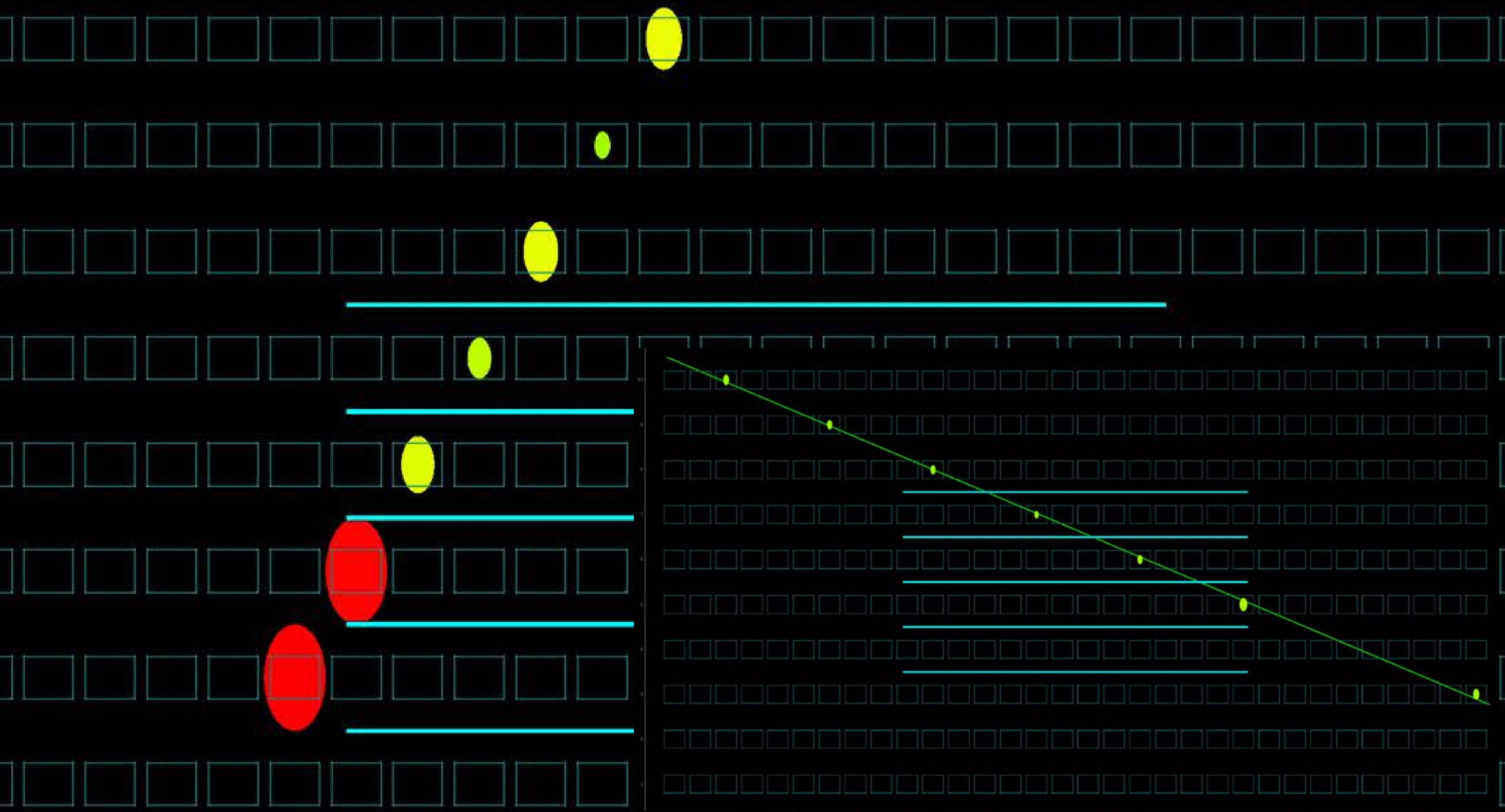


Fluctuations in energy loss
Landau distribution

Lecture 3

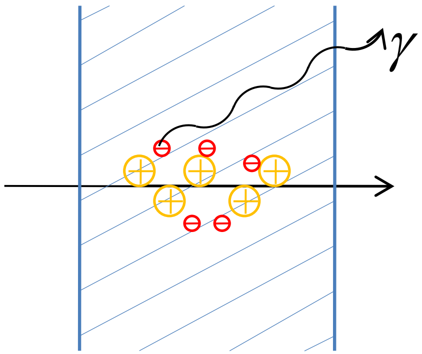
Measure the energy loss of 10'000 "identical" charged heavy particles (same mass and same energy) traversing a layer of material. What is the distribution of the amount of energy loss?

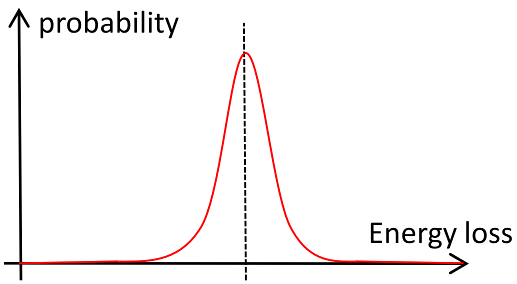
Bethe-Bloch only gives the mean dE/dX !

Thick absorber: Always contributions from many exchanges to total energy loss in a layer

