

Neutronics Exercises

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10 Multigroup Theory

10.1 Study of a very large reactor

Exercise description:

Calculate the infinite multiplication constant and the relative group fluxes in a very large fuel assembly with the four-group constants given below:

Group Index	1	2	3	4
$\nu\Sigma_f$	0.0096	0.0012	0.0177	0.1851
Σ_a	0.0049	0.0028	0.0305	0.1210
$\Sigma_s (g \rightarrow g+1)$	0.0831	0.0585	0.0651	-
χ	0.575	0.425	0	0

The fast flux is normalized to 1.

Use your favorite programming language to solve the linear system of equations

Knowledge to be applied: u

Expected results: $k_{inf}=1.21500$; $\Phi_1=1$; $\Phi_2=2.4167$; $\Phi_3=1.4788$; $\Phi_4=0.7956$;

Exercise solution:

Due to its large size, the leakage term is neglected.

According to the multigroup theory, the four coupled equations to solve are then:

$$\begin{aligned}\Sigma_{r,1}\Phi_1 &= \frac{\chi_1}{k} (\nu\Sigma_{f,1}\Phi_1 + \nu\Sigma_{f,2}\Phi_2 + \nu\Sigma_{f,3}\Phi_3 + \nu\Sigma_{f,4}\Phi_4) \\ \Sigma_{r,2}\Phi_2 - \Sigma_{s,1 \rightarrow 2}\Phi_1 &= \frac{\chi_2}{k} (\nu\Sigma_{f,1}\Phi_1 + \nu\Sigma_{f,2}\Phi_2 + \nu\Sigma_{f,3}\Phi_3 + \nu\Sigma_{f,4}\Phi_4) \\ \Sigma_{r,3}\Phi_3 - \Sigma_{s,2 \rightarrow 3}\Phi_2 &= 0 \\ \Sigma_{r,4}\Phi_4 - \Sigma_{s,3 \rightarrow 4}\Phi_3 &= 0\end{aligned}$$

The removal and scattering terms are gathered in the A matrix:

0.088	0	0	0
-0.0831	0.0613	0	0
0	-0.0585	0.0956	0
0	0	-0.0651	0.121

The fission source terms are gathered in the F matrix:

0.00552	0.00069	0.010178	0.106433
0.00408	0.00051	0.007523	0.078668
0	0	0	0
0	0	0	0

Using either a direct or iterative solution, the largest eigenvalue and the associated fluxes are:

$k_{inf}=1.21500$; $\Phi_1=1$; $\Phi_2=2.4167$; $\Phi_3=1.4788$; $\Phi_4=0.7956$

10.2 Two-group analytical model of a bare cylindrical core

Exercise description:

Calculate in two-group theory the critical radius of a 3.5m high bare cylindrical core with the cross sections given below:

Group Index	1	2
$\nu\Sigma_f$	0.0085	0.1851
Σ_a	0.0121	0.121
$\Sigma_s (g \rightarrow g+1)$	0.0241	-
χ	1	0
D	1.267	0.354

The cross sections are given in cm^{-1} . We can assume that both fast and thermal fluxes have the same spatial shape, e.g. the geometrical buckling is energy independent.

Expected results: $R=29.6\text{cm}$; $B^2 = 0.00668$

Exercise solution:

According to the multigroup theory, the coupled equations to solve are:

$$\begin{aligned} -D_1 \nabla^2 \Phi_1 + \Sigma_{r,1} \Phi_1 &= \frac{\chi_1}{k} (\nu \Sigma_{f,1} \Phi_1 + \nu \Sigma_{f,2} \Phi_2) \\ -D_2 \nabla^2 \Phi_2 + \Sigma_{r,2} \Phi_2 &= \frac{\chi_2}{k} (\nu \Sigma_{f,1} \Phi_1 + \nu \Sigma_{f,2} \Phi_2) + \Sigma_{s,1 \rightarrow 2} \Phi_1 \end{aligned}$$

For a bare uniform reactor, the vanishing of the flux on the boundary requires that the neutron flux in both groups satisfies:

$$\nabla^2 \Phi_i + B^2 \Phi_i = 0$$

Where B is the geometrical buckling of the system. The leakage terms are replaced by an accrued removal term:

$$\begin{aligned} (\Sigma_{r,1} + D_1 B^2) \Phi_1 &= \frac{1}{k} (\nu \Sigma_{f,1} \Phi_1 + \nu \Sigma_{f,2} \Phi_2) \\ (\Sigma_{r,2} + D_2 B^2) \Phi_2 &= \Sigma_{s,1 \rightarrow 2} \Phi_1 \end{aligned}$$

Solving for k:

$$k = \frac{\nu \Sigma_{f,1}}{\Sigma_{r,1} + D_1 B^2} + \frac{\Sigma_{s,1 \rightarrow 2}}{\Sigma_{r,1} + D_1 B^2} \frac{\nu \Sigma_{f,2}}{\Sigma_{r,2} + D_2 B^2}$$

For a critical system, the buckling B is solution of the polynomial $aB^4 + bB^2 + c = 0$ with:

$$\begin{aligned} a &= D_1 D_2 \\ b &= (\Sigma_{r,1} - \nu \Sigma_{f,1}) D_2 + \Sigma_{r,2} D_1 \\ c &= \Sigma_{r,1} \Sigma_{r,2} - \nu \Sigma_{f,1} \Sigma_{r,2} - \Sigma_{s,1 \rightarrow 2} \nu \Sigma_{f,2} \end{aligned}$$

Since is a positive value, $B^2 = \frac{-b + (b^2 - 4ac)}{2a} = 0.00668$

For a finite cylinder, the geometrical buckling is given by: $B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$

So finally, $R = \sqrt{\frac{2.405^2}{B^2 - \left(\frac{\pi}{H}\right)^2}} = 29.61\text{cm}$.

The axial leakage is practically negligible as one might expect.