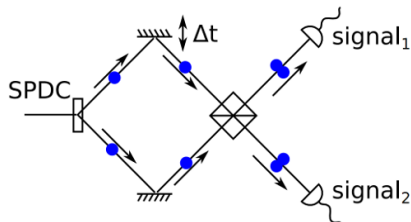


Quantum Electrodynamics and Quantum Optics: Lecture 5

Fall 2024

Hong-Ou-Mandel effect



Joint detection probability

$$P(x_1, x_2) \propto \langle \hat{E}^-(x_1) \hat{E}^-(x_2) \hat{E}^+(x_2) \hat{E}^+(x_1) \rangle$$

$$\hat{E}^+(x_1) = \frac{\epsilon}{\sqrt{2}} (i\hat{a}_1 e^{ik_1 x_1} + \hat{a}_2 e^{ik_2 x_1})$$

$$\hat{E}^+(x_2) = \frac{\epsilon}{\sqrt{2}} (\hat{a}_1 e^{ik_1 x_2} + i\hat{a}_2 e^{ik_2 x_2})$$

Hong-Ou-Mandel effect

Quantum mechanical calculation:

$$\hat{E}^+(x_b)\hat{E}^+(x_a)|1\rangle_a|1\rangle_b = \frac{\epsilon^2}{2}(e^{i(k_1x_2+k_2x_1)} - e^{i(k_1x_1+k_2x_2)})|0_a,0_b\rangle$$

$$\langle 1|_b\langle 1|_a\hat{E}^-(x_a)\hat{E}^-(x_b) = \frac{\epsilon^2}{2}(e^{-i(k_1x_2+k_2x_1)} - e^{-i(k_1x_1+k_2x_2)})\langle 0_a,0_b|$$

$$P_{a,b} = \frac{\epsilon^4}{2}(1 - \cos((k_1 - k_2) \cdot (x_1 - x_2)))$$

$$P_{\max} = \epsilon^4, \quad P_{\min} = 0, \quad \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = 1$$

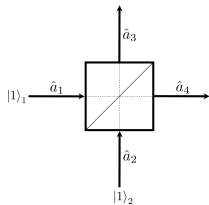
But in the classical case:

$$P_{a,b} \propto \langle (I_1 + I_2)^2 \rangle - 2\langle I_1 I_2 \rangle \cos(\pi(x_a - x_b)), \quad \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} < 1$$

Beam Splitter and Indistinguishable Photons

By injecting **indistinguishable** single photons to each port of the beam splitter, we will have a **pair** of photons in the output ports. The state $|1\rangle_3|1\rangle_4$ does not appear!

$$|\psi\rangle_{\text{in}} = |1\rangle_1|1\rangle_2 = \hat{a}_1^\dagger \hat{a}_2^\dagger |0\rangle_1|0\rangle_2$$
$$\begin{pmatrix} \hat{a}_3 \\ \hat{a}_4 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$



and the fact that the vacuum state is the same before and after BS:

$$\begin{aligned} |\psi\rangle_{\text{out}} &= \frac{1}{2} (\hat{a}_3^\dagger - \hat{a}_4^\dagger) (\hat{a}_3^\dagger + \hat{a}_4^\dagger) |0\rangle_3|0\rangle_4 \\ &= \frac{1}{2} (\hat{a}_3^{+2} + \cancel{\hat{a}_3^\dagger \hat{a}_4^\dagger} - \cancel{\hat{a}_4^\dagger \hat{a}_3^\dagger} - \hat{a}_4^{+2}) |0\rangle_3|0\rangle_4 \\ &= \frac{1}{\sqrt{2}} (|2\rangle_3|0\rangle_4 - |0\rangle_3|2\rangle_4), \text{ where } [\hat{a}_3, \hat{a}_4] = 0 \end{aligned}$$