-> Central limit theorem applies

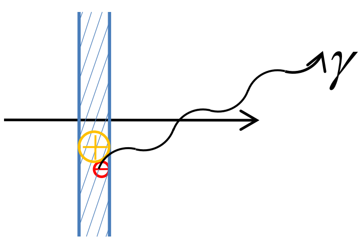
-> Gaussian distribution of energy loss

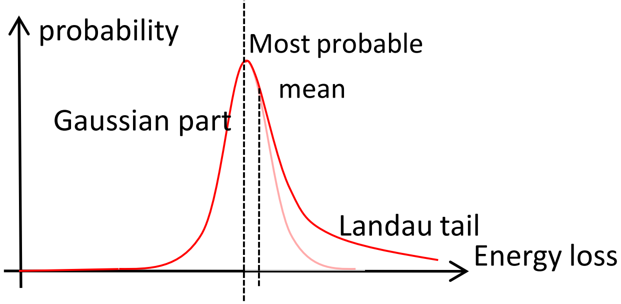




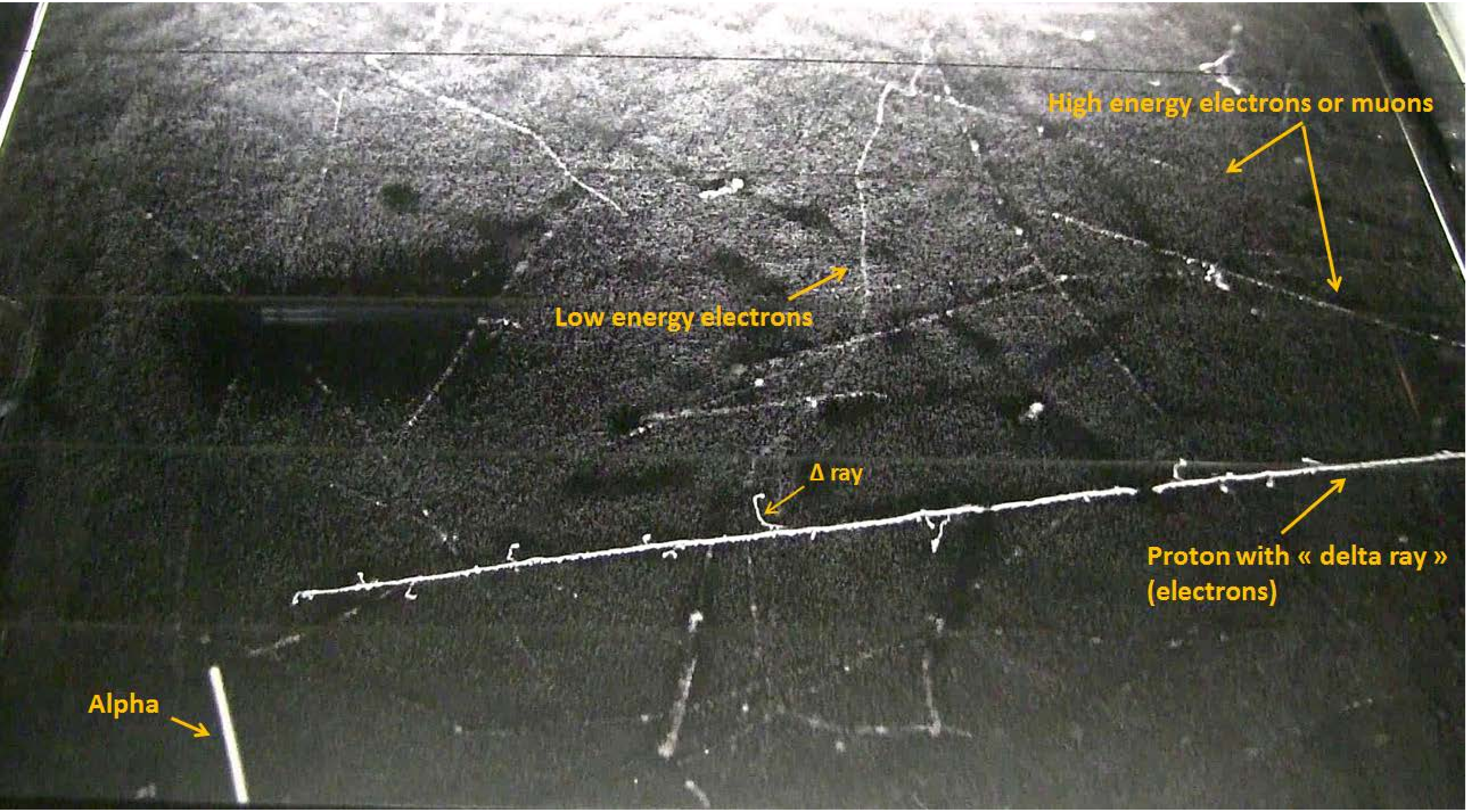
Thin absorber: Most of the time, still many exchanges of low energy photons (those have the highest probability) -> Gaussian part

However, sometimes one high-energy exchange producing δδ-rays can dominate -> large tail, together: Landau distribution





