Neutronics Exercises

14 Depletion

14.1 *Fuel evolution*

Exercise description:

For a reactor fuelled with low enriched uranium, show that the ^{239}Pu concentration (relative to that of ^{235}U), after an irradiation corresponding to a fluence of θ n/cm^2 may be expressed as: $\frac{N_9(\theta)}{N_5(\theta)} = \frac{N_{8,0}}{N_{5,0}}\frac{\sigma_{a,8}}{\sigma_{a,9}}\frac{1-e^{-\sigma_{a,9}\theta}}{e^{-\sigma_{a,5}\theta}}$

$N_{5,0}$ being the initial ^{235}U concentration and $N_{8,0}$ being that of ^{238}U. It is assumed that the ^{238}U concentration remains constant (it indeed does not vary significantly). $\sigma_{a,5}$, $\sigma_{a,8}$, and $\sigma_{a,9}$ are the microscopic (1-group) absorption cross-sections of ^{235}U , ^{238}U and ^{239}Pu, respectively.

(a) Calculate the $^{239}Pu/^{235}U$ ratio for a reactor operating during 2yrs with a constant flux of 4.10^{13}n/cm^2.s, given the following data:

σ_{a5} = 344 b $\quad$ σ_{a8} = 1.5 b $\quad$ σ_{a9} = 820 b

$N_{50} = 1.7 \cdot 10^{20}$ cm^{-3}, $\quad$ $N_{80} = 6.7 \cdot 10^{21}$ cm^{-3}

(b) What is the ratio of the fission rate of ^{239}Pu to that of ^{235}U at the end of the 2-year period? Take $\sigma_{f,5}$= 293b and $\sigma_{f,9}$= 607b.

Knowledge to be applied: $\theta = \int_t \Phi(t)\mathrm{dt}$, $\frac{N_9(\theta)}{N_5(\theta)} = \frac{N_{8,0}}{N_{5,0}}\frac{\sigma_{a,8}}{\sigma_{a,9}}\frac{1-e^{-\sigma_{a,9}\theta}}{e^{-\sigma_{a,5}\theta}}$, $R = \Phi N\sigma$

Expected results: (a) $\frac{N_9(\theta)}{N_5(\theta)} = 0.15$ (b) $\frac{F_9}{F_5} = 0.31$

14.2 *Average burn-up*

Exercise description:

(a) Let's consider a $100 MW_{th}$ reactor made of 120 fuel assemblies. Knowing that each fuel element contains 10 kg of fuel, what is the average burn-up in MWd/kg after a 1-year operational period?

(b) The initial percentage of fissile atoms in the fuel is 15.2%. What is the fraction of fissile material which, on average, (i) has undergone fission, (ii) has been destroyed?

(Consider that 10^6 MWd correspond to the fissioning of 1t of fissile material and use $\sigma_f = 582$ b, $\sigma_a = 681$ b for the fissile nuclide.)

Expected results: (a) $Burnup = 30.4 \frac{MWd}{kg}$ (b) (i) $F_{fissioned}$=20% (ii) $F_{destroyed}$=23.4%

14.3 *Xe poisoning I*

Exercise description:

(a) ^{135}Xe is the fission product with the largest poisoning effect in a thermal reactor. The direct production rate from fission is relatively low (~ 0.3%) and it is primarily formed by the β^--decay of the much more abundant fission product, ^{135}I (production rate ~ 6.1%): $I\xrightarrow{\beta^-, T_{1/2}=6.6h}\mathrm{Xe}\xrightarrow{\beta^-, T_{1/2}=9.1h}\mathrm{Cs(stable)}$

For a reactor at steady-state with a flux Φ, show that the equilibrium xenon concentration (relative to the number density of fissile atoms) is given by: $\frac{N_x}{N_f}=\frac{(\gamma_i+\gamma_x)(\sigma_f/\sigma_x)}{1+\lambda_x/(\Phi\sigma_x)}$, where γ_i is the fission yield for ^{135}I, γ_x that for ^{135}Xe, σ_f the microscopic cross-section for thermal fission, σ_x that for Xe absorption, and λ_x is the disintegration constant for ^{135}Xe. (N.B.: One may neglect absorptions in ^{135}I.)

