Quantum State Reconstruction of the Single-Photon Fock State

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Introduction

- measurement of the complete Wigner function of the single-photon state $|1\rangle$
- method : homodyne tomography

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Quantum State Reconstruction of the Single-Photon Fock State

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We have reconstructed the quantum state of optical pulses containing single photons using the method of phase-randomized pulsed optical homodyne tomography. The single-photon Fock state $|1\rangle$ was prepared using conditional measurements on photon pairs born in the process of parametric down-conversion. A probability distribution of the phase-averaged electric field amplitudes with a strongly non-Gaussian shape is obtained with the total detection efficiency of $(55 \pm 1)\%$. The angle-averaged Wigner function reconstructed from this distribution shows a strong dip reaching classically impossible negative values around the origin of the phase space.

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Optical Quadratures

- definition :

$$\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger) \qquad [\hat{X}_1, \hat{X}_2] = \frac{i}{2}$$
$$\hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$$

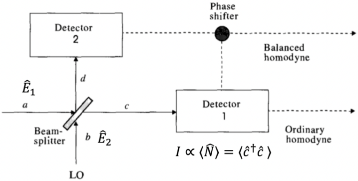
- or more generally :

$$\hat{X}_\varphi = \hat{a}e^{i\varphi} + \hat{a}^\dagger e^{-i\varphi} \qquad [\hat{X}_\varphi, \hat{X}_{\varphi+\frac{\pi}{2}}] = 2i$$

Balanced Homodyne detection

- 50:50 beam splitter
- mode $\hat{b}$: $|\beta\rangle = \left||\beta|e^{i\varphi}\right\rangle$
- output operators :

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b})$$
$$\hat{d} = \frac{1}{\sqrt{2}}(\hat{a} - \hat{b})$$



- Difference in photocurrent :

$$I_c - I_d \propto \left\langle \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} \right\rangle = |\beta| \underbrace{\langle \hat{a} e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi} \rangle}_{\propto \langle \hat{X}_\varphi \rangle}$$

Wigner Function

- phase-space quasi-probability density
- uniquely defines the state
- Definition :

$$W(p,q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ipx} \left\langle q - \frac{x}{2} \middle| \hat{\rho} \middle| q + \frac{x}{2} \right\rangle dx$$

- Marginal distributions :

$$|\psi(p)|^2 = \int_{-\infty}^{\infty} W(p,q) dq$$
$$|\psi(q)|^2 = \int_{-\infty}^{\infty} W(p,q) dp$$

Wigner Function

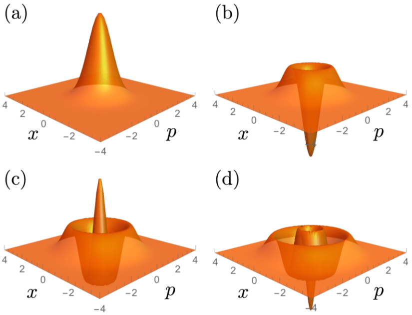


Figure 1: Wigner Functions of the four fock states. (a) n = 0 , (b) n = 1 , (c) n = 2 , (d) n = 3.

Reference: Andreas Ketterer (Oct. 2016). "Modular variables in quantum information". PhD thesis

Wigner Function

- Physical intuition / meaning of Wigner Function :

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Wigner function as the expectation value of a parity operator

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(Received 30 August 1976)

It is pointed out that the Wigner function $f(r, p)$ is $2/h$ times the expectation value of the parity operator that performs reflections about the phase-space point r, p. Thus $f(r, p)$ is proportional to the overlap of the wave function ψ with its mirror image about r, p; this is clearly a measure of how much ψ is *centered* about r, p, and the Wigner distribution function now appears physically more meaningful and natural than it did previously.

Wigner Function

- Wigner function :

$$f(r,p) = \frac{2}{h}\int \mathrm{d}s \exp\frac{-2ips}{\hbar}\psi^*(r-s)\psi(r+s)$$
$$= \frac{2}{h}\int \mathrm{d}k \exp\frac{-2ikr}{\hbar}\tilde{\psi}(p+k)\tilde{\psi}^*(p-k)$$

- Operator Π_{rp} :

$$\Pi_{rp} = \int \mathrm{d}s \exp\frac{-2ips}{\hbar}\left|r-s\right\rangle\left\langle r+s\right|$$
$$= \int \mathrm{d}k \exp\frac{-2ikr}{\hbar}\left|p+k\right\rangle\left\langle p-k\right|$$

- we get :

$$f(r,p) = \frac{2}{h}\left\langle\psi|\Pi_{rp}|\psi\right\rangle$$

Wigner Function

- $r = 0,\ p = 0$

$$\Pi \equiv \Pi_{00} = \int \mathrm{d}r \,|-r\rangle \langle r| = \int \mathrm{d}p \,|p\rangle \langle -p|$$

