
Quantum Information and Quantum Computing, Problem set 10

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Problem 1 : A depolarising channel

Take a two-dimensional Hilbert space $\mathcal{H}$ and a completely positive trace preserving (CPTP) map $\mathcal{E} : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H})$ given by

$$\mathcal{E}(\hat{\rho}) := p \frac{\hat{\mathbb{1}}}{2} + (1 - p) \hat{\rho} \quad (1)$$

for $p \in [0, 1]$.

This map describes the action of what is known as a depolarizing channel.

- (a) Any quantum channel given by a CPTP map $\mathcal{E}$ can be written in terms of Kraus operators M_a as

$$\mathcal{E}(\hat{\rho}) = \sum_a \hat{M}_a \hat{\rho} \hat{M}_a^\dagger \quad (2)$$

where $\sum_a \hat{M}_a^\dagger \hat{M}_a = \mathbb{1}$.

Find a Kraus-operator representation for $\mathcal{E}$.

Hint: You might want to prove (and then use) the identity $\mathbb{1} = \frac{1}{2}(\hat{\rho} + \hat{\sigma}_x \hat{\rho} \hat{\sigma}_x + \hat{\sigma}_y \hat{\rho} \hat{\sigma}_y + \hat{\sigma}_z \hat{\rho} \hat{\sigma}_z)$, where $\sigma_{x,y,z}$ are the Pauli-matrices.

- (b) Any two-dimensional density matrix can be written in the Bloch representation

$$\rho = \frac{1}{2}(\mathbb{1} + \mathbf{r} \cdot \hat{\boldsymbol{\sigma}}) \quad (3)$$

where $\hat{\boldsymbol{\sigma}} = [\hat{\sigma}_x \ \hat{\sigma}_y \ \hat{\sigma}_z]^T$ is the vector of pauli matrices and $\mathbf{r} \in \mathbb{R}^3, \|\mathbf{r}\| \leq 1$ is called Bloch vector.

Calculate the action of the depolarizing channel $\mathcal{E}$ on such a bloch state for a given $\mathbf{r}$. How is the radius $r = \|\mathbf{r}\|$ affected?

Bonus: What happens when the depolarizing channel is applied to a pure state?

Problem 2 : Noise models in Qiskit

In this hands-on exercise we will see how to create custom noise models in Qiskit.

- (a) Learn how to save and plot density matrices in Qiskit using the classical density matrix simulator
- (b) Learn how to configure a noise model in Qiskit and use it to create a bit-flip, a phase-flip and a depolarising channel.

- (c) Apply your noise model to a circuit of choice and plot the final density matrices. What happens to the purity of the matrix?
- (d) Learn how to perform a tomography experiment to evaluate the density matrix of the system on the quantum device