(b) Show that the reactivity effect of ^{135}Xe at equilibrium in a large thermal reactor fuelled with ^{235}U, is given approximately by: $\rho=-\frac{\gamma_i+\gamma_x}{\bar{\nu}p\varepsilon(1+\lambda_x/\Phi\sigma_x)}$, where $\bar{\nu}$, p, ε have their usual meaning. Calculate the reactivity effects for a reactor operating with a thermal neutron flux of (i) 10^{10}, (ii) 10^{12}, (iii) 10^{14} n/cm^2.s , using the data from table.

(c) What is theoretically the maximal reactivity effect?

$\bar{\nu}=2.42$,	$p=0.925$,	$\varepsilon=1.040$	
$\gamma_1=0.061$,	$\gamma_x=0.003$,	$\sigma_x=2.6\cdot10^6$ b,	$\lambda_x=2.1\cdot10^{-5}\mathrm{s}^{-1}$

Knowledge to be applied: $\rho=\frac{k-k_0}{k}\simeq\frac{f-f_0}{f}$, $f=\frac{\Sigma_{a,c}}{\Sigma_{a,c}+\Sigma_{a,m}+\Sigma_{a,\mathrm{Xe}}}$ $k_0=k_\infty=1=\eta fp\varepsilon$,

Expected results: (b) ρ= (i) -3.4.10^{-5}, (ii) -3.0.10^{-3}, (iii) -2.5% (c) ρ_{max}=-2.75%

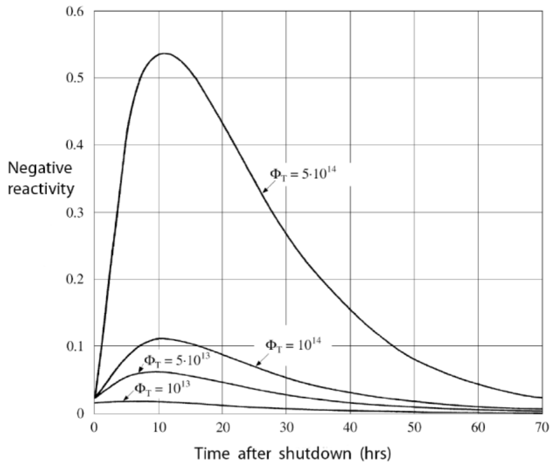
14.4 *Xe poisoning II*

Exercise description:

A reactor is in steady-state operation with equilibrium concentrations $N_{i,0}$ and $N_{x,0}$ of ^{135}I and ^{135}Xe, respectively. The reactor is shutdown.

(a) Show that the ^{135}Xe concentration varies as: $N_x(t) = N_{x,0}e^{-\lambda_x t} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}\left(e^{-\lambda_x t} - e^{-\lambda_i t}\right)$

Considering that $N_{i,0}$ and $N_{x,0}$ depend on the flux level, it is clear that the increase in the Xe concentration following the shutdown can prevent restarting the reactor during a certain time, in the case of a high-flux reactor. The figure alongside indicates the evolution of the Xe-concentration (through the negative reactivity in the system due to the presence of Xe-135) after reactor shutdown for different operational flux values. Consider now the case of a nominal flux value of 10^{14} n/cm^2.s.



(b) Estimate the waiting time necessary in order to be able to restart the reactor, if the maximal reactivity reserve held by the control system is: (i) 10%, (ii) 5%, (iii) 2%. (Assume that it is not possible to restart before the Xe concentration has passed its peak value.)

Knowledge to be applied: After t=0, $\frac{dN_x(t)}{dt} = -\lambda_x N_x(t) + \lambda_i N_i(t)$,

Expected results: (a) $N_x(t) = N_{x,0}e^{-\lambda_x t} + \frac{\lambda_i}{\lambda_i - \lambda_x} N_{i,0}\left(e^{-\lambda_x t} - e^{-\lambda_i t}\right)$ (b) reading off from the curve $\Phi=10^{14}$, the waiting time is: (i) $t \approx 10h$, (ii) $t \approx 31h$, (iii) $t \approx 44h$,