Broad topic	**Lecture title**
Basic principles of NPP	Introduction / Review of nuclear physics
	Interaction of neutrons with matter
	Nuclear fission
	Fundamentals of nuclear reactors
	LWR plants
Modeling the beast	The diffusion of neutrons - Part 1
	The diffusion of neutrons - Part 2
	Neutron moderation without absorption
	Neutron moderation with absorption
	Multigroup theory
	Element of lattice physics
	Neutron kinetics
	Depletion
Reactor Concepts Zoo	Advanced LWR technology
	Breeding and LFR
	AGR, HTGR
	Channels, MSR and thorium fuel
Review session	

THIS LESSON…

- Background
- Formulation of the depletion problem
  - Formulation of the Bateman depletion equations
  - Matrix exponential solution
- Effects of burnup on neutronics and reactor operation
  - Reactivity
  - Reactor safety
  - Fission product poisons
  - Decay heat and radioactive inventory
- Burnup algorithms
  - Burnup calculation as an example of a non-linear coupled problem
  - Explicit Euler and predictor-corrector methods

BACKGROUND

- A single LWR fuel assembly spends 3 to 4 years in the reactor. During this time, the neutronics properties of the fuel change due to various changes in the isotopic composition:
  - U-235 is depleted and replaced by Pu-239 as the primary fissile isotope
  - Non-fissile plutonium and minor actinides and fission products are accumulated in the fuel
  - Burnable absorber used for reactivity control is depleted
- These changes are directly reflected in reactivity and the safety parameters of the reactor core.
- The accumulation of radioactive fission products also forms the source term for accident analyses and final disposal of spent fuel.
- Fuel utilization is measured in units of burnup, which refers to the amount of extracted energy per uranium or heavy metal mass (for example, 40 MWd/kgU).

- When a material consisting of several nuclides is subjected to neutron irradiation, the changes in the nuclide composition over time are characterized by the Bateman depletion equations:

$$\frac{dN_j}{dt} = \underbrace{\sum_{i \neq j} S_{i \to j}}_{(A)} - \underbrace{\lambda_j N_j}_{(B)} - \underbrace{\Phi \sigma_j N_j}_{(C)} \qquad (1)$$

- Where:
  - ➢ A is the production rate of nuclide *j* from nuclide *i* by decay transmutation and fission
  - ➢ B is the decay rate of the nuclide *j*
  - ➢ C is the total transmutation and fission rate of nuclide *j*

- The source term can be written as the sum of decay transmutation and fission terms:

$$S_{i \to j} = \lambda_{i \to j} N_i + \Phi \sigma_{i \to j} N_i + \Phi \gamma_{i \to j} \Sigma_{f,i}$$

- The RHS of the source term couple the production rate of nuclide *j* with the loss rate of nuclide *i*.
- Obtaining the neutron-induced source and loss terms requires resolving the neutron flux, which couples the depletion problem to the transport problem, as will be discussed later on

- A typical unit for measurement of the energy released from the fuel per unit mass is the fuel burnup

- Fuel burnup:

(contributions of all fissionable isotopes)

$$B(t) = \frac{1}{\rho_{\text{fuel}}} \int_0^t \underbrace{\left(\sum_{i=1}^{\text{All Fissionables}} \underbrace{E_i}_{\text{Energy per Fission}} \underbrace{N_i \sigma_f^i \Phi(t')}_{\text{Fission Rate}} \right)}_{\text{Power generated per cm}^3} dt'$$

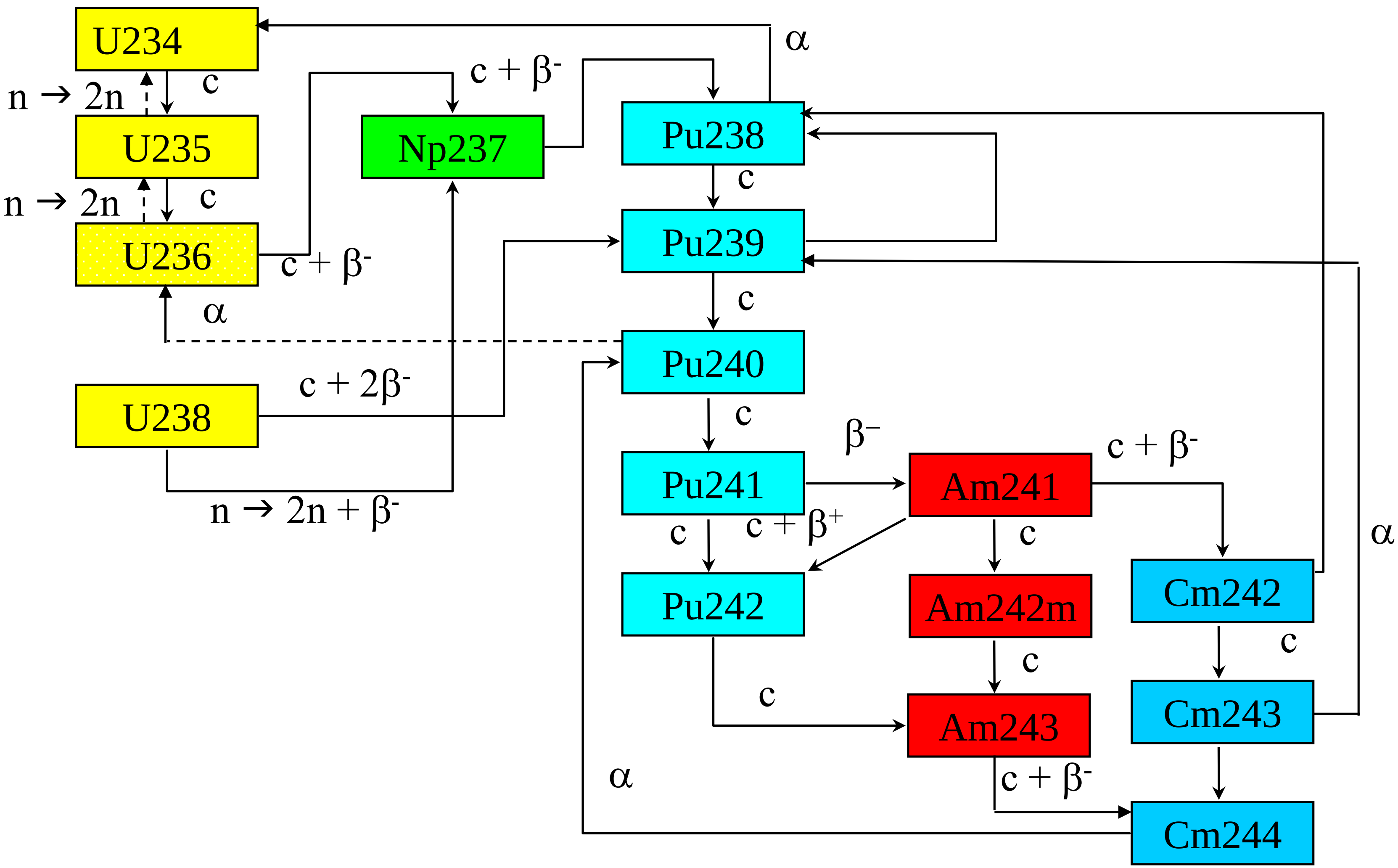
  – Units: J/g or MWd/t

- Using burnup instead of time to represent the residence of the fuel in the reactor is advantageous as it allows handling the variations in flux level over time.