Hong-Ou-Mandel effect with differently polarized photons

Output state

Consider the case where two single photon states with orthogonal polarization (H, V) enter a 50:50 lossless beamsplitter from port 1 and port 2 individually. The output joint state in port 3 and port 4 can be written as:

$$|\psi\rangle_{out} = \hat{a}_{1,H}^\dagger \hat{a}_{2,V}^\dagger |0\rangle_1 |0\rangle_2 \quad (1)$$

$$= \frac{1}{2} (\hat{a}_{3,H}^\dagger + i\hat{a}_{4,H}^\dagger) (i\hat{a}_{3,V}^\dagger + \hat{a}_{4,V}^\dagger) |0\rangle_1 |0\rangle_2 \quad (2)$$

$$= \frac{1}{2} (i\hat{a}_{3,H}^\dagger \hat{a}_{3,V}^\dagger + \hat{a}_{3,H}^\dagger \hat{a}_{4,V}^\dagger - \hat{a}_{4,H}^\dagger \hat{a}_{3,V}^\dagger + i\hat{a}_{4,H}^\dagger \hat{a}_{4,V}^\dagger) |0\rangle_1 |0\rangle_2 \quad (3)$$

$$= \frac{1}{2} (i|1,H\rangle_3 |1,V\rangle_3 + |1,H\rangle_3 |1,V\rangle_4 - |1,V\rangle_3 |1,H\rangle_4 + i|1,H\rangle_4 |1,V\rangle_4) \quad (4)$$

Wigner Function (E. Wigner, Phys.Rev. 40, 1932)

JUNE 1, 1932

PHYSICAL REVIEW

VOLUME 40

On the Quantum Correction For Thermodynamic Equilibrium

By E. WIGNER

Department of Physics, Princeton University

(Received March 14, 1932)

The probability of a configuration is given in classical theory by the Boltzmann formula $\exp [-V/kT]$ where V is the potential energy of this configuration. For high temperatures this of course also holds in quantum theory. For lower temperatures, however, a correction term has to be introduced, which can be developed into a power series of $\hbar$. The formula is developed for this correction by means of a probability function and the result discussed.

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E. WIGNER

transformations, one can choose any matrix or operator-representation for the Q and H . In building the exponential of H one must, of course, take into account the non-commutability of the different parts of H .

2

It does not seem to be easy to make explicit calculations with the form (4) of the mean value. One may resort therefore to the following method.

If a wave function $\psi(x_1 \cdots x_n)$ is given one may build the following expression²

$$P(x_1, \cdots, x_n; p_1, \cdots, p_n) \\ = \left(\frac{1}{h\pi}\right)^n \int_{-\infty}^{\infty} \cdots \int dy_1 \cdots dy_n \psi(x_1 + y_1 \cdots x_n + y_n)^* \\ \psi(x_1 - y_1 \cdots x_n - y_n) e^{2i(p_1 y_1 + \cdots + p_n y_n)/\hbar} \quad (5)$$

and call it the probability-function of the simultaneous values of $x_1 \cdots x_n$ for the coordinates and $p_1 \cdots p_n$ for the momenta. In (5), as throughout this paper, $\hbar$ is the Planck constant divided by 2π and the integration with respect to the y has to be carried out from $-\infty$ to ∞ . Expression (5) is, real, but not everywhere positive. It has the property, that it gives, when integrated with respect to the p , the correct probabilities $|\psi(x_1 \cdots x_n)|^2$ for the different values of the coordinates and also it gives, when integrated with respect to the x , the correct quantum mechanical probabilities

$$\left| \int_{-\infty}^{\infty} \cdots \int \psi(x_1 \cdots x_n) e^{-i(p_1 x_1 + \cdots + p_n x_n)/\hbar} dx_1 \cdots dx_n \right|^2$$

for the momenta $p_1, \cdots, p_n$. The first fact follows simply from the theorem about the Fourier integral and one gets the second by introducing $x_k + y_k = u_k$; $x_k - y_k = v_k$ into (5).