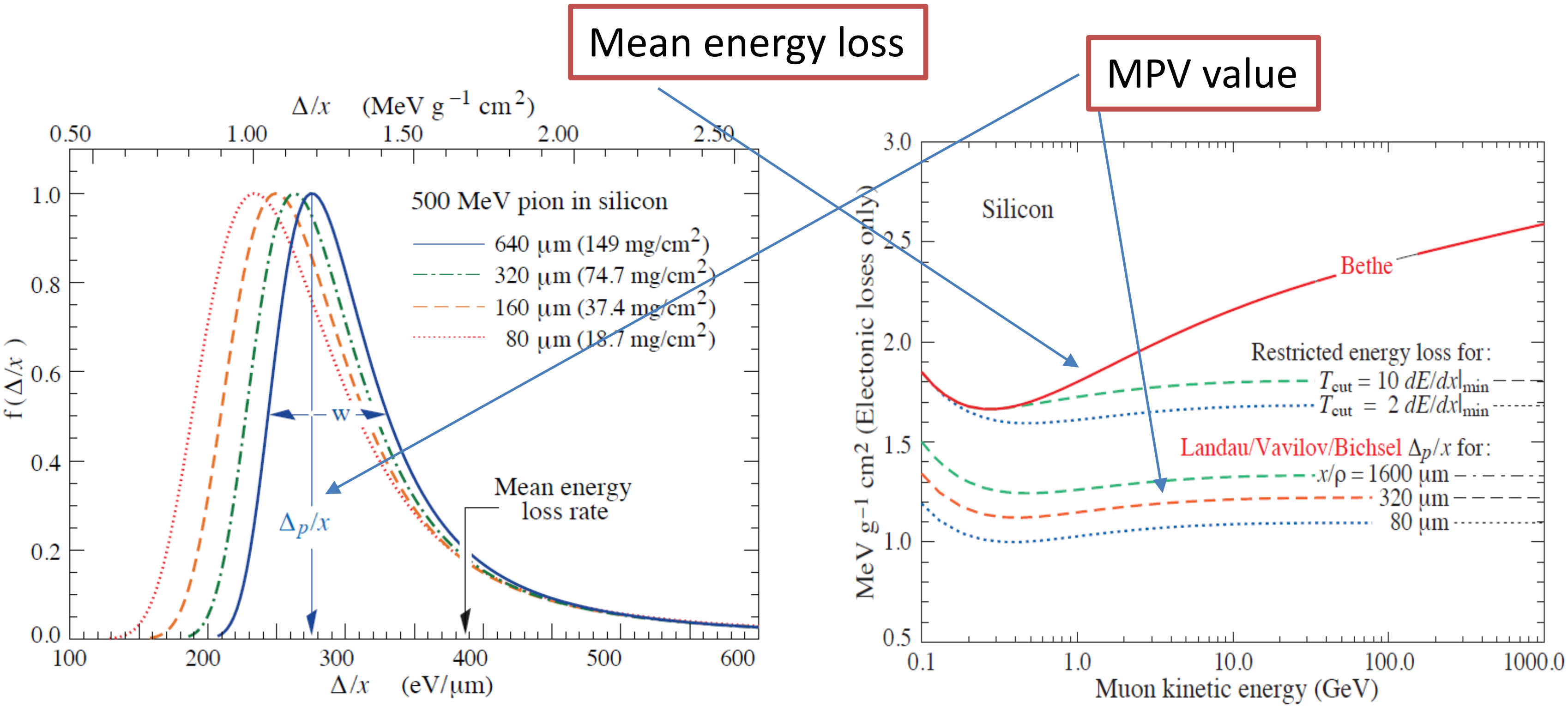
Cloud chamber can reveal "Delta rays"



Landau distribution

The fluctuations of energy loss of a charged particle in a thin layer (concerns tracking detectors) was described by Landau (1944). The Landau distribution is an asymmetric probability distribution.

- Small energy transfers are most likely
- Tail is caused by rate collisions with small impact parameter. In these collisions, electrons with high energies (keV) are produced called δ-electrons



Energy loss of e^+ and e^-

They are special because of their low mass (m_e=511keV/c^2)
Two main effects for energy loss: ionization and radiation (Bremsstrahlung)
Ionization: e^- + atom -> e^- + ion + e^-
The Bethe-Bloch formula needs modification:

- Identical mass for target electrons and ionizing particle
- Scattering of identical, undistinguishable particles
- electrons have very small mass => β->1

$$\left\langle -\frac{dE}{dX}\right\rangle_{tot} = \left\langle -\frac{dE}{dX}\right\rangle_{ion} + \left\langle -\frac{dE}{dX}\right\rangle_{Brems}$$

$$\left\langle -\frac{dE}{dX}\right\rangle_{ion} = K\frac{Z}{A}\frac{1}{\beta^2}\left[\ln\frac{m_e c^2\beta^2\gamma^2 T}{2I^2} + F(\gamma)\right]$$

