Neutronics Exercises

11 Elements of lattice physics

11.1 *“sandwich” reactor*

Exercise description:

The fuel elements of a mobile reactor are made up of thin plates with a “sandwich” structure as indicated in the figure: The cladding is of zirconium. The central part contains a mixture of highly enriched (nearly pure ^{235}U) uranium and zirconium, in the ratio 150 Zr-atoms per U-atom. The cladding and the central part have the same thickness, viz. 0.05 cm. The plates are immerged in water, serving as both moderator and coolant, such that the volumes occupied by the plates and by water are identical. At ambient temperature:

(a) What is the mean free path of thermal neutrons in the cladding and the central part?

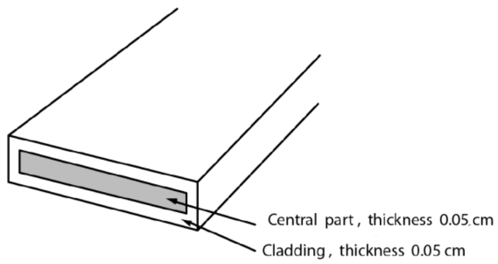
(b) What is the k_∞ value for the reactor core?

Use the data given below:

H_2O… Mol. density = $0.0334.10^{24}$ cm^{-3}

Zr… At. density = $0.0429\ .10^{24}$ cm^{-3} , $\sigma_t = 6.6$ b

^{235}U… $\sigma_a = 666$ b, $\sigma_s = 8.9$ b , $\eta_{th} = 2.065$



The absorption cross-section indicated for ^{235}U is the value at 0.0253 eV. The corresponding values for Zr and for H_2O (per molecule) are, respectively, 0.185 b and 0.664 b.

Knowledge to be applied: $\lambda_t = 1/\Sigma_t$, $k_\infty = \eta_{\mathrm{th}} f$, expression for f

Expected results: (a) cladding $\lambda_t = 3.5\mathrm{cm}$, fuel $\lambda_t = 2.1\mathrm{cm}$ (b) $k_\infty = 1.40$

Exercise solution:

(a) The mean free path in the cladding is $\lambda_t = 1/\Sigma_t = 1/(0.0429 \times 6.6) = 3.5\mathrm{cm}$. For determining the mean free path in the central fuel-containing region, one needs to calculate Σ_t for the mixture of zirconium and ^{235}U. Since the uranium is present essentially as an “impurity”, the zirconium concentration in the central part is practically the same as in the cladding, i.e. $N_{Zr}=0.0429.10^{24}$ cm^{-3}. The atomic concentration of ^{235}U in the central part is thus N_{U5} = $0.0429.10^{24}$ cm^{-3} ÷ 150 = $(2.86.10^{-4}).10^{24}$ cm^{-3}.

For ^{235}U, $\sigma_t = \sigma_a + \sigma_s = 681 + 8.9 \simeq 675b$.

Hence, for the central part, $\Sigma_t = 2.86 \times 10^{-4} \times 675 + 0.0429 \times 6.6 = 0.476\mathrm{cm}^{-1}$

The mean free path is thus: $\lambda_t = 1/\Sigma_t = 2.1\mathrm{cm}$

The mean free path in the cladding is 3.5/0.05 = 70 times larger than the thickness of the cladding, while, in the central part, it is 2.1/0.05 = 42 times larger than the corresponding region thickness. The reactor core is thus clearly quasi-homogeneous.

(b) We have $k_\infty = \eta_{\mathrm{th}} f$, where $\eta_{\mathrm{th}} = 2.065$ for ^{235}U. For calculating f, one needs to consider the homogenized mixture of the atomic densities (H_2O, Zr, ^{235}U). Since the water and the Zr each occupy half the volume, their atomic densities are exactly half their normal individual values. In units of 10^{24} cm^{-3}:

$N_{H_2O} = 0.5 \times 0.0334 = 0.0167\mathrm{cm}^{-3}$ and $N_{\mathrm{Zr}} = 0.5 \times 0.0429 = 0.0215\mathrm{cm}^{-3}$

The uranium is present in one-third of the volume of the sandwich, i.e in one-sixth of the total volume, and its density is $2.86.10^{-4}$ in the central part, so that for the homogeneous core mixture: $N_U = 02.86 \times 10^{-4}/6 = 4.77 \times 10^{-5}\mathrm{cm}^{-3}$

At 0.0253 eV, the microscopic absorption cross-sections of ^{235}U, Zr and H_2O (per molecule) are 681 b, 0.185 b and 0.664 b, respectively. The value of f is:

$$f = \frac{4.77\times10^{-5}\times666}{4.77\times10^{-5}\times666+0.0215\times0.185+0.0167\times0.664} = 0.678$$

so that $k_\infty = 2.065 \times 0.678 = 1.40$

11.2 *Microscopic capture cross section for U-238*

Exercise description:

We want to compute the effective microscopic capture cross section of U-238 in the energy group 50 of the WIMS-69 structure, ranging from 4.0 to 9.87 eV. U-238 is mixed homogeneously with moderator nuclei (H-1 and O-16). The composition of the system is such that the dilution cross section is equal to 50b.