- Average values:

$$\bar{B}(t) = \frac{1}{M_{\text{fuel}}} \int_0^t P(t')\, dt'$$

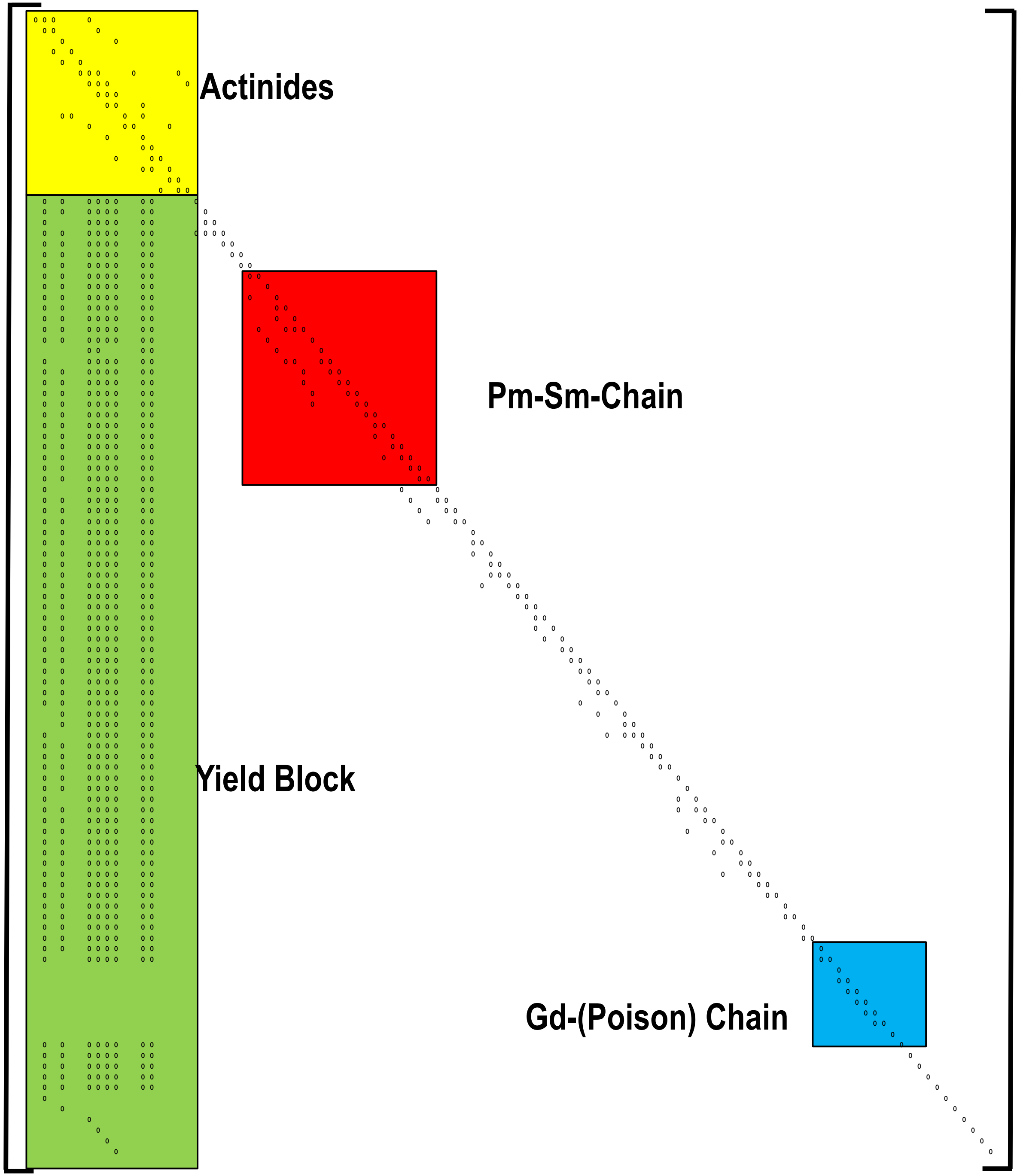
  - The thermal power P is a directly measurable quantity at the plant
  - One can express all parameters (nuclide composition, k_{eff}, reactivity coefficients , etc.) in function of Burnup…


U234
U235
U236
U238
Np237
Pu238
Pu239
Pu240
Pu241
Pu242
Am241
Am242m
Am243
Cm242
Cm243
Cm244
n → 2n
n → 2n
n → 2n + β-
c
c + β-
c + 2β-
c + β+
β−
α

- The Bateman depletion equations form a system of coupled first order differential equations, which can be written in matrix form:

$$\frac{dN}{dt} = \boldsymbol{A}N, \quad N(0) = N_0$$

- Where the matrix **A** contains the coefficients (loss terms on the diagonal, production terms off-diagonal) and the vector N contains the nuclide concentrations. The number of equations can reach 1500 to 2000 depending on the number of nuclides considered



Non-zero elements in a coefficient matrix for a system of ~ 100 nuclides, indexed according to decreasing isotope mass.
The vertical columns on the left are formed by the fission product distributions of actinides

Matrix exponential method

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- The solution of the Bateman depletion equations in matrix form can be written formally as:

$$N(t) = N_0 \exp(\mathbf{A}t)$$

- The matrix exponential can in principle be evaluated from a Taylor-series expansion (ORIGEN approach) :

$$N(t) = \exp(\mathbf{A}t)\, N_0 = \left(\mathbf{1} + \frac{1}{1!}\mathbf{A}t + \frac{1}{2!}(\mathbf{A}t)^2 + \frac{1}{3!}(\mathbf{A}t)^3 + \ldots.\right) N_0$$

- The problem is that A has very difficult numerical characteristics:
  - Its size
  - Its norm ~ 10^{21}
  - Magnitude of eigenvalues varies between 0 and ~ 10^{21}
  - Time steps, t varies from 10 to 10^6 s
- Other (better) approaches have been developed recently (CRAM method)

