

Introduction to Particle Accelerators, Tutorial 11

EPFL – *Teaching Assistants:*

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Exercise 1. Consider a storage ring composed of $N = 24$ identical FODO cells with the parameters given in Tab. 1.

Parameter		Value	
Half-cell length	l_d	6	m
Quadrupole length	l_q	0.705	m
Quadrupole strength	k	0.3659	m^{-2}

Table 1: FODO parameters.

- (a) Is the transverse particle motion in such a structure stable? Justify your answer.
- (b) Calculate the betatron tunes of the machine for the given parameters. Assume equal quadrupole strengths in both planes.
- (c) Consider the thin-lens approximation and calculate the values of the β -function at the center of the focusing and defocusing quadrupoles.

Exercise 2. The LHC lattice in the arcs consists of a symmetric FODO structure with an elliptical pipe cross-section with physical aperture dimensions of $r_x = 22$ mm and $r_y = 18$ mm. Table 2 provides typical values of the main optics parameters in the arcs at injection energy. All values are given in units of m. The total energy at injection is 450 GeV and the proton rest mass is $938.272 \text{ MeV}/c^2$.

- (a) Where are the minima and maxima of β_x , β_y , and D_x ?
- (b) Calculate the horizontal and vertical acceptance at injection energy considering monochromatic beam.

Max. β_x	Min. β_x	Max. β_y	Min. β_y	Max. D_x	Min. D_x
172.3	14.8	176.5	13.2	2.4	-0.1

Table 2: Main optics parameters at injection energy. All values in units of m.

- (c) Compute the maximum number of rms beam sizes that fit in the vacuum chamber for both transverse planes at injection energy. Consider an energy spread of $\sigma_\delta = 1.129 \times 10^{-4}$. The normalized transverse beam emittance is $\epsilon_n = 2.5 \mu\text{m rad}$ in both planes.

Exercise 3. Consider a 5 TeV electron storage ring built around the Earth's equator (diameter $D = 11000 \text{ km}$). Assume that the storage ring is completely filled with bending magnets.

- Calculate the momentum, Lorentz factor γ , the electron velocity β , and the revolution period T_0 of the beam.
- Estimate the magnetic field strength in the bending magnets.
- Calculate the energy loss per turn U_0 due to synchrotron radiation.
- Compute the transverse damping time in seconds and in number of turns.

A more moderate proposal: use only the Earth's magnetic field ($B = 3 \times 10^{-5} \text{ T}$, with direction parallel to the Earth's rotational axis) instead of bending magnets and let the electrons circulate in the same vacuum pipe as above.

- Determine the new energy of the electron beam.
- Evaluate again the energy loss per turn due to synchrotron radiation.
- Calculate the critical energy of the synchrotron radiation photons.
- Assuming an energy acceptance of the machine of $\Delta E/E = 1 \%$, how many full revolutions will the beam perform if no RF acceleration is provided?

Exercise 4. Protons are accelerated in a two-gap cyclotron whose magnet has a diameter of $D = 2 \text{ m}$ and a magnetic dipole field of $B = 1 \text{ T}$.

- What is the maximum kinetic energy that can be achieved with this accelerator?
- Assume that the maximum energy is achieved after 100 turns. What is the energy gain at every passage through the cavity gap?
- A proton with an orbit of $R = 0.5 \text{ m}$ is synchronous with the RF system. How many times did the proton travel across the RF gap?
- What is the oscillation period of the RF system?

Exercise 5. An electron beam of 5 GeV is sent through a long 2 T bending magnet, and through an undulator with a period of 15 mm. At the given magnetic gap (separation of undulator jaws), the undulator provides a magnetic field of 0.5 T on axis.

- (a) What is the critical photon wavelength of the synchrotron radiation emitted in the bending magnet?
- (b) What is the main frequency of the synchrotron radiation produced in the undulator in the forward direction?
- (c) Describe qualitatively what happens to the photon energy (of the main harmonic) when we increase the undulator gap (*hint: how does the magnetic field of the undulator change?*).
- (d) You would like to produce a broader spectrum of synchrotron radiation and hence want to turn your undulator ($K \leq 1$) into a wiggler ($K > 1$). How could you potentially achieve that?