In this energy range, the resonance integrals are computed with NJOY using the NR and approximation of the flux and tabulated against dilution. Additionally, the potential scattering cross section of U-238 is measured to be equal to 8 barns.

(a) Using the NR approximation, write down the expression of the flux and resonance integrals as well as the microscopic cross section in this energy range.

(b) Determine the following relationship between microscopic cross section and resonance integral: $\sigma_{c,g}^{U-238} = \frac{RI_{c,g}^{U-238}}{\frac{-RI_{c,g}^{U-238}}{\left(\sigma_p^{U-238}+\sigma_0\right)}+\Delta u_g}$

(c) Using the resonance integrals tabulated for various dilutions below (produced with the NJOY code), determine the resulting microscopic cross section.

(d) Now, instead of a homogeneous mixture, we consider that U-238 is located in an infinite array of fuel rods of radius 0.5 cm. Compute the updated microscopic cross section

The tabulated RI for U-238 and the N.R. approximation are given below:

σ_0 (barn)	$RI_{c,g}^{U-238}$ (s^{-1})
45	6.22
46	6.28
47	6.34
48	6.40
49	6.46
50	6.52
51	6.58
52	6.64
53	6.70
54	6.75
55	6.81
56	6.87
57	6.92
58	6.98
59	7.03

Exercise solution:

(a) The NR approximation leads to the flux integral:

$$\phi_g = \int_{E_{g-1}}^{E_g} \frac{\left(\sigma_p^{U-238}+\sigma_0\right)}{\left(\sigma_c^{U-238}(E)+\sigma_p^{U-238}+\sigma_0\right)} \frac{dE}{E}$$

The associated resonance integral:

$$RI_{c,g}^{U-238} = \int_{E_{g-1}}^{E_g} \frac{\sigma_c^{U-238}(E)\left(\sigma_p^{U-238}+\sigma_0\right)}{\left(\sigma_c^{U-238}(E)+\sigma_p^{U-238}+\sigma_0\right)} \frac{dE}{E}$$

The resulting microscopic cross section is

$$\sigma_{c,g}^{U-238} = \frac{RI_{c,g}^{U-238}}{\phi_g}$$

(b) Noticing that the flux integral can be expressed as:

$$\phi_g = \int_{E_{g-1}}^{E_g} \frac{\sigma_c^{U-238}(E)}{\left(\sigma_c^{U-238}(E) + \sigma_p^{U-238} + \sigma_0\right)} \frac{dE}{E} - \int_{E_{g-1}}^{E_g} \frac{\sigma_c^{U-238}(E)}{\left(\sigma_c^{U-238}(E) + \sigma_p^{U-238} + \sigma_0\right)} \frac{dE}{E} + \int_{E_{g-1}}^{E_g} \frac{\left(\sigma_p^{U-238} + \sigma_0\right)}{\left(\sigma_c^{U-238}(E) + \sigma_p^{U-238} + \sigma_0\right)} \frac{dE}{E}$$

Then it comes:

$$\phi_g = \frac{-1}{\left(\sigma_p^{U-238} + \sigma_0\right)} \int_{E_{g-1}}^{E_g} \frac{\sigma_c^{U-238}(E)\left(\sigma_p^{U-238} + \sigma_0\right)}{\left(\sigma_c^{U-238}(E) + \sigma_p^{U-238} + \sigma_0\right)} \frac{dE}{E} + \int_{E_{g-1}}^{E_g} \frac{dE}{E}$$

And finally:

$$\phi_g = \frac{-RI_{c,g}^{U-238}}{\left(\sigma_p^{U-238} + \sigma_0\right)} + \Delta u_g$$

With Δu_g the lethargy width of the group considered.
Then the microscopic cross section can be expressed as:

$$\sigma_{c,g}^{U-238} = \frac{RI_{c,g}^{U-238}}{\frac{-RI_{c,g}^{U-238}}{\left(\sigma_p^{U-238} + \sigma_0\right)} + \Delta u_g}$$