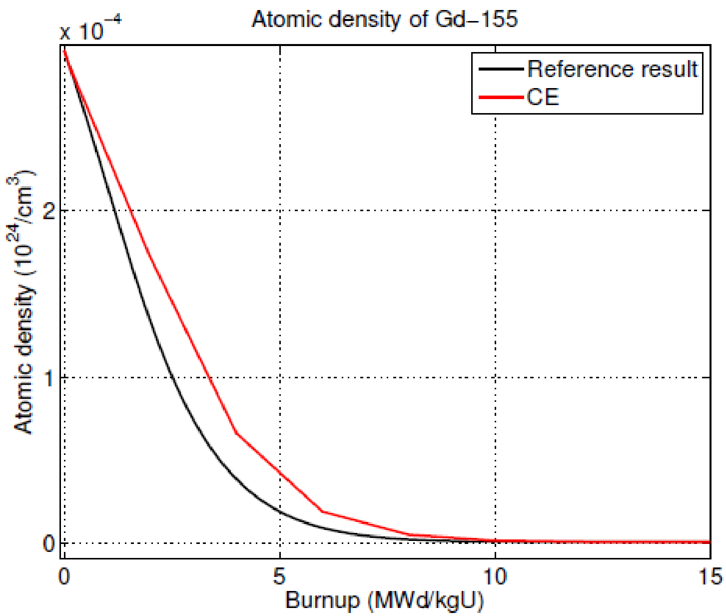
Non-linearity of the depletion problem

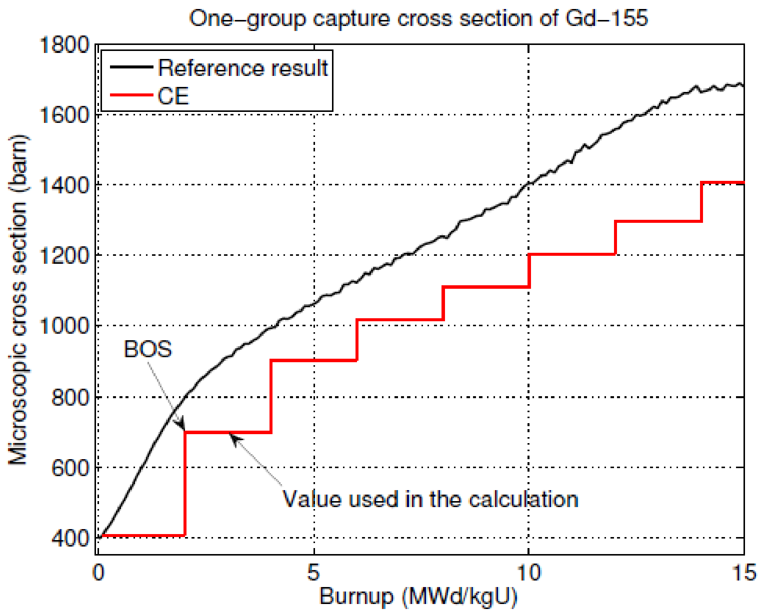
- In the formulation of the Bateman depletion equations it was assumed that neutron-induced (microscopic) reaction rates remain constant in time. A similar assumption is made for the (macroscopic) cross sections in the formulation of the neutron transport equation. In reality, neither is true, because:
  - Changes in flux spectrum are reflected in the spectrum-averaged one-group cross sections used for solving the depletion problem.
  - Changes in nuclide concentrations are reflected in the macroscopic cross sections used for solving the transport problem.
- The result is that the coupling of two linear problems forms a non-linear system
- There exists methods for solving the coupled equations as a single problem, but in practice the usual approach is to divide the time interval into discrete depletion steps and apply operator splitting:
  - Transport problem is solved assuming that reaction rates remain constant over the time interval. The calculation produces flux spectrum, which is used for calculating the microscopic transmutation cross sections for the depletion equations.
  - Depletion problem is solved assuming that flux spectrum remains constant over the time interval. The calculation produces material compositions for the next transport solution.

Time-step discretization: Euler method

- The simplest depletion algorithm is the explicit Euler method, in which the beginning-of-step (BOS) flux spectrum and reaction rates are used directly for solving the depletion equations.

