

PHYS-448 Introduction to Particle Accelerators

Tutorial 4

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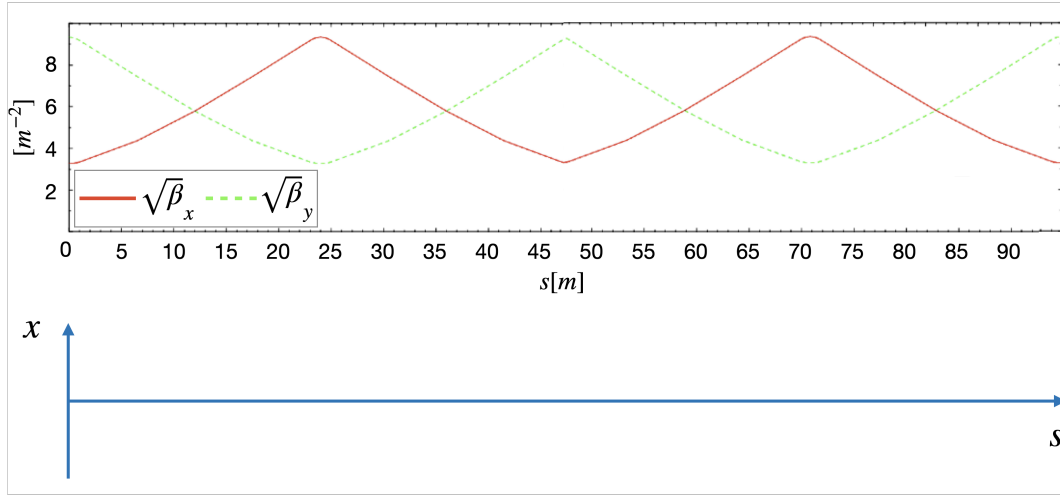
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Exercise 1. Consider the initial transverse coordinates in the horizontal plane (x_0, x'_0) of a particle. Assuming the thin lens approximation:

- Calculate the final coordinates of the particle passing through a drift of length s . Create a drawing to illustrate the transformation of the phase space ellipse, starting from the beam waist and continuing along the drift. (Hint: the beam waist occurs when the beam size is minimum)
- Calculate the final coordinates of the particle passing through a focusing quadrupole. How does it change if the quadrupole is defocusing? Create a drawing to illustrate the evolution of the phase space, starting from a diverging beam and passing through a focusing quadrupole.

Exercise 2.

- Can you explain in your own words the meaning of:
 - β -function
 - Emittance
 - Phase advance
 - Betatron tune
- Concerning the two parameters β -function and beam emittance ϵ , they both determine the beam envelope. Can you explain the difference?
- Consider the FODO structure in the figure: The phase advance per cell is 90 degrees and the red and green curves are the horizontal and vertical β -functions respectively. Can you draw in the plot below two particle trajectories that propagate through the two cells, one starting with $x = 0, x' > 0$ and the other one starting with $x > 0, x' = 0$?



- d) Assume somewhere in the storage ring there is a position where $\alpha = 0$. Where should such a situation occur typically? How will the phase space ellipse look like? Can you give a physical interpretation of the β -function at such place? (Hint: find the relation between the β -function and physical parameters: beam size and beam divergence)

inspired by an exercise from CAS school 2014

Exercise 3. Let's compare thick and thin lenses numerically now! **In this exercise calculate matrix arrays with five-digit precision after the decimal point.**

- Evaluate the matrix for a (thick) quadrupole magnet of length 3.05 m with a field gradient of $\partial B_y / \partial x = 250 \text{ T m}^{-1}$, given a traversing particle with a momentum of $p_0 = 20 \text{ TeV}/c$. (Hint: The normalised strength of a quadrupole for a particle of positive charge yields $k = \frac{1}{B\rho} \frac{\partial B}{\partial x}$)
- Compare with the product of thin lens matrices over the same length and explore different configurations of thin lenses at varying positions. Compute the matrix for two thin lenses (e.g. one with $f' = 2f$ at the beginning and an equally strong lens at the end, separated by a drift of length $L = 3.05 \text{ m}$) – what happens when you increase the number of thin lenses?

Exercise 4. The transfer matrix for one turn around a storage ring is:

$$M_{rev} = \begin{bmatrix} 4 & 170 \\ -\frac{1}{10} & -4 \end{bmatrix}$$

Evaluate the fractional part of the tune and the Twiss parameters (α , β and γ) at this point in the lattice. What is the matrix for two turns? And for four turns? Can you make any conclusion out of it?