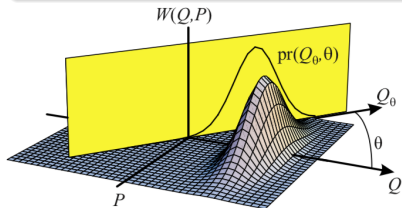
Wigner Function¹

Wigner function, the phase-space quasi-probability density

$$W_{\hat{\rho}}(q, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle q + \frac{1}{2}q' \left| \hat{\rho} \right| q - \frac{1}{2}q' \right\rangle e^{-ipq'} dq'$$

This function uniquely defines the state and directly relates to the quadrature histograms measured experimentally via

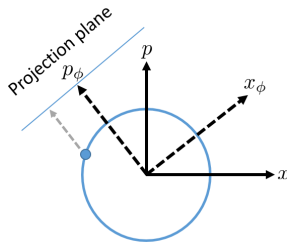
$$\text{Pr}(q_{\theta}, \theta) = \int_{-\infty}^{\infty} W_{\text{det}}(q_{\theta} \cos \theta - p_{\theta} \sin \theta, q_{\theta} \sin \theta + p_{\theta} \cos \theta) dp_{\theta}.$$



The experimentally measured probability density $\text{Pr}(q_{\theta}, \theta)$ is the integral projection of the Wigner function $W_{\hat{\rho}}(q, p)$ onto a vertical plane defined by the phase of the local oscillator.

¹Lvovsky, Alexander I., and Michael G. Raymer. "Continuous-variable optical quantum-state tomography." *Reviews of Modern Physics* 81.1 (2009): 299.

Quantum State Tomography



$W(p, q)$ is a joint probability function for the $\hat{p}$ and $\hat{q}$ operators:

Marginals

$$\Pr(q_\phi) = \langle q_\phi | \hat{\rho} | q_\phi \rangle = \int_{-\infty}^{\infty} W(p, q) dp_\phi, \text{ where}$$

$$\hat{q}_\phi = \hat{q} \cos(\phi) + \hat{p} \sin(\phi), \quad \hat{p}_\phi = -\hat{q} \sin(\phi) + \hat{p} \cos(\phi)$$

Quantum State Tomography

Motivation

To reconstruct a quantum state of light, we cannot directly measure ρ_{nn} with a photo-detector but we can measure $\Pr(X_\theta)$ and reconstruct the full Wigner function. For rotationally symmetric states e.g. Fock states the reconstruction becomes Abel transform²:

$$X_\theta = \langle X_\theta | \rho | X_\theta \rangle = \langle X | U_\theta^\dagger \rho U_\theta | X \rangle$$

$$\Pr(X_\theta) = \int_{-\infty}^{\infty} W(p_\theta, q_\theta) dp_\theta$$

$$W(r) = -\frac{1}{\pi} \int_r^\infty \frac{d\Pr(q_\phi)}{dq_\phi} (q_\phi^2 - r^2)^{-1/2} dq_\phi.$$

²Vogel, W., Welsch, D.G. "Quantum Optics" (2001). Chapter 7

Wigner Function and Parity Operator

Alternative expression for the Wigner function³

$$\begin{aligned} W(r, p) &= \frac{1}{h^2} \int dk \int ds e^{-i \frac{kr+sp}{h}} \langle \psi | e^{-i \frac{k\hat{R}+s\hat{P}}{h}} | \psi \rangle \\ &= \frac{2}{h} \langle \psi | \hat{\Pi}_{rp} | \psi \rangle \end{aligned}$$

Where $\hat{\Pi}_{rp} = \hat{D}(r, p) \hat{\Pi} \hat{D}^{-1}(r, p)$ is a displaced parity operator $\hat{\Pi}$, which acts as follows

$$\hat{\Pi} \hat{R} \hat{\Pi} = -\hat{R}$$

$$\hat{\Pi} \hat{P} \hat{\Pi} = -\hat{P}$$

Wigner function as the expectation value of a parity operator

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(Received 30 August 1976)

It is pointed out that the Wigner function $f(r, p)$ is $2/h$ times the expectation value of the parity operator that performs reflections about the phase-space point r, p . Thus $f(r, p)$ is proportional to the overlap of the wave function ψ with its mirror image about r, p ; this is clearly a measure of how much ψ is *centered* about r, p , and the Wigner distribution function now appears physically more meaningful and natural than it did previously.