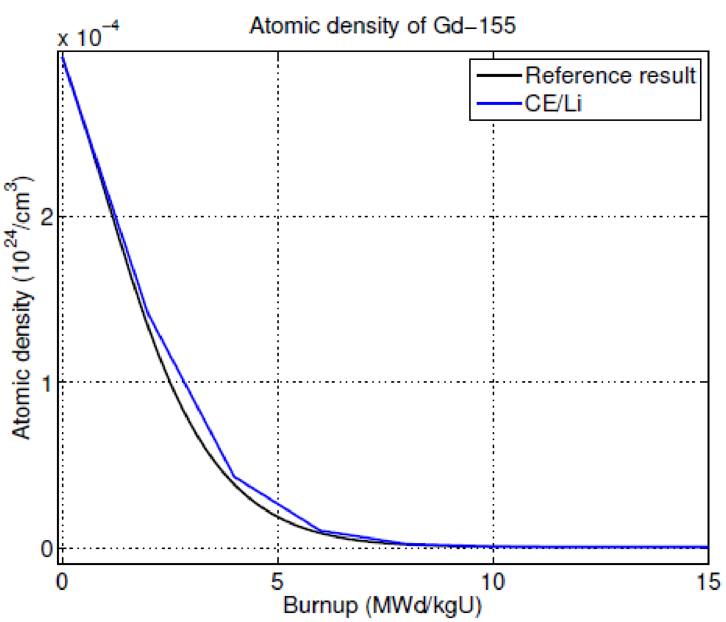
Gd-155 depletion in a PWR assembly

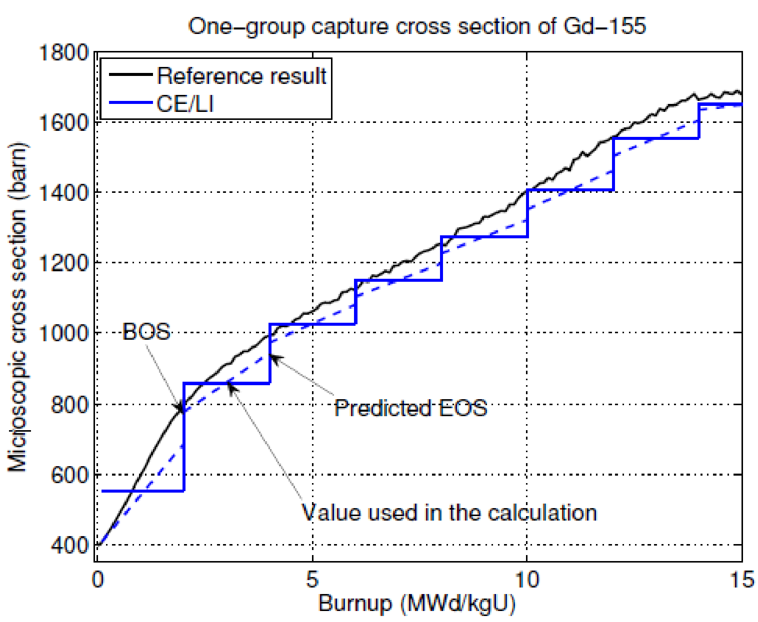




Time-step discretization: Predictor-corrector method

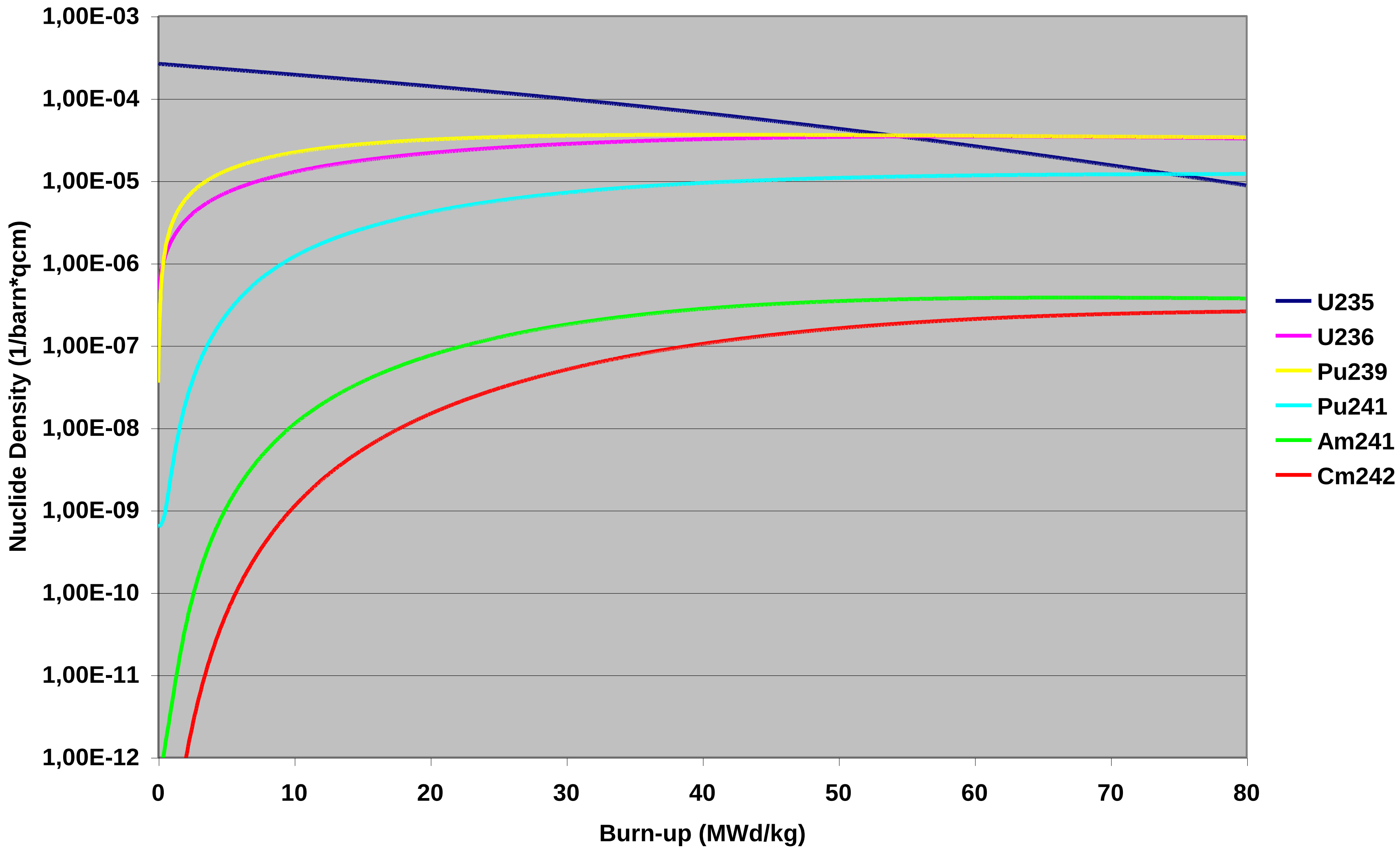
- The changes in the neutron-induced reaction rates within the step can be accounted for to some extent, by using constant values that represent the average behavior, rather than the BOS state.
- This is the basis of predictor-corrector methods:
  - 1) Predictor calculation – reaction rates are calculated for the BOS composition.
  - 2) Corrector calculation – the material is depleted over the interval, and new reaction rates calculated for the end-of-step (EOS) composition.
  - The final burnup calculation is carried out from BOS to EOS, using the average of the predictor and corrector reaction rates, which corresponds to linear interpolation between the two points.
- The method usually results in improved accuracy, but two transport solutions are required per time step resulting in larger computational costs



