ± 0.013
z_{re}	7.68	7.67 ± 0.73
$100\theta_*$	1.041085	1.04110 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26



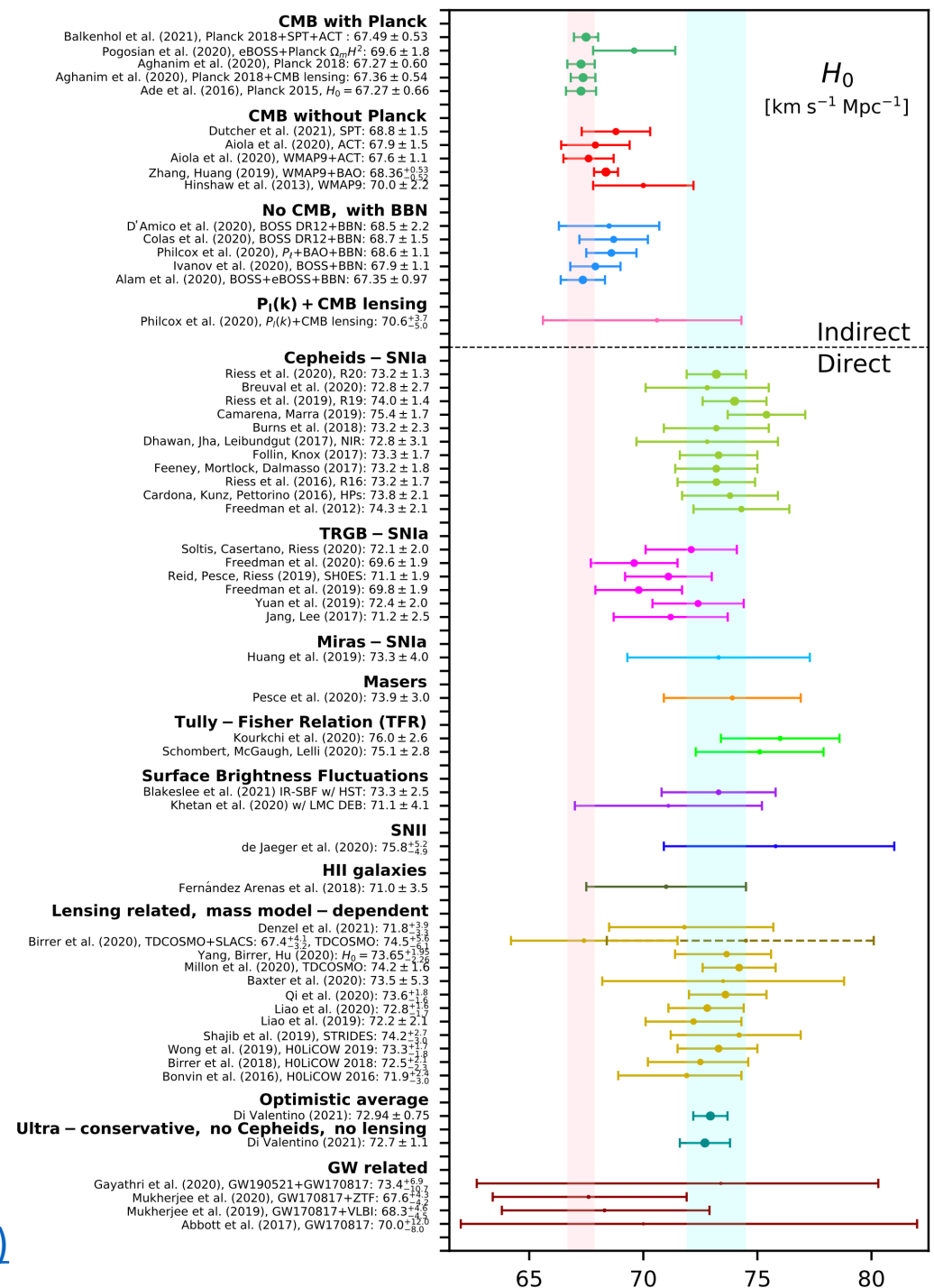
$$H_0 = 67.36 \pm 0.54 \left[\frac{\text{km}}{\text{s Mpc}} \right]$$

Planck Collaboration ([2018](#))

The Hubble tension

- What's different now?
 - Data homogeneity
 - Accurate CCD photometry
 - Distances & parallaxes
 - Sample sizes
- Which measurements are independent?
- How can you determine an average in this situation?
- This is very exciting to theorists!

[Di Valentino et al. \(2021\)](#)



The Early Universe

H_0 as a cosmological end-to-end test

Today's Universe

$$H_0 = 73.04 \pm 1.04$$

$$8.0\% / \text{yr} \quad \text{drag} / 5.0\sigma$$

$$67.4 \pm 0.5 \text{ Mpc}$$

New Physics???

Planck VI (2018)

Riess et al. (2022)