³Moyal JE. Quantum mechanics as a statistical theory. Mathematical Proceedings of the Cambridge Philosophical Society. 1949;45(1):99-124.
doi:10.1017/S0305004100000487

State Reconstruction

Inverse Radon transformation

$$W_{\text{det}}(q, p) = \frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^\infty \text{Pr}(q_\theta, \theta) \times K(q \cos \theta + p \sin \theta - q_\theta) dq_\theta d\theta$$

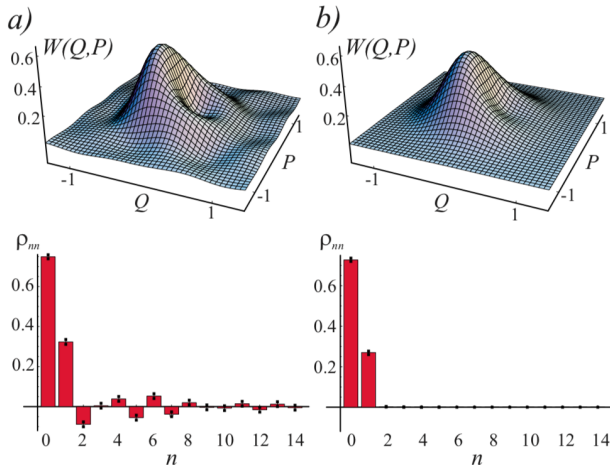
with the integration kernel $K(x) = \frac{1}{2} \int_{-\infty}^\infty |\xi| e^{i\xi x} d\xi$. The density matrix can then be reconstructed using the pattern function method.

Maximum likelihood reconstruction

$$L = \Pi_i \text{Pr}_{\hat{\rho}}(q_i, \theta_i)$$

is the likelihood function given the measured data set $\{(q_i, \theta_i)\}$ where $\hat{\rho}$ is the density matrix to be optimized.

State Reconstruction



Quantum optical state estimation from a set of 14152 experimental homodyne measurements by means of (a) the inverse Radon transformation and the pattern-function method and (b) the likelihood maximization algorithm. The Wigner function and the diagonal elements of the reconstructed density matrix are shown.

The original Radon transformation paper in German.

SITZUNG VOM 30. APRIL 1917.

Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten.

Von
JULIAN RADON.

Integriert man eine gegebene Regularitätsbedingungen unterworfenen Funktion einer Veränderlichen x, y — eine Punktfunktion $f(P)$ in der Ebene — längs aller beliebigen Geraden g , so erhält man in den Integralwerten $F(g)$ eine Geradenfunktion. Das in Abschnitt A vorstehender Abhandlung gelöste Problem ist die Umkehrung dieser linearen Funktionaltransformation, d. h. es werden folgende Fragen beantwortet: kann jede, gegebene Regularitätsbedingungen genügende Geradenfunktion auf diese Weise entstanden gedacht werden? Wenn ja, in dem f durch F eindeutig bestimmt und wie kann es ermittelt werden?

In Abschnitt B gelangt das dann in gewisse Hinsicht ideale Problem der Bestimmung einer Geradenfunktion $F(g)$ aus ihren Punktwertfunktionen $f(P)$ zur Lösung.

Schließlich werden in Abschnitt C gewisse Verallgemeinerungen kurz angedeutet, es dienen insbesondere die Behandlung abwickelbarer Mannigfaltigkeiten sowie höherer Räume Anhaltspunkt.

Die Behandlung dieser an sich interessanten Probleme gewinnt ein erhöhtes Interesse durch die zahlreichen Beziehungen, die zwischen diesem Gegenstande und der Theorie der logarithmischen und Euklidischen Potentiale bestehen, auf die an den betreffenden Stellen zu verweisen sein wird.