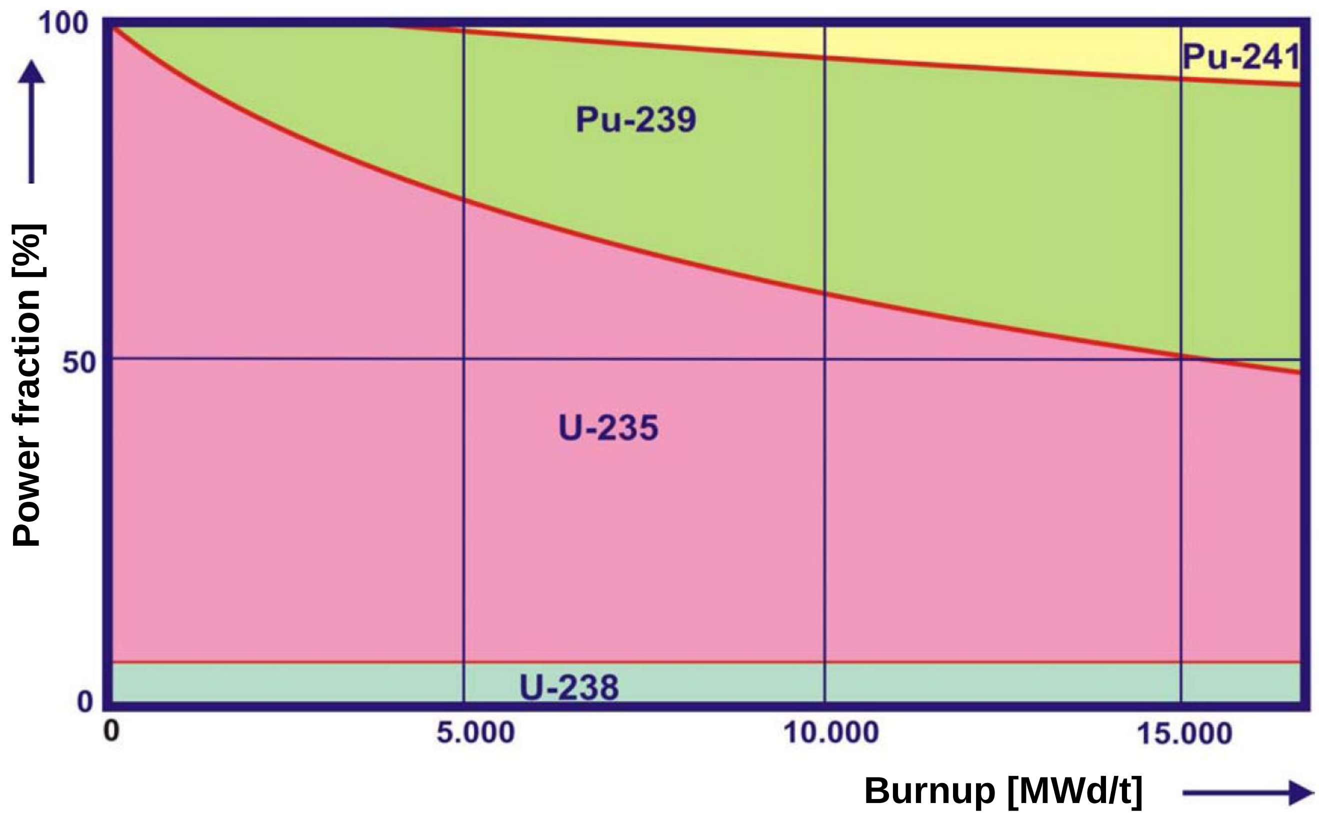


Actinide composition

- Actinide concentrations in a 4wt% enriched UO_2 in a LWR
  - K-inf decreases with exposure

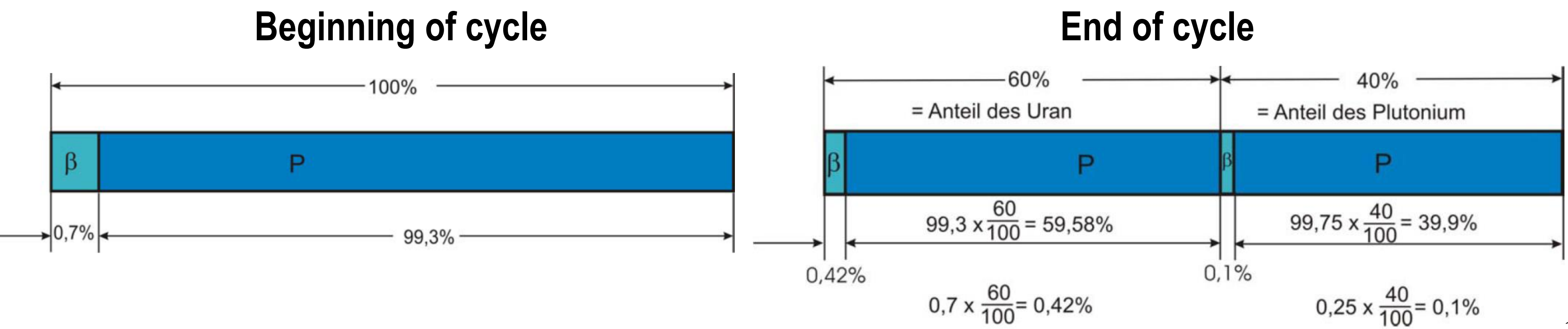


- Fission power evolution of the main uranium and plutonium isotopes for a PWR with respect to burnup



- Since Pu-239 is produced from U-238, mainly from captures in the low lying resonance range,
  - The amount of moderation has a significant impact in plutonium production.
  - In BWR's, more plutonium is produced for given fuel burnup in the upper part of the assembly

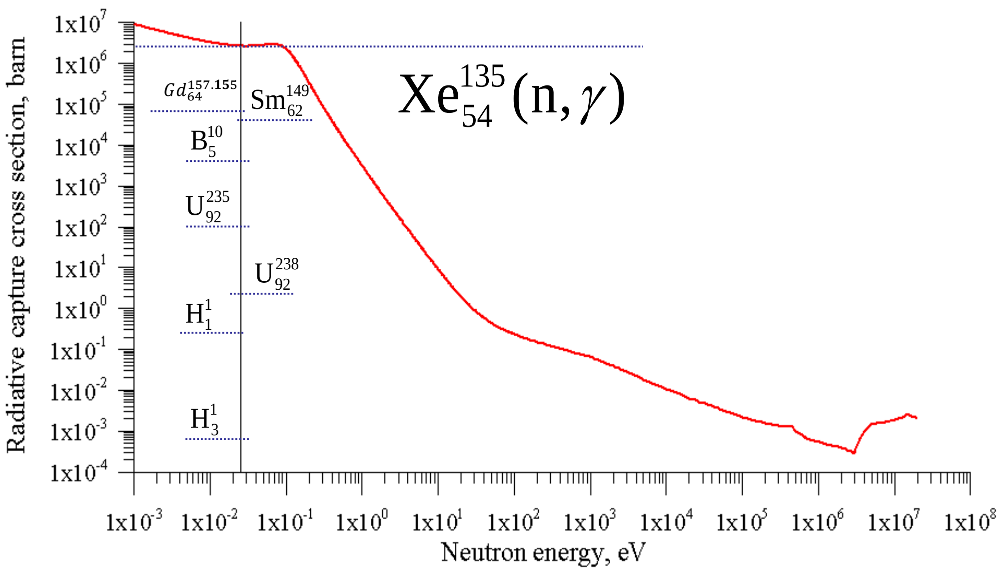
- The cross sections of plutonium isotopes are higher compared to uranium in the thermal energy range. Increasing parasitic absorption and the contribution of Pu-239 in total fission rate is also reflected in the safety characteristics:
  - Average number of prompt neutrons emitted in Pu-239 thermal fission is larger than for U-235
    - ✓ Effect on reactivity
  - High Pu-239 content lead to lower delayed neutron fraction
  - Reduction of CR and soluble boron worths due to spatial self-shielding and spectrum hardening.
  - General changes in reactivity feedback coefficients
    - ✓ complex underlying mechanisms