T: kinetic energy of e, W_{max}=1/2T

Bremsstrahlung: e^- + field (nucleus) -> e^- + γ
Deceleration in Coulomb-field of nuclei => radiation
Dominates above a certain energy, the critical energy E_c (O(10Mev) for solids)
Approx. :

$$-\left(\frac{dE}{dX}\right)_{Brems} = 4\alpha N_A r_e^2 K\frac{Z^2}{A}E\ln\frac{183}{\sqrt[3]{Z}} \equiv \frac{E}{X_0}$$

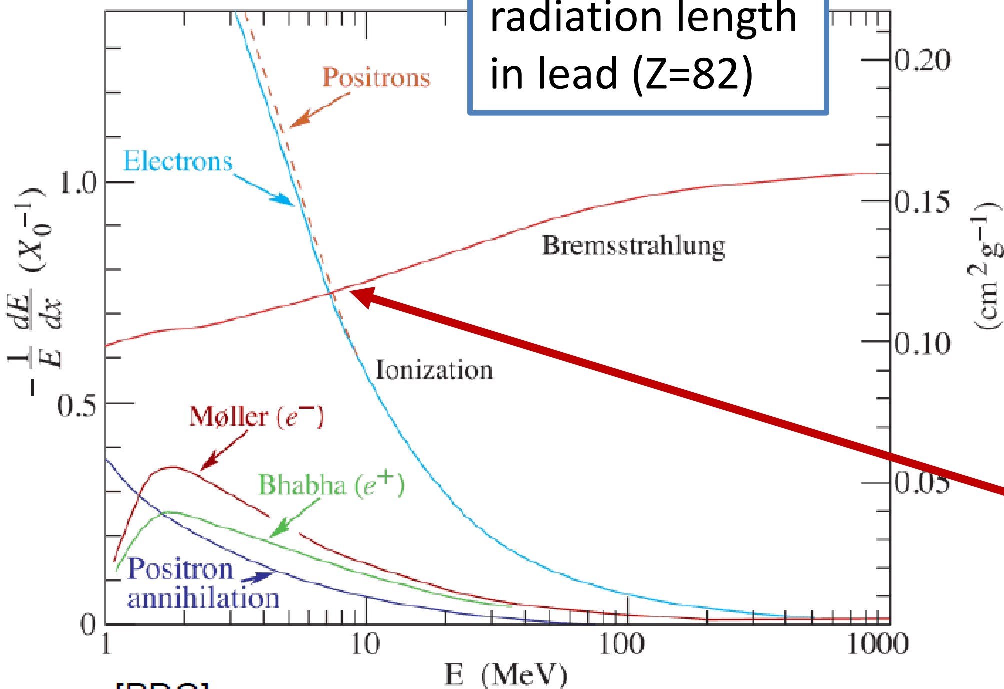
Energy loss of electrons and E_c

Critical Energy E_c:

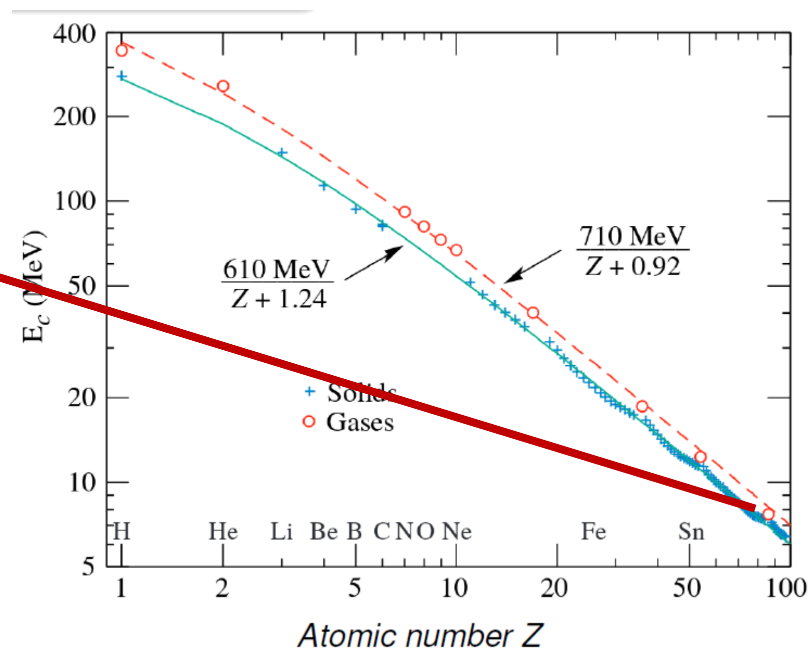
$$\left(\frac{dE}{dx}\right)_{Brems} = \left(\frac{dE}{dx}\right)_{ion}$$