JULIAN RADON: DETERMINATION OF FUNCTIONS FROM INTEGRAL VALUES. 253

A. Bestimmung einer Punktfunktion in der Ebene aus ihren Integralwerten.

i. Es sei $f(x, y)$ eine für alle reellen Punkte $P = [x, y]$ erklärte reelle Funktion, die folgende Regularitätsbedingungen erfüllt:

a₁) $f(x, y)$ ist stetig.

b₁) Es konvergiert das über die ganze Ebene zu entnehmende Doppelintegral $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$.

c₁) Wird für einen beliebigen Punkt $P = [x, y]$ und jedes $r \geq 0$

$$f_r(r) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} f(x + r \cos \varphi, y + r \sin \varphi) d\varphi$$

gebildet, so gelte für jeden Punkt P :

$$\lim_{r \rightarrow 0} f_r(r) = 0.$$

Dann gelten folgende Sätze:

Satz I: Der geradlinige Integralwert von f für die Gerade g mit der Gleichung $x \cos \varphi + y \sin \varphi = r$, der durch

$$(I) F(g) = F(-r, \varphi + \pi) = \int_{-\infty}^{\infty} f(x \cos \varphi + r \sin \varphi, y \sin \varphi + r \cos \varphi) dx$$

gegeben ist, ist „im allgemeinen“ vorhanden; das soll heißen: für jeden Punkt bilden die Randwertsequenzen jener Tangenten, für welche F nicht existiert, eine Menge vom inneren Maße Null.

Satz II: Bildet man den Mittelwert von $F(g, \varphi)$ für die Tangenten des Kreises mit dem Zentrum $P = [x, y]$ und dem Radius r

$$(II) F_r(x, y) = \frac{1}{2\pi} \int_0^{2\pi} F(x \cos \varphi + y \sin \varphi + r, \varphi) d\varphi,$$

so konvergiert dieses Integral für alle P, r absolut.

Satz III: Der Wert von f ist durch F eindeutig bestimmt und läßt sich folgendermaßen berechnen:

$$(III) f(P) = -\frac{1}{\pi} \int_0^{2\pi} F_r(x \cos \varphi + y \sin \varphi + r, \varphi) d\varphi$$

254

Julian Radon:

Dabei ist das Integral im dreifachen Sinne zu verstehen und kann auch durch die Formel:

$$(III') f(P) = -\frac{1}{\pi} \lim_{\epsilon \rightarrow 0} \left(\frac{F_r(P)}{\epsilon} - \int_0^{2\pi} F_r d\varphi \right)$$

dargestellt werden.

Indem wir nun Beweis schreiten, bemerken wir vorweg, daß die Bedingungen a₁ — c₁ gegebener Bewegungen der Ebene invariant sind. Wir können also den Punkt $[x, y]$ immer als Repräsentanten irgendeines Punktes der Ebene betrachten.

Man erkennt nun das Doppelintegral:

$$(I) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$$

als absolut konvergent. Verwende der Transformation

$$x = g \cos \varphi - r \sin \varphi, \quad y = g \sin \varphi + r \cos \varphi$$

geht dasselbe über in:

$$\int_{-\infty}^{\infty} \int_0^{2\pi} f(g \cos \varphi - r \sin \varphi, g \sin \varphi + r \cos \varphi) dg d\varphi = \int_{-\infty}^{\infty} \int_0^{2\pi} f(g \cos \varphi - r \sin \varphi, g \sin \varphi + r \cos \varphi) dg d\varphi,$$

so daß man seinen Wert auch durch

$$\frac{1}{2} \int_{-\infty}^{\infty} \int_0^{2\pi} f(g \cos \varphi - r \sin \varphi, g \sin \varphi + r \cos \varphi) dg d\varphi = \frac{1}{2} \int_{-\infty}^{\infty} \int_0^{2\pi} f(g, \varphi) dg d\varphi$$

ausdrücken kann. Nach bekannten Eigenschaften der absolut konvergierenden Doppelintegrale folgen hieraus die Behauptungen der Sätze I und II.

Um auf die Formel (III) zu kommen, kann man folgenden Weg einschlagen: Einführung von Polarkoordinaten in (I) liefert

$$\int_{-\infty}^{\infty} \int_0^{2\pi} f\left(\frac{r}{\sqrt{x^2+y^2}} \cos \varphi, \frac{r}{\sqrt{x^2+y^2}} \sin \varphi\right) dg d\varphi$$

255

Julian Radon:

Dabei ist das Integral im dreifachen Sinne zu verstehen und kann auch durch die Formel:

$$(III') f(P) = -\frac{1}{\pi} \lim_{\epsilon \rightarrow 0} \left(\frac{F_r(P)}{\epsilon} - \int_0^{2\pi} F_r d\varphi \right)$$

dargestellt werden.