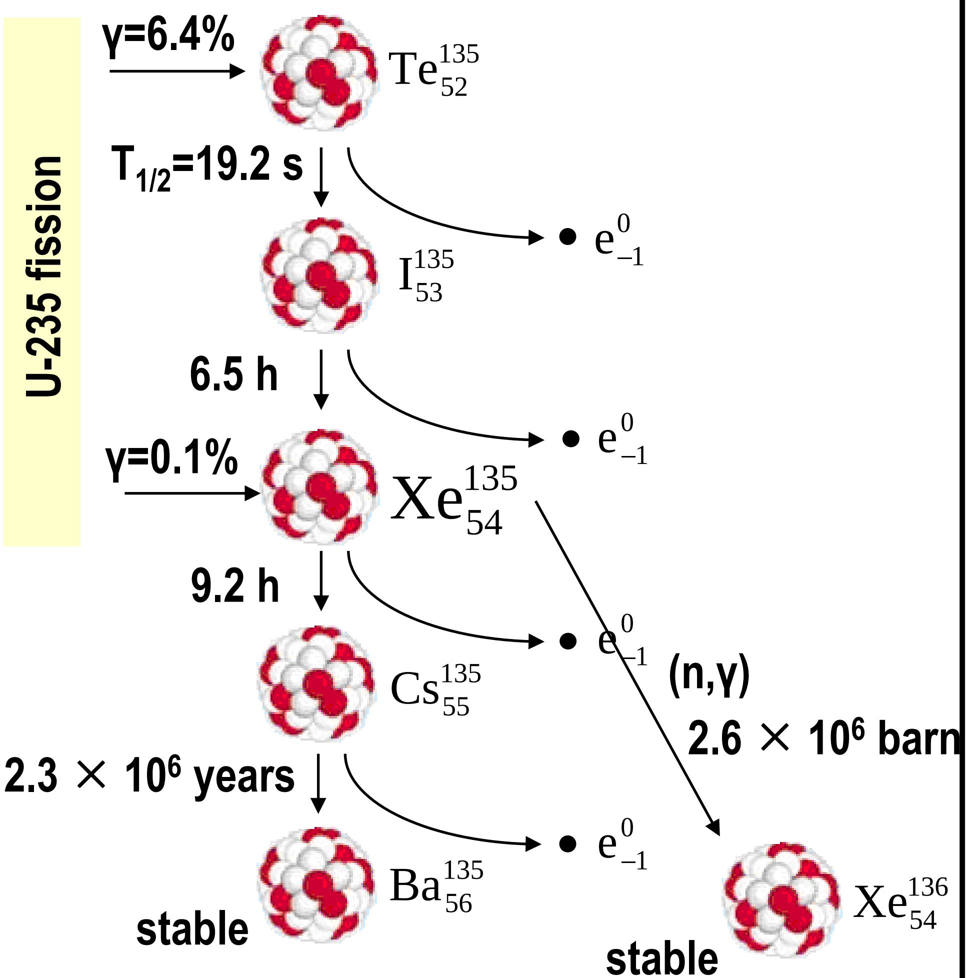
- Hundreds of intermediate-mass nuclides are produced in the fuel during irradiation by fission reactions.
  - ➢ Increase in parasitic neutron absorption
  - ➢ Similar deterioration of reactivity and spectrum-hardening than non-fissile actinides.

- Poison Fission Products: Xe-135 and Sm-149
  - ➢ Exceptionally high capture cross sections in the thermal region
  - ➢ Produced by fission and decay of precursor isotopes
  - ➢ During reactor operation, the dominant removal mechanism is neutron capture

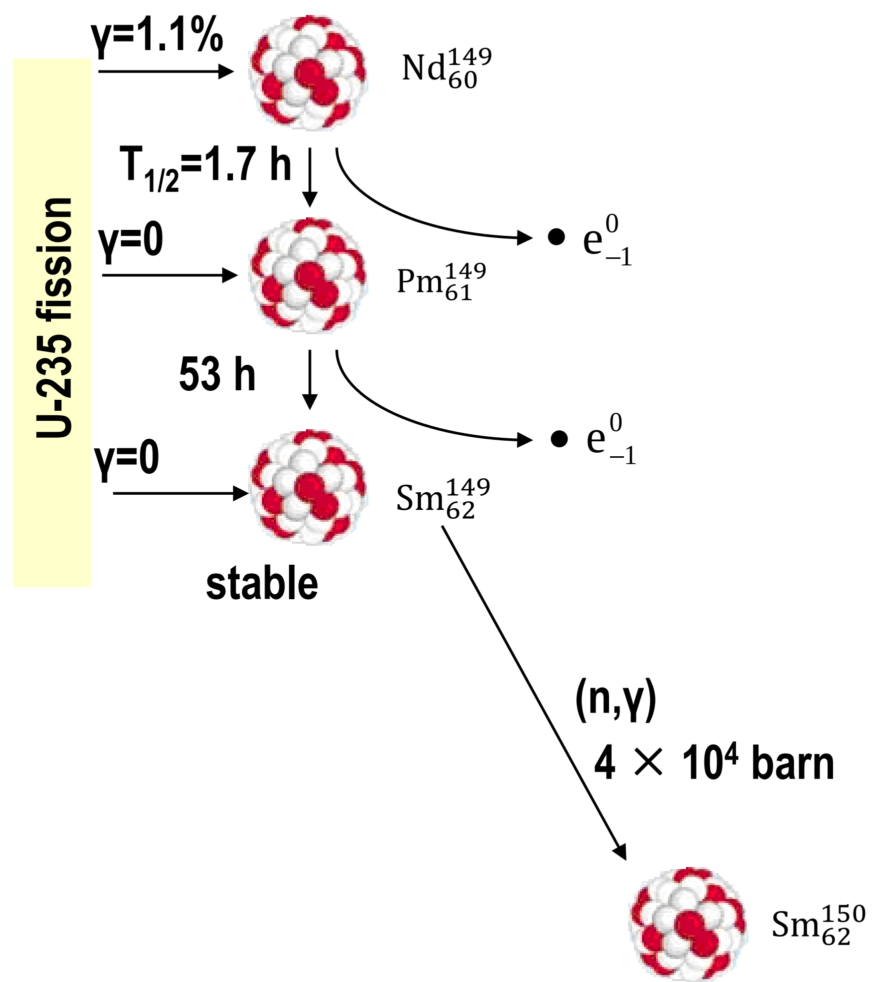
- Xe-135 decays into low-absorbing Cs-135 with a half-life of 9 hours
- Sm-149 is stable, and only removed by capture



- Xenon-135



- Samarium-149



Xenon Equilibrium

- By lumping the short-lived precursors (Te-135) in the fission yield of I-135
- By assuming the capture rate in I-135 is negligible, the Bateman equations can be written as:

$$\frac{dN_I(t)}{dt} = \gamma_I \bar{\Sigma}_f \Phi - \lambda_I N_I(t)$$

$$\frac{dN_{Xe}(t)}{dt} = \lambda_I N_I(t) + \gamma_{Xe} \bar{\Sigma}_f \Phi - \lambda_{Xe} N_{Xe}(t) - \bar{\sigma}_{a,Xe} \Phi N_{Xe}(t)$$

- ~ 2 - 3 days after start-up, one obtains an "equilibrium"

$$\frac{dN_I^\infty}{dt} = \gamma_I \bar{\Sigma}_f \Phi - \lambda_I N_I^\infty \overset{!}{=} 0$$

$$\frac{dN_{Xe}^\infty}{dt} = \lambda_I N_I^\infty + \gamma_{Xe} \bar{\Sigma}_f \Phi - \lambda_{Xe} N_{Xe}^\infty - \bar{\sigma}_{a,Xe} \Phi N_{Xe}^\infty \overset{!}{=} 0$$