- $D(r,p) \equiv \exp \frac{i}{\hbar}(p\hat{R} - r\hat{P})$

$$D^{-1}(r,p)\hat{R}D(r,p) = \hat{R} + r$$
$$D^{-1}(r,p)\hat{P}D(r,p) = \hat{P} + p$$

  and

$$\Pi_{rp} = D(r,p)\Pi D^{-1}(r,p)$$

- we get :

$$\Pi_{rp}(\hat{R} - r)\Pi_{rp} = -(\hat{R} - r)$$
$$\Pi_{rp}(\hat{P} - p)\Pi_{rp} = -(\hat{P} - p)$$

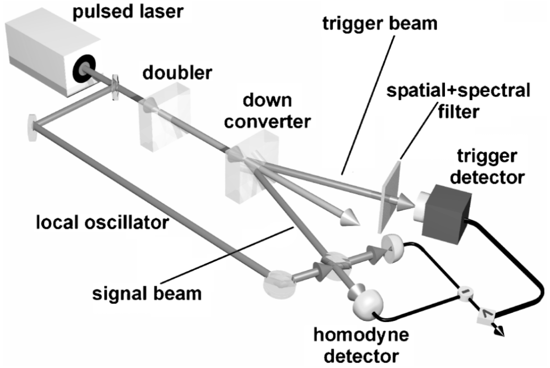
Wigner Function

$$f(r,p) = \frac{2}{h} \langle \psi | \Pi_{rp} | \psi \rangle$$

- measure of how much ψ is centered about (r, p)

Preparation of the single-photon state

- Experimental setup :



Preparation of the single-photon state

- Two-photon down conversion :

$$|\Psi\rangle = N\left(|0,0\rangle + \int d\vec{k}_s d\vec{k}_t \Phi(\vec{k}_s, \vec{k}_t) \left|1_{\vec{k}_s}, 1_{\vec{k}_t}\right\rangle\right)$$

- State ensemble selected by the trigger :

$$\hat{\rho}_t = \int d\vec{k}_t \, T(\vec{k}_t) \left|1_{\vec{k}_t}\right\rangle \left\langle 1_{\vec{k}_t}\right|$$

  where $T(\vec{k}_t)$ is the transmission function of the filter.
- Signal state :

$$\hat{\rho}_s = \mathrm{Tr}_t[|\Psi\rangle \langle\Psi| \, \hat{\rho}_t]$$

- Tight filtering : pure single-photon state

$$\hat{\rho}_s = |1\rangle \langle 1|$$

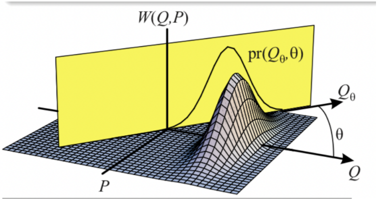
Measurement of the single-photon state

- perfect experiment : $\hat{\rho}_{\text{meas}} = |1\rangle \langle 1|$
- inneficiencies : $\hat{\rho}_{\text{meas}} = \eta |1\rangle \langle 1| + (1 - \eta) |0\rangle \langle 0|$
  where η the measurement efficiency.
- Balanced homodyne detection : $\Delta I \propto \hat{X}_\theta \equiv \hat{X} \cos\theta + \hat{P} \sin\theta$
- For each phase θ , large number of measurement to get $\text{pr}(\hat{X}_\theta)$

Reconstruction of the single-photon state

- Wigner function :

$$\mathrm{pr}(\hat{X}_\theta) = \int_{-\infty}^{\infty} W(X\cos\theta - P\sin\theta, X\cos\theta + P\sin\theta)dP$$



- WF can be reconstructed from $\mathrm{pr}(\hat{X}_\theta)$ for a large number of θ

Reconstruction of the Wigner function

- θ varied randomly
- single phase-randomized marginal distribution

$$\mathrm{pr}_{\mathrm{av}}(X) = \left\langle \mathrm{pr}(\hat{X}_\theta) \right\rangle_\theta$$

- fine for rotationally symmetric Wigner functions
- phase-averaged Wigner function :

$$W(R) = \frac{-1}{\pi} \int_R^\infty \frac{d\mathrm{pr}_{\mathrm{av}}(X)}{dX} (X^2 - R^2)^{-1/2} dX$$

Reconstruction of the density matrix

- diagonal elements in Fock basis :

$$\rho_{nn} = \pi \int_{-\infty}^{\infty} \mathrm{pr}_{\mathrm{av}}(X) f_{nn}(X) dX$$

where $f_{nn}(X)$ are the amplitude pattern functions (independent of optical state).