Indem wir nun Beweis schreiten, bemerken wir vorweg, daß die Bedingungen a₁ — c₁ gegebener Bewegungen der Ebene invariant sind. Wir können also den Punkt $[x, y]$ immer als Repräsentanten irgendeines Punktes der Ebene betrachten.

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geht dasselbe über in:

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so daß man seinen Wert auch durch

$$\frac{1}{2} \int_{-\infty}^{\infty} \int_0^{2\pi} f(g \cos \varphi - r \sin \varphi, g \sin \varphi + r \cos \varphi) dg d\varphi = \frac{1}{2} \int_{-\infty}^{\infty} \int_0^{2\pi} f(g, \varphi) dg d\varphi$$

ausdrücken kann. Nach bekannten Eigenschaften der absolut konvergierenden Doppelintegrale folgen hieraus die Behauptungen der Sätze I und II.

Um auf die Formel (III) zu kommen, kann man folgenden Weg einschlagen: Einführung von Polarkoordinaten in (I) liefert

$$\int_{-\infty}^{\infty} \int_0^{2\pi} f\left(\frac{r}{\sqrt{x^2+y^2}} \cos \varphi, \frac{r}{\sqrt{x^2+y^2}} \sin \varphi\right) dg d\varphi$$

State Reconstruction Experiment⁴

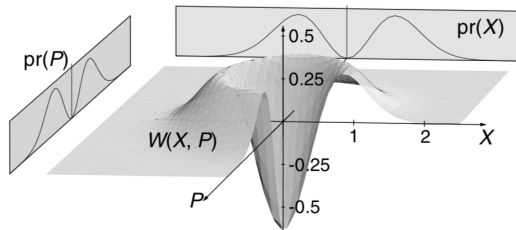


FIG. 1. Theoretical phase space quasiprobability density (Wigner function) of the single-photon state $|1\rangle$: $W(X, P) = \frac{2}{\pi}[4(X^2 + P^2) - 1]e^{-2(X^2 + P^2)}$. $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ and $\hat{P} = (\hat{a} - \hat{a}^\dagger)/\sqrt{2}i$ are normalized noncommuting electric field quadrature observables. Single-quadrature probability densities (*marginal distributions*) are also displayed.

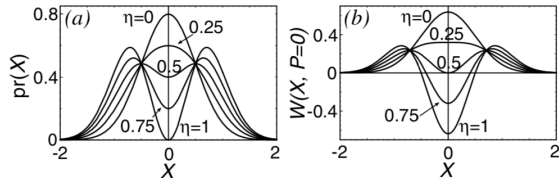


FIG. 3. Effect of the nonperfect measurement efficiency η on the marginal distribution (a) and the reconstructed WF (b). For the WF, cross sections by the plane $P = 0$ are shown. Negative values require $\eta > 0.5$.

⁴Lvovsky, Alexander I., et al. "Quantum state reconstruction of the single-photon Fock state." *Physical Review Letters* 87.5 (2001): 050402.

State Reconstruction Experiment

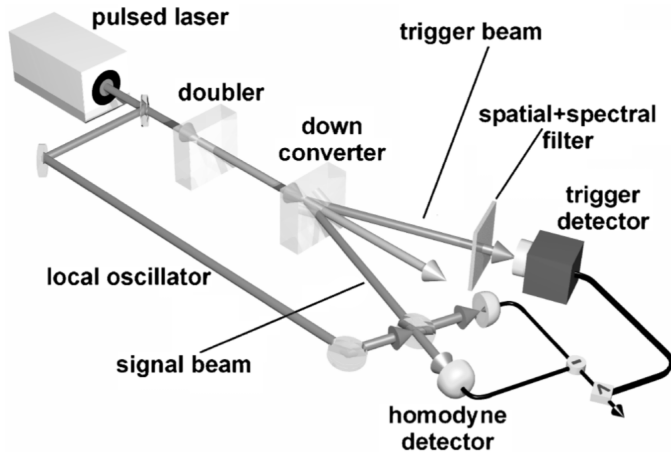


FIG. 2. Simplified scheme of the experimental setup.

