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Series 6: Plackett-Burman & Full factorial designs

1. Sensitivity of a Plackett-Burman design

To analyse the sensitivity of a Plackett-Burman design, the measurement of a linear system with first-order interactions is simulated and the impact of interactions and uncertainties on the main effects is observed. Proceed then the following way:

a) Create a Matlab function $Y = measurement(E, s\ [, alpha])$ simulating a series of experiments whose *true* model is

$$y = a_o + \sum_{i=1}^{5} a_i x_i + \sum_{i \leq j}^{5} a_{ik} x_j x_i \tag{1}$$

and whose experimental uncertainty is modeled by the distribution $N(\mu = 0, \sigma^2)$. The parameters of this function must be the matrix of essays E and the standard error σ. The coefficients alpha of the true model can be given as parameters of the function or given by default within the function (use the logical variable *nargin* to test if you need to use the default values).

b) In the main part of the script :

- prepare a Plackett-Burman design,
- simulate the essays with the routine *measurement*,
- analyze the results to determine the effets (*fitlm()*).

c) Perform the analysis for a *true* model without interaction with important (10%), medium (5%) and small effects (1%) varying the uncertainty of the measurement between 1% and 10%. Observe the evolution of the estimated coefficients, of the confidence intervals and of the p-values of the ANOVA.

d) Perform the analysis for a *true* model whose level of interaction varies and with the measurement uncertainty varying between 1% and 10%. Observe the evolution of the estimated coefficients and confront these results with the matrix of alias between the main effects and the interactions.

e) Finally perform a *full foldover* to get rid of the influence of the interactions.

2 Analysis of a full factorial design

Table 1 presents the data of a full factorial design 2^4 with the results obtained for each experiment with four factors.

1. Determine the significant half-effects using a *normal plot* or a *half-normal plot*
2. Perform an analysis of the variance
3. Prepare conclusions based on these experimental evidences

Table 1: Data from a set of experiments following a design 2^4 on a process with four4 factors (standardized)

Run	x_1	x_2	x_3	x_4	Y
1	-1	-1	-1	-1	19.67
2	-1	-1	-1	1	-3.12
3	-1	-1	1	-1	23.55
4	-1	-1	1	1	2.46
5	-1	1	-1	-1	47.18
6	-1	1	-1	1	36.7
7	-1	1	1	-1	61.23
8	-1	1	1	1	19.25
9	1	-1	-1	-1	-18.59
10	1	-1	-1	1	40.72
11	1	-1	1	-1	-29.31
12	1	-1	1	1	42.6
13	1	1	-1	-1	-29.61
14	1	1	-1	1	-1.45
15	1	1	1	-1	-26.47
16	1	1	1	1	4.92