Quantum Electrodynamics and Quantum Optics

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE (EPFL)

Exercise No.11

11.1 Weak continuous measurement with parametrically coupled resonant cavity

1. Since $\theta = 4A\omega_c\hat{\sigma}_z/\kappa$, we have

$$S_{\hat{\sigma}_z\hat{\sigma}_z} = S_{\theta\theta}/(4A\omega_c/\kappa)^2 \tag{1}$$

2. The SNR is gvien by

$$\mathrm{SNR} = \frac{\langle\hat{\sigma}_z\rangle^2}{\Delta\hat{\sigma}_z^2} = \frac{t}{S_{\hat{\sigma}_z\hat{\sigma}_z}} \tag{2}$$

and the measurement rate is given by

$$\Gamma_{\mathrm{meas}} = \frac{\partial\mathrm{SNR}}{2\partial t} = \frac{1}{2S_{\hat{\sigma}_z\hat{\sigma}_z}} \tag{3}$$

To get meaningful information, we have to measure for $t = 1/\Gamma_{\mathrm{meas}}$.

3. Given that $n = a^\dagger a = \dot{N}t_{WD} = 4\dot{N}/\kappa$ and $F = -\partial H/\partial\sigma_z = -(A\hbar\omega_c)\,n = -(A\hbar\omega_c t_{WD})\,\dot{N}$, we obtain

$$\Gamma_\varphi = \frac{2}{\hbar^2}S_{FF} = \frac{2}{\hbar^2}(A\hbar\omega_c t_{WD})^2 S_{\dot{N}\dot{N}} = 2(A\omega_c t_{WD})^2 S_{\dot{N}\dot{N}} \tag{4}$$

4. From the results of previous questions, we find

$$\frac{\Gamma_\varphi}{\Gamma_{\mathrm{meas}}} = \frac{2(A\omega_c t_{WD})^2 S_{\dot{N}\dot{N}}}{1/2S_{\sigma_z\sigma_z}} = \frac{2(A\omega_c t_{WD})^2 S_{\dot{N}\dot{N}}}{1/2(A\omega_c t_{WD})^{-2} S_{\theta\theta}^{-1}} = 4S_{\dot{N}\dot{N}}S_{\theta\theta} = 1 \tag{5}$$