(c) Using the tabulated resonance intervals, $RI_{c,g}^{U-238} = 6.52$ s^{-1}; then

$$\sigma_{c,g}^{U-238} = \frac{6.52}{\frac{-6.52}{(50+8)} + ln\left(\frac{9.87}{4.0}\right)} = 8.25b$$

(d) Considering now a heterogeneous system, one can use average chord length method to determine the escape cross section and increase the dilution of the system. For the infinite array of fuel rods, $\Sigma_e = \frac{1}{\bar{l}} = \frac{S}{4V} = \frac{2\pi R}{4\pi R^2} = \frac{1}{2R} = 1$ cm^{-1}: Given its atomic concentration (10^{24} at. cm^{-3}), the dilution of the system is increased by 1b due to its heterogeneous nature. Using the tabulated resonance intervals, $RI_{c,g}^{U-238} = 6.58$ s^{-1}; then

$$\sigma_{c,g}^{U-238} = \frac{6.58}{\frac{-6.58}{(50+1+8)} + ln\left(\frac{9.87}{4.0}\right)} = 8.31b$$

11.3 *Absorption of low lying U-238 resonances*

Exercise description:

We want to compute the contribution of each of the first five capture resonance of U-238 to the multigroup effective microscopic capture cross section for an energy range going from 4 to 367.3eV. U-238 is mixed homogeneously with moderator nuclei (H-1 and O-16). The composition of the system is such that the dilution cross section is equal to 75b.

For each of those resonances, an energy group is defined as to include each resonance independently of the others. The capture integrals at 75b for each energy group are computed with NJOY using the NR and WR approximations. For the whole range of energy, the potential scattering cross section of U-238 is measured to be equal to 8 barns.

(a) Using the relationship between resonance integral and cross section derived in the previous exercise for NR, compute the effective cross section for each energy group for each approximation. Derive a similar relationship for the WR approximation and perform the same task.

(b) Using the information provided on the resonance parameters, please justify the relative performance of WR /NR with respect to the exact solution obtained by solving numerically the slowing down equations.

The tabulated RIs for U-238 as well as some resonance specific parameters are given below:

Grp ID	Emin	Emax	RI(s^{-1}) – NR	RI(s^{-1}) – WR	Effective Width (Γ_p)	$(1-\alpha_{res})E_0$
1	4	9.87	25.634	24.414	1.26	0.110
2	9.87	27.7	3.796	3.636	1.95	0.348
3	27.7	48.05	3.1	2.969	3.65	0.612
4	48.05	148.7	0.557	0.534	1.32	0.966
5	148.7	367.3	0.281	0.269	2.63	1.73

The resonance absorption probabilities (1-p) computed with NJOY are provided below:

Grp ID	Emin	Emax	1-p		
			NR	WR	Exact
1	4	9.87	0.2376	0.1998	0.1963
2	9.87	27.7	0.0745	0.0706	0.0675
3	27.7	48.05	0.0474	0.0611	0.0582
4	48.05	148.7	0.0090	0.0095	0.0092
5	148.7	367.3	0.0044	0.0077	0.0050

Exercise solution:

(a) Using the expression for the microscopic cross section derived above, for the NR approximation,

$$\sigma_{c,g}^{U-238,NR} = \frac{RI_{c,g}^{U-238,NR}}{\frac{-RI_{c,g}^{U-238,NR}}{\left(\sigma_p^{U-238} + \sigma_0\right)} + \Delta u_g}$$

In the WR approximation, the same approach is taken but this time σ_p^{U-238} is set to 0 as we neglect any scattering on a resonant nucleus.

Finally it comes: $\sigma_{c,g}^{U-238,WR} = \frac{RI_{c,g}^{U-238,WR}}{\frac{-RI_{c,g}^{U-238,WR}}{(\sigma_0)}+\Delta u_g}$

Finally for each energy group and the respective resonance approximations, one can obtain the following results:

Grp ID	Emin	Emax	σ_c (b) – NR	σ_c(b) – WR
1	4	9.87	39.008	37.708
2	9.87	27.7	4.313	4.139
3	27.7	48.05	3.495	3.351
4	48.05	148.7	0.61	0.585
5	148.7	367.3	0.307	0.294

(b) Looking at the parameters of the resonance, it is clear that when the effective resonance width is larger than the maximum energy loss per collision on the resonant nucleus, the WR approximation predicts better results than the NR approximation. It is the case for the low lying resonances, e.g. for group 1 2 and 3. For the group 5, NR performs better.