Quantum Information and Quantum Computing, Problem set 5

Assistant : sara.alvesdossantos@epfl.ch, clemens.giuliani@epfl.ch, khurshed.fitter@epfl.ch

Problem 1 : Random number generation

We want to design a random number generator using readout on a quantum circuit.

1. Design a quantum circuit that generates random integers uniformly distributed between 0 and $2^N - 1$. Use only the universal set of gates H, S, T, $CNOT$
2. Design a quantum circuit that generates random integers with the following property. The probability of generating even integers is uniformly distributed, as well as the probability of generating odd integers. However, even integers are $x = \frac{2+\sqrt{2}}{2-\sqrt{2}}$ more likely than odd integers. Again, use only gates from the universal set.
3. Show that this result can be generalized to an arbitrary value of x, if we use an arbitrary phase gate $R_z(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$.
4. Think about the possible generalization to an arbitrary distribution of integers, and what would be the associated challenges.

Problem 2 : Quantum Teleportation

Design a circuit to perform the quantum teleportation protocol on a quantum circuit of 3 qubits: one is the qubit whose state we want to send the other two are those entangled and shared between the sender and the receiver.

Problem 3 : Oracle for the Deutsch-Jozsa quantum algorithm

Design a circuit that executes the quantum oracle for a three-qubit Deutsch-Jozsa quantum algorithm, for the following cases:

- A constant function $f(x) = 0$.
- A constant function $f(x) = 1$.
- A balanced function $f(x)$ which takes the value $f(x) = 1$ for $\{x = 001, 100, 010, 111\}$.
- A balanced function $f(x)$ which takes the value $f(x) = 1$ for $\{x = 000, 011, 101, 110\}$.