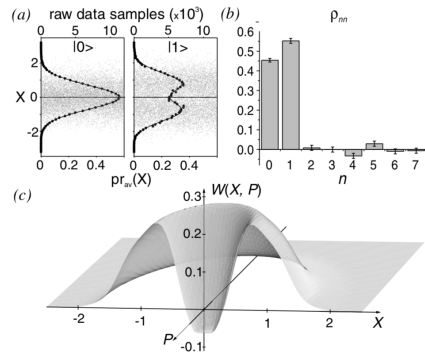


FIG. 4. Experimental results: (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their histograms corresponding to the phase-randomized marginal distributions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.

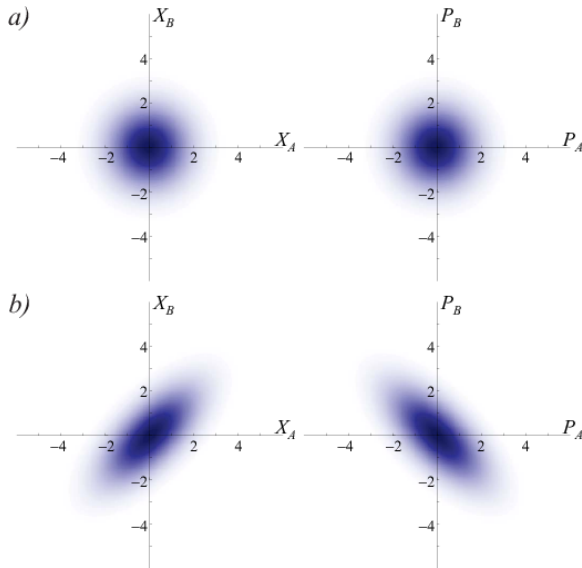
Two Mode Squeezed Vacuum State

$$\hat{S}_{\text{two-mode}} = e^{r\hat{a}^\dagger\hat{b}^\dagger - r\hat{a}\hat{b}}$$
$$|\Psi\rangle = \frac{1}{\cosh(r)} \sum_{n=0}^{\infty} (\tanh(r))^n |n, n\rangle$$

Wave-function associated with the state:

$$\langle Q_1, Q_2 | \Psi \rangle = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{4}e^{2r}(Q_1 - Q_2)^2 - \frac{1}{4}e^{-2r}(Q_1 + Q_2)^2\right)$$
$$\langle P_1, P_2 | \Psi \rangle = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{4}e^{2r}(P_1 - P_2)^2 - \frac{1}{4}e^{-2r}(P_1 + P_2)^2\right)$$

Two mode squeezed vacuum state



Covariance matrix

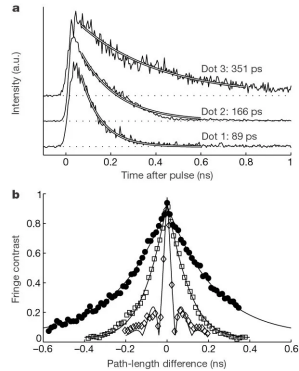
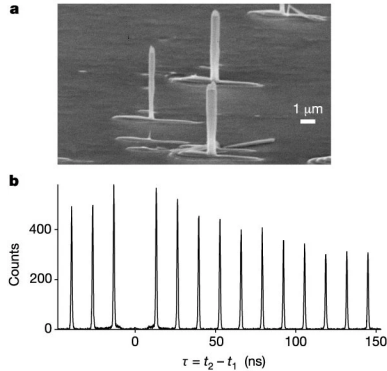
Since squeezed states are Gaussian states (with Gaussian statistics), 4 numbers characterize their full state: $\{V_{xx}, V_{xy}, V_{yx}, V_{yy}\}$