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$$\frac{dN_I^\infty}{dt} = \gamma_I \overline{\Sigma}_f \Phi - \lambda_I N_I^\infty \overset{!}{=} 0 \quad \Rightarrow \quad N_I^\infty = \frac{\gamma_I \overline{\Sigma}_f \Phi}{\lambda_I}$$

$$\frac{dN_{Xe}^\infty}{dt} = \lambda_I N_I^\infty + \gamma_{Xe} \overline{\Sigma}_f \Phi - \lambda_{Xe} N_{Xe}^\infty - \overline{\sigma}_{a,Xe} \Phi N_{Xe}^\infty \overset{!}{=} 0 \quad N_{Xe}^\infty = \frac{\lambda_I N_I^\infty + \gamma_{Xe} \overline{\Sigma}_f \Phi}{\lambda_{Xe} + \overline{\sigma}_{a,Xe} \Phi} = \frac{(\gamma_I + \gamma_{Xe}) \overline{\Sigma}_f \Phi}{\lambda_{Xe} + \overline{\sigma}_{a,Xe} \Phi}$$

- In the 4 factor formula, the poison FPs mainly influence f

- Reactivity insertion due to Xe-135 buildup is

$$\Rightarrow \quad \rho = \frac{k - k_0}{k} \cong \frac{f - f_0}{f}$$

- Expressing f, it comes

$$f_0 = \frac{\Sigma_{ac}}{\Sigma_{ac} + \Sigma_{am}}, \quad f = \frac{\Sigma_{ac}}{\Sigma_{ac} + \Sigma_{am} + \Sigma_{aP}} \qquad \rho = -\frac{\Sigma_{aP}}{\Sigma_{ac} + \Sigma_{aM}} \quad \ldots (1)$$

- For a large reactor,
$$k_{eff} = k_{\infty} = 1 = \eta \cdot f \cdot p \cdot \varepsilon = \left(\bar{\nu} \frac{\Sigma_{fc}}{\Sigma_{ac}} \right) \cdot \left(\frac{\Sigma_{ac}}{\Sigma_{ac} + \Sigma_{aM}} \right) \cdot p \cdot \varepsilon$$
$$\Rightarrow \quad \Sigma_{ac} + \Sigma_{aM} = \Sigma_{fc} \cdot \bar{\nu} \cdot p \cdot \varepsilon \quad \text{... (2)}$$

- From (1) and (2),
$$\Rightarrow \quad \rho \cong -\frac{\Sigma_{aP} / \Sigma_{fc}}{\bar{\nu} p \varepsilon}$$

- At equilibrium,
$$\frac{N_{Xe}}{N_f} = \frac{(\gamma_I + \gamma_{Xe}) \bar{\sigma}_f / \sigma_{a,Xe}}{1 + \lambda_{a,Xe} / (\bar{\sigma}_{a,Xe} \Phi)}$$
$$\Rightarrow \rho = -\frac{\Sigma_{aP} / \Sigma_{fc}}{\bar{\nu} p \varepsilon} = -\frac{\sigma_{a,Xe} N_{Xe} / \sigma_f N_f}{\bar{\nu} p \varepsilon} = \frac{(\gamma_I + \gamma_{Xe})}{\bar{\nu} p \varepsilon \left(1 + \lambda_{a,Xe} / (\bar{\sigma}_{a,Xe} \Phi)\right)}$$

- Equilibrium-Xe effect depends on Φ.
$$\text{For } \Phi >> \frac{\lambda_{Xe}}{\sigma_{a,Xe}} \qquad \rho \to \frac{(\gamma_I + \gamma_{Xe})}{\bar{\nu} p \varepsilon} \quad (\text{Saturation})$$

- Ex. For a system with U-235 as fuel (p = ε = 1), $\rho_{\max} \cong -\frac{(0.061 + 0.003)}{2.42} = -2.6\%$

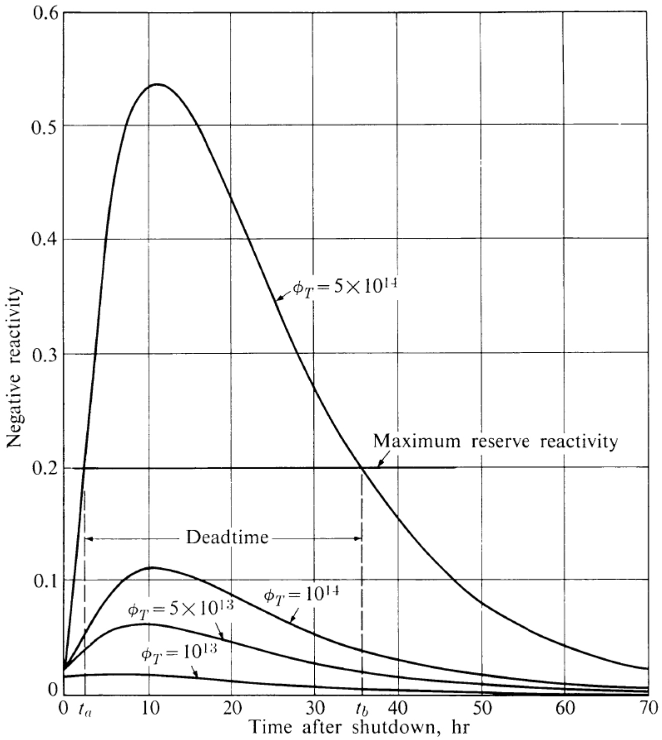
- By far, most important FP effect in a thermal reactor…

Xenon dead-time

- The temporal behavior of Xe-135 concentration in fuel is characterized by two physical factors:
  - ➢ The source term is dominated by the decay of I-135,
    - ✓ the production rate of Xe-135 follows changes in power level with a delay (precursor half-life - 6.6 h)
  - ➢ The loss term is dominated by neutron capture
    - ✓ loss rate of Xe-135 follows changes in power level instantaneously

- Increasing Xe-135 concentration lead to a condition where the negative reactivity exceeds the positive reactivity reserve, making start-up impossible until the isotope has decayed from the fuel.
- This period is known as the *Xenon dead time*

During reactor shutdown



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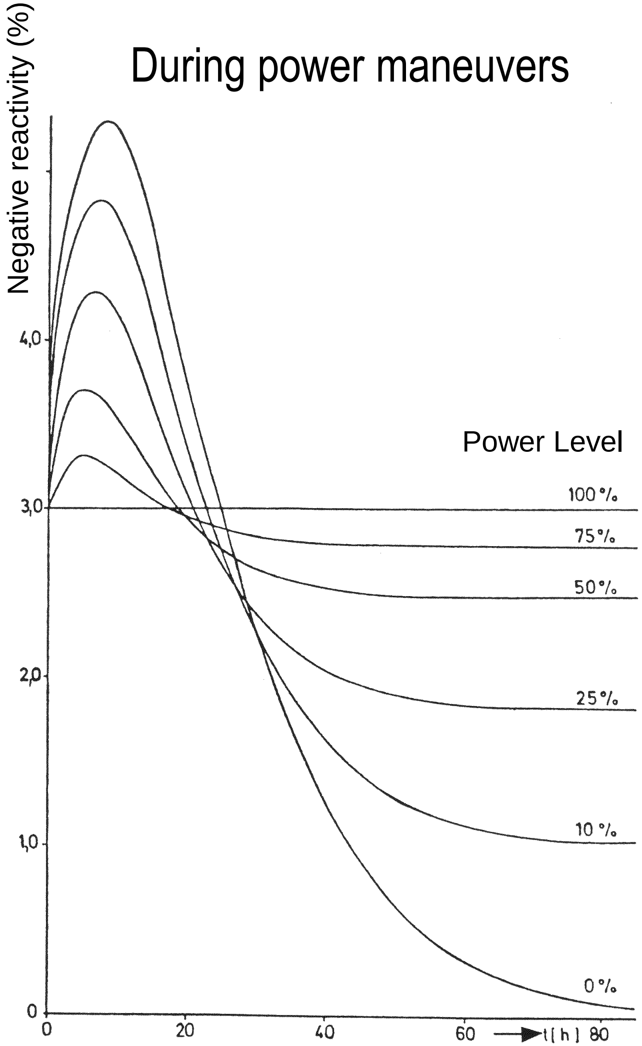
During power maneuvers



