Solutions serie 3

1 Primary ionization / Efficiency

Avec $\langle n_p \rangle \approx 22\ \mathrm{cm}^{-1}$, for a layer with the thickness of 1mm, $\langle n_p \rangle = 2.2$. the maximum efficiency is

$$\epsilon_{\mathrm{det}} = 1 - e^{-\langle n_p \rangle} = 1 - e^{-2.2} = 88.9\%$$

2 Photon interactions

1. With the formula given in the notes, the efficiency for the different processes as a function of the energy of the Z and the reduced photon energy $\epsilon = E_\gamma / m_e$:

	Z	ϵ	σ_{ph} [b]	σ_{c} [b]	σ_{p} [b]	dominant process
a)	13	2	$5 \cdot 10^{-4}$	2.74	0	Compton
b)	1	0.2	$3 \cdot 10^{-6}$	0.49	0	Compton
c)	26	0.2	35.7	12.8	0	Photo elctric
d)	6	20	$1 \cdot 10^{-6}$	0.30	0.07	Compton
e)	82	20	0.53	4.14	13.4	pair production

2. With $P = (k, \vec{k})$, $P' = (k', \vec{k}')$ the four vector of the incident photon and the diffused photon are: $P_0 = (m_e, \vec{0})$, $P_e = (E, \vec{p})$ the one for the electron in rest and the recoiled electron are also: $T = E - m_e = k - k'$ the kinetic energy of this electron.

   With conservation:

$$\begin{aligned} P &= P' + P_e - P_0 \\ k &= k' + E - m_e = k' + T \\ \vec{k} &= \vec{k}' + \vec{p} \end{aligned}$$

   One can express p^2 in two different ways :

$$\begin{aligned} p^2 &= k^2 + k'^2 - 2\vec{k} \cdot \vec{k}' = k^2 + k'^2 - 2kk' \cos\theta \\ p^2 &= E^2 - m_e^2 = T^2 + 2m_e T \\ &= (k - k')^2 + 2m_e(k - k') \\ k^2 + k'^2 - 2kk' \cos\theta &= k^2 + k'^2 - 2kk' + 2m_e T \\ kk' \cos\theta &= kk' - m_e T \end{aligned}$$

   One can replace k' par $k - T$,

$$m_e T = (1 - \cos\theta)(k^2 - kT)$$

$$T(m_e + k(1 - \cos\theta)) = k^2(1 - \cos\theta)$$

$$T = \frac{k^2(1-\cos\theta)}{m_e + k(1-\cos\theta)} = \frac{E_\gamma^2(1-\cos\theta)}{m_e + E_\gamma(1-\cos\theta)}$$

The energy of the electron is minimal for $\theta = 0$, soit $T \approx 0$. It is maximal therefore at $\theta = \pi$, which is when the photon direction is backwards. This is the so called Compton Edge

$$T = \frac{2E_\gamma^2}{m_e + 2E_\gamma} = E_\gamma \frac{2E_\gamma}{m_e + 2E_\gamma} = E_\gamma \frac{1}{1 + \frac{m_e}{2E_\gamma}} \xrightarrow{E_\gamma \gg m_e} E_\gamma - \frac{m_e}{2} \quad \text{donc} \quad k' = \frac{m_e}{2}$$