Definition of covariance matrix

$$V_{xy} = \frac{\langle \hat{X}\hat{Y} + \hat{Y}\hat{X} \rangle - 2\langle \hat{X} \rangle \langle \hat{Y} \rangle}{2\Delta\hat{X}_{vac}^2}$$

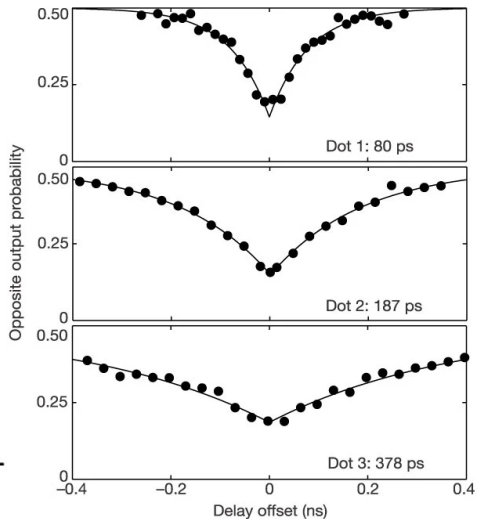
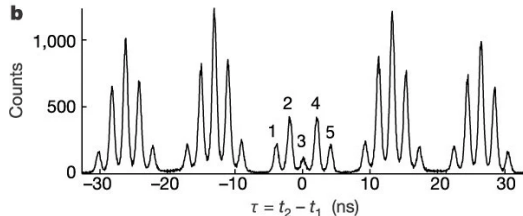
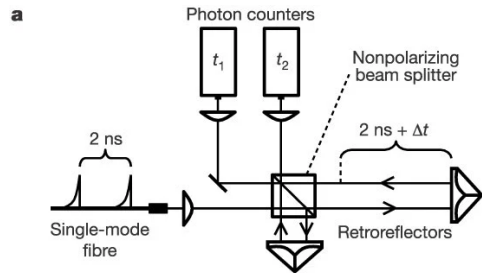
From covariance matrix one can determine the amount of entanglement ("Logarithmic negativity" : The logarithmic negativity is an entanglement measure which is easily computable and an upper bound to the distillable entanglement).

Indistinguishable Photons From a Single-Photon Device⁵



⁵Santori, C., Fattal, D., Vučković, J. et al. Indistinguishable photons from a single-photon device. *Nature* 419, 594–597 (2002)

Indistinguishable Photons From a Single-Photon Device



PHYSICAL REVIEW A **76**, 042319 (2007)

Charge-insensitive qubit design derived from the Cooper pair box

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Short dephasing times pose one of the main challenges in realizing a quantum computer. Different approaches have been devised to cure this problem for superconducting qubits, a prime example being the operation of such devices at optimal working points, so-called “sweet spots.” This latter approach led to significant improvement of T_2 times in Cooper pair box qubits [D. Vion *et al.*, *Science* **296**, 886 (2002)]. Here, we introduce a new type of superconducting qubit called the “transmon.” Unlike the charge qubit, the transmon is designed to operate in a regime of significantly increased ratio of Josephson energy and charging energy E_J/E_C . The transmon benefits from the fact that its charge dispersion decreases exponentially with E_J/E_C , while its loss in anharmonicity is described by a weak power law. As a result, we predict a drastic reduction in sensitivity to charge noise relative to the Cooper pair box and an increase in the qubit-photon coupling, while maintaining sufficient anharmonicity for selective qubit control. Our detailed analysis of the full system shows that this gain is not compromised by increased noise in other known channels.

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Questions for this week's paper

- What's the expression for Wigner function the authors use?
- What measurement data is taken and how is it used to reconstruct the Wigner function?
- How does measurement efficiency impact the result?
- How does signal-LO mode-matching influence efficiency in homodyne detection?
- Why a single laser is used for both the local-oscillator and the signal?
- Why do they use a doubler and down-converter as a source of single photons?
- How does the spatial-temporal pulse shape of single photon match LO?
- How is the density matrix reconstructed? What is the physical meaning of the off-diagonal values of the density-operator?