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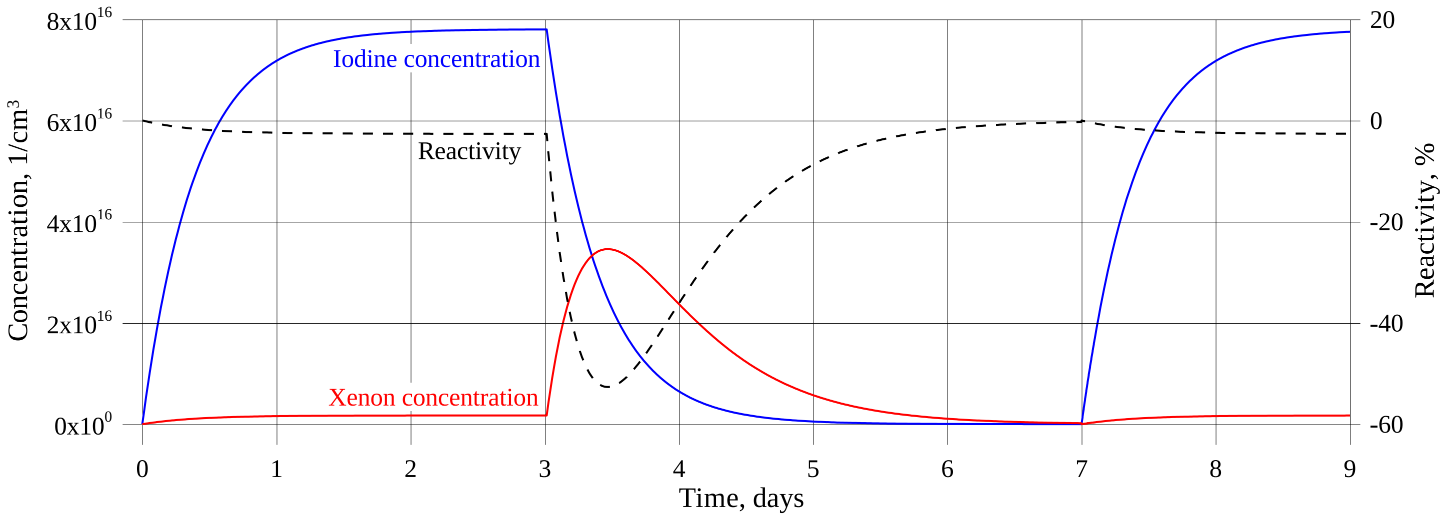
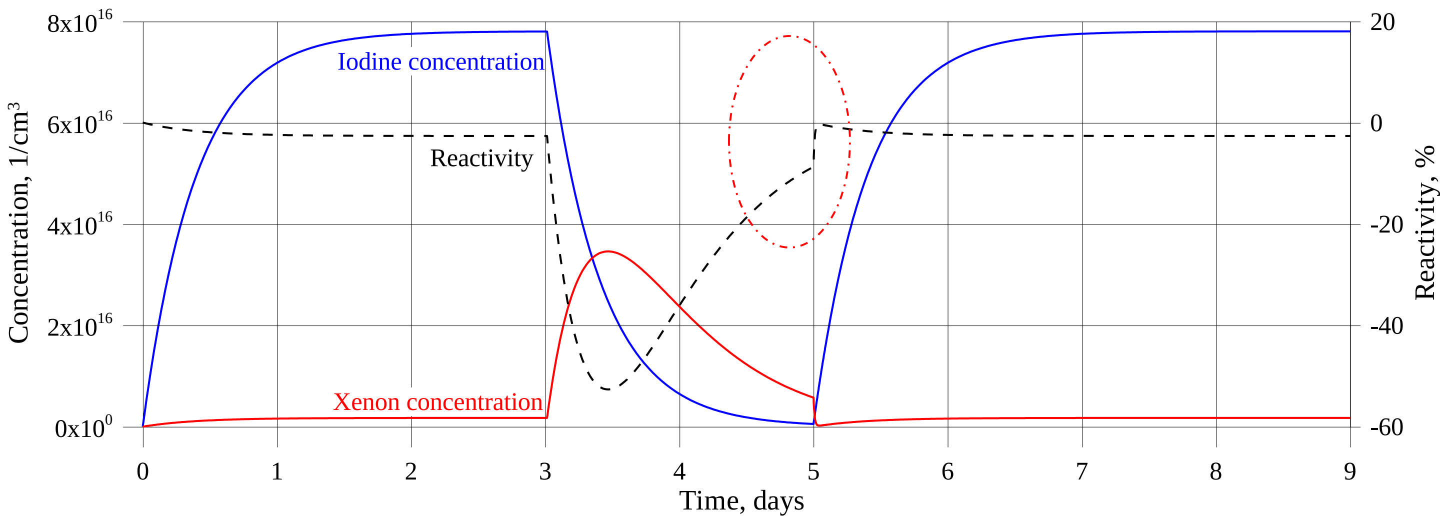
XENON CONCENTRATION DURING POWER MANEUVERS





SUMMARY

- Nuclear fuel is irradiated in the reactor core for an extended period of time, during which the physical characteristics change due to depletion of fissile uranium and build-up of plutonium and fission products.
- These changes cause various effects in reactivity and reactor safety parameters, which must be taken into account in reactor operation.
- The physical changes are caused by radioactive decay and neutron-induced transmutation and fission reactions. The evolution of isotopic composition is characterized by the Bateman depletion equations.
- The solution of depletion equations is a non-trivial problem due to the difficult numerical characteristics of the system
- The solution is typically based on the calculation of the matrix exponential
- The depletion problem is coupled to the transport problem by the changes in nuclide compositions and neutron flux.
- The coupled problem is non-linear, and the solution typically relies on operator splitting and consecutive solution of two linear problems. This introduces errors in the solution, which depend on the depletion algorithm and the step length used in the iteration.