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Series 1: Constant coefficient models

1 Corrosion

Five different treatments against corrosion have been applied to a collection of samples. The tests have been performed four times. After the tests, the samples have been analyzed by an expert who has determined their level of corrosion and has attributed a mark to each sample. The marks are reproduced in table 1.

a) Consider that the data would fit a constant coefficient model $y = \mu + \alpha_i + \epsilon_{ij}$ and infer the corresponding coefficients,

b) Produce a *dotplot* to represent graphically the effects,

c) Perform an ANOVA to determine which treatments can be differenciated between them,

d) How many replicates would permit to observe a significative difference between treatment B and C.

Table 1: Experimental data corresponding to observed corrosion levels

Replicates	Treatments				
	A	B	C	D	E
1	1	9	6	3	14
2	3	5	6	3	10
3	5	5	3	0	18
4	3	5	3	6	14

2 Production rubbish

Table 2 gives the rates of rubbish resulting of the production of an industrial process taking place in different production units of a company. These different units are organized in *standard cells* or in *free cells* (a production method in which the task of each worker is not strictly defined). The shift of the teams can also vary, meaning that the shift can be *Morning*, *Afternoon* or *Night* depending if the team start its work at 7 a.m., 2 p.m. or 10 p.m.
Determine if the work shift or the work organization are factors influencing the rate of rubbish. In this perspective :

a) Consider the hypothesis that the data would fit a *constant coefficient model* $y = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$ and infer the corresponding coefficients,

b) Produce a *dotplot* to represent graphically the effects,

c) Perform an ANOVA to determine the confidence to attribute to the model and to each of its factors with a confidence level $\alpha = 5\%$.

d) Analyse the effects by pairs and determines the significant differences, factor by factor.

Table 2: Rubbish rates (nbr of pieces)

	Free cells			Standard cells		
	Shift M	Shift A	Shift N	Shift M	Shift A	Shift N
1	26	41	36	51	39	42
2	41	82	87	96	104	92
3	14	26	39	35	114	133
4	16	86	99	36	92	124

3 Productivity in a mechanical workshop

Table 3 gives the number of pieces produced per day in a mechanical workshop by different operators using different machines and different drills. With the objective to determine the dominant factors and the best set of factors:

a) Consider the hypothesis that the data would fit a *constant coefficient model* $y = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}$ and infer the corresponding coefficients,

b) Produce a *dotplot* to represent graphically the effects,

c) Perform an ANOVA to determine the confidence to attribute to the model and to each of its coefficients.

Table 3: Daily production (pieces per day)

		Machine		
Operator	Drill	Mx3	Zublin	Bosch
Charly	D45	40	55	49
Charly	Cv23	33	46	41
Charly	Kh200	43	56	53
Pedro	D45	53	54	55
Pedro	Cv23	44	50	48
Pedro	Kh200	44	51	53
Luis	D45	43	52	56
Luis	Cv23	45	43	42
Luis	Kh200	44	43	50

4 Analysis of a production process with a Latin square

Table 4 gives the data of production using three different processes in three different workshops for three different work shifts. Tow replicates have been realized. The experiment has been managed using a *latin square* 3×3 whose structure is:

$$\begin{array}{ccc} A & B & C \\ B & C & A \\ C & A & B \end{array}$$

so that A corresponds to the workshop of Biel, B to the one of Singapour and C to the one of Tartu. With the objective to determine the dominant factors and the best set of factors:

a) Consider the hypothesis that the data would fit a *constant coefficient model* $y = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}$ and infer the corresponding coefficients,

b) Produce a *dotplot* to represent graphically the effects,

c) Perform an ANOVA to determine the confidence to attribute to the model and to each of its coefficients.

Table 4: Production in nbr of lots. The third variable, the workshop, follows the Latin square given above.

	shift 1	shift 2	shift 3
Process 1	40	53	58
Process 2	36	80	60
Process 3	27	58	43
Process 1	24	52	57
Process 2	45	95	58
Process 3	20	64	38

5 Design of an experiment with an hyper-Graeco-Latin square

$$\begin{array}{ccccc} A & B & C & D & E \\ C & D & E & A & B \\ E & A & B & C & D \\ B & C & D & E & A \\ D & E & A & B & C \end{array} \qquad \begin{array}{ccccc} A & B & C & D & E \\ D & E & A & B & C \\ B & C & D & E & A \\ E & A & B & C & D \\ C & D & E & A & B \end{array} \qquad \begin{array}{ccccc} A & B & C & D & E \\ E & A & B & C & D \\ D & E & A & B & C \\ C & D & E & A & B \\ B & C & D & E & A \end{array}$$

a) Describe succinctly the experimental situation by defining the objective, the factors, the response, the model,

b) Built the matrix of experiments,

c) Compute the LSD for 1, 2 and 3 replications, and compare the results with an OFAT design (each factor analyzed independently).