Reference: G. M. D'Ariano, U. Leonhardt, and H. Paul (Sept. 1995). "Homodyne detection of the density matrix of the radiation field". In: *Phys. Rev. A* 52 (3), R1801–R1804. DOI: 10.1103/PhysRevA.52.R1801. URL: https://link.aps.org/doi/10.1103/PhysRevA.52.R1801

Effect of measurement efficiency

- $\hat{\rho}_{\text{meas}} = \eta \left|1\right\rangle \left\langle 1\right| + (1-\eta) \left|0\right\rangle \left\langle 0\right|$

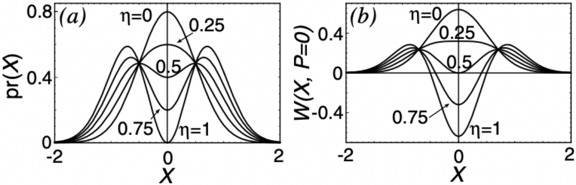


Figure 2: Effect of the nonperfect measurement efficiency η on the marginal distribution (a) and the reconstructed WF (b).

- negative values require $\eta > 0.5$

Results

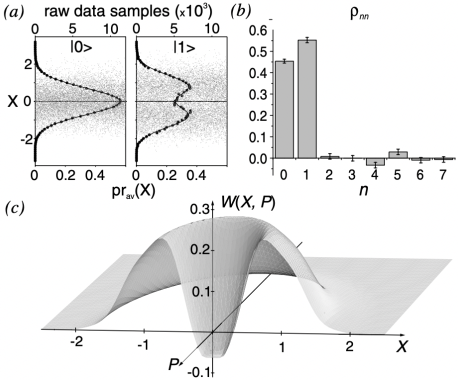


Figure 3: Experimental results : (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their his- tograms corresponding to the phase-randomized marginal distri- butions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.

Signal-LO mode-matching

- $\hat{\rho}_{\text{meas}} = \eta \left|1\right\rangle \left\langle 1\right| + (1-\eta) \left|0\right\rangle \left\langle 0\right|$
- Homodyne detection : interference between signal and LO fields at the BS
- overlap : spatial , temporal and spectral

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Effective quantum efficiency in the pulsed homodyne detection of a n-photon state

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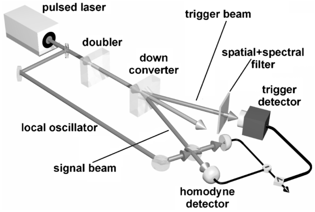
Abstract. Homodyne detection can be used to perform measurements on various quantum states of the light, such as conditional single photon states produced by parametric fluorescence processes. In the pulsed regime, the time and frequency overlap between the single photon wave packet and the local oscillator field plays a crucial role. We show in this paper that this overlap can be characterized by an effective quantum efficiency, which is explicitly calculated in various situations of experimental interest.

- effective quantum efficiency : $\eta_{\text{eff}} = \dfrac{\left|\langle \varepsilon_{\text{LO}} | \varepsilon \rangle\right|^2}{\langle \varepsilon_{\text{LO}} | \varepsilon_{\text{LO}} \rangle \langle \varepsilon | \varepsilon \rangle}$

Spectral mode matching : Use of the doubler

- signal photon has to have same frequency as local oscillator
- need to double the frequency before SPDC
- Second Harmonic Generation (SHG)



SHG

- Linear optics :

$$P = \epsilon_0 \chi^{(1)} E$$

- Non-linear optics :

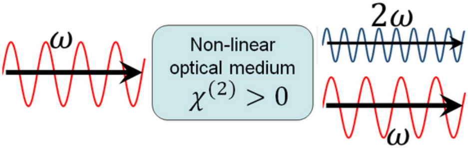
$$P_i = \epsilon_0 \left(\chi^{(1)}_{ij} E_i E_j + \chi^{(2)}_{ijk} E_i E_j E_k + \chi^{(3)}_{ijkl} E_i E_j E_k E_l + ... \right)$$

- monochromatic incoming wave :

$$E \propto \cos(\omega t - kx)$$

- Second order non-linear polarizaion :

$$P^{(2)} \propto \frac{1}{2} \epsilon_0 \chi^{(2)} (cos(2\omega t - 2kx) + 1)$$



Conclusion

- first quantum tomography measurement of a highly nonclassical state of the electromagnetic field
- Preparation of the single-photon state in a well-defined mode thanks to measurement on photon pairs
- reconstruction of the phase-averaged Wigner function and density matrix diagonal elements of a single-photon Fock state with 55% measurement efficiency
- non gaussian shape and negative values around the origin (signature of non-classiality)