SMALL CALCULATIONS ON THE CRITICALITY ACCIDENT AT TOKAI MURA

A.1.

$$N_H = 2N_{H2O} = 2 \times \frac{\rho_{H2O} \cdot N_A}{M_{H2O}} = 2 \times \frac{1 \times 6.022 \cdot 10^{23}}{18} = 6.69 \cdot 10^{22} cm^{-3}$$

- uranium aqueous solution: homogeneous medium with fissile material

- ε = 1, because of the one-group approximation so only the thermal neutrons are fissioning.

- p = 1, because we are considering all n to be thermal

-$f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{tot}} = \frac{eN_U\sigma_a^5}{eN_U\sigma_a^5 + eN_H\sigma_a^H} \rightarrow f_{5\%} = 0.2208, \;\; f_{20\%} = 0.5314$

-$\eta = \frac{\nu\Sigma^f}{\Sigma_a^{fuel}} = \frac{2.4 \times eN_U\sigma_f^5}{N_U\sigma_a^5} = 2.047$

$k_\infty = \varepsilon p f \eta \quad \rightarrow k_{\infty,5\%} = 0.451, k_{\infty,20\%} = 1.088 > 1$

The risk is limited in the dissolution column which is prone to high leakage.

A.2.

$$k_{eff} = \frac{k_\infty}{1 + L^2Bg^2} \;\; , valid \; if > 1$$

Bg: geometrical buckling
k_{eff}/k_∞ represents the fast and thermal leakages.
L: diffusion length of neutrons in solution.
L_H: diffusion length of neutrons in light water.

$$L^2 = \frac{D}{\Sigma_a^{tot}}, \; {L_H}^2 = \frac{D}{\Sigma_a^H} \Rightarrow \frac{L^2}{L_H^2} = \frac{\Sigma_a^{tot}}{\Sigma_a^H} = 1 - f \;\Rightarrow\; L^2 = L_H^2(1-f) = 3.69 \; cm^2$$

$$Bg^2 = \chi_R^2 + \chi_H^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\Pi}{H}\right)^2 = 593.8 \; m^{-2}$$

$$\Rightarrow \; k_{eff} = \frac{k_\infty}{1 + (1-f){L_H}^2Bg^2} = 0.310 \; (5\%), 0.892 \; (20\%)$$

B.1.

$V_t = 51522\ cm^3 = H \times \Pi R^2$

$V_c = 25133\ cm^3 = h_0 \times \Pi R^2 \ \Longrightarrow \dfrac{V_c}{V_t} = 0.4878 = \dfrac{h_0}{H} \Longrightarrow h_0 = 19.512\ cm$

$$k_{eff} = \frac{k_\infty}{1 + (1-f){L_H}^2 Bg^2}\ with\ Bg^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{h_0}\right)^2 = 403.84\ m^{-2}$$

So, with e = 20%, k_{eff} = 0.950

-"filling continues until 4/5..."

$\Longrightarrow h = \dfrac{4}{5} \cdot H = 32\ cm \ \Longrightarrow\ Bg^2 = 240.98\ m^{-2} \ \Longrightarrow k_{eff} = 0.999$

B.2.

$\phi_{rz} = \phi(r,z) = \phi_{00} \cdot J_0(\alpha R) \cdot \cos(\beta z)\,, \qquad with\ 0 < r < R\ \ and\ -\dfrac{h}{2} < z < \dfrac{h}{2}$

$\alpha = \dfrac{2.405}{R}\ and\ \beta = \pi/H$

$J_0(\alpha R)$ *is* max *when* $r = 0$

$\dfrac{\partial}{\partial z}[\cos(\beta z)] = 0 \ \Longrightarrow -\beta \sin(\beta z) = 0 \ \Longrightarrow z = 0\ or\ \beta z = \ \pi \ \Longrightarrow \dfrac{\pi}{H} z = \pi \Longrightarrow z = H$

But $-\frac{h}{2} < z < \frac{h}{2}$, so the only possible solution is z = 0.

Therefore, the flux is maximal at the center of the tank:

$$\phi_{max} = \phi_{00} = \ \phi(r = 0, z = 0)$$

-Total fission power: $P_{volumic} = Q\Sigma_f \phi_{rz}$

$\Longrightarrow P \cdot dV = Q\Sigma_f \phi_{rz} \cdot 2\pi R dR dz$

-"the total fission power is only about 1W":

$$1[J.s^{-1}] = \phi_{00} 2\pi Q \Sigma_f \int_0^R J_0(\alpha R)\, r dr \int_{-\frac{\beta H}{2}}^{\frac{\beta H}{2}} \cos(\beta z)\, dz$$

$x = \alpha R \ \ and \ \ y = \beta z$:

$$1 = \phi_{00} 2\pi Q \Sigma_f \int_0^{\alpha R} J_0(x) \frac{x dx}{\alpha^2} \int_{-\frac{H}{2}}^{\frac{H}{2}} \cos(y) \frac{dy}{\beta} = \phi_{00} \frac{2\pi Q \Sigma_f}{\alpha^2 \beta}\ [x J_1(x)]_0^{\alpha R} \cdot [\sin\,(y)]_{\frac{-\beta H}{2}}^{\frac{\beta H}{2}}$$

$$= \phi_{00} \frac{2\pi Q \Sigma_f}{\alpha^2 \beta} \cdot\ \alpha R \times 0.5 \cdot 2 \sin \sin \frac{\beta H}{2} = \phi_{00} \frac{2 Q \Sigma_f}{2.405} R^2 H$$

with Q = 200 MeV and $\Sigma_f = e N_U \sigma_f^5$*:* $\qquad \phi_{00} = 1.21 \cdot 10^8\ cm^{-2}\, s^{-1}$

C.1.

Infinite reflector: thickness about 2 to 3 L
$R = 23\ cm \implies Bg^2 = 222.72\ m^{-2} \implies k_{eff} = 1.006$

For U-235, $\beta_{delayed} = 0.00640$ so it corresponds to a +1$ reactivity insertion.

$$n(t) = n_0 \exp\left(\frac{k_{eff} - 1}{l_p + \beta_d \tau_d} t\right) = n_0 \exp(0.0187t)$$

$with\ l_p = 10^{-4}s$ which means that prompt neutrons are emitted $10^{-4}s$ after a neutron hit a U-235 nucleus, and $\tau_d = 50s$ being the delayed neutrons mean emission time.

So, the mean time between 2 generations of neutrons is $\tau = l_p + \beta_d \tau_d$

$$\Rightarrow \frac{n(t)}{n_0} = 2 = \exp(0.0187 t_{double}) \Rightarrow t_{double} = 37s$$

With such a reactivity insertion, the power is doubled every 37 seconds!

C.2.

We want $k_\infty = 0.3$ by adding Boric acid:

$$k_\infty = \varepsilon p f \eta = f\eta = \frac{\Sigma_a^{fuel}}{\Sigma_{tot}^{fuel}} \cdot \frac{\upsilon \Sigma_f}{\Sigma_a^{fuel}} = \frac{\upsilon \Sigma_f}{\Sigma_a^{tot}}$$

$$= \frac{\upsilon N_5^{homogeneous} \sigma_f^5}{N_H \sigma_a^H + N_5^{homogeneous} \sigma_a^5 + N_{10}^{homogeneous} \sigma_a^{10}} = 0.3$$

Where: $N_5^{homogeneous} = eN_U$ and $N_{10}^{homogeneous} = 0.2 N_B$

$$\implies N_B = 10^{10} cm^{-3}$$