$$E_c \approx \frac{610MeV}{Z + 1.24}$$

Energy loss per radiation length in lead (Z=82)



[PDG]



- Ionisation decreases logarithmically with E
- Bremsstrahlung increases linear with E
- Bremsstrahlung dominates for E>1GeV

Radiation length X_0

Radiation length X_0 is a characteristic length for both Bremsstrahlung and Pair production!

Bremsstrahlung

High energy electrons: $-\left.\frac{dE}{dz}\right|_{rad} = \frac{E}{X_0}$

$$\langle E(z) \rangle = E_0 \cdot \exp\left(-\frac{z}{X_0}\right)$$

X_0 = average distance needed to reduce the energy of a high-energy electron by a factor of 1/e

$$i.e.\ \ \langle E(X_0) \rangle = 36.8\% E_0$$

Pair production

High energy photons: $\sigma_{pp} \approx \frac{7}{9} \frac{M}{\rho N_A X_0}$

$$\langle I(z) \rangle = I_0 \cdot \exp\left(-\frac{7}{9} \frac{z}{X_0}\right)$$

X_0 = 7/9 of the mean free path for pair production by a high-energy photon

$$i.e.\ \ \langle I(X_0) \rangle = 45.9\% I_0$$

Calculation: $$X_0 \rho\, [g/cm^2] = \frac{716.4 g/cm^2 A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

ρ = density, Z = atomic number, A = mass number

	$X_{0*}\rho$ [g/cm²]	X_0[cm]	E_c[MeV]
H_2	63	700000 (gas)	340
H_2O	36	36	93
Pb	6.3	0.56	6.9
Si	21.82	9.37	40.19
PS*	43.79	41.31	93.11

*Scintillating plastic fibres made of polystyrene $(C_6H_5CHCH_2)_n$

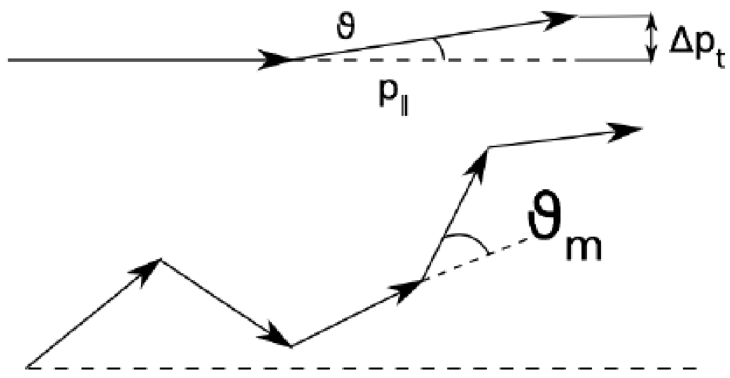
The thickness of materials (detectors or absorbers) are often given in units of X_0, d=300µm of Si is equal to X/X_0=0.32% or d=1.35mm polystyrene plastic fibres X/X_0=0.33 %, (comparison of 0.3mm Si and 1.35mm PS)

http://pdg.lbl.gov/2019/AtomicNuclearProperties/HTML/polystyrene.html

Multiple (Coulomb) scattering

Incident particle can also scatter in the Coulomb field of the NUCLEUS !

The consequence is a deflection of trajectory which depends on the factor Z !



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4(\Theta/2)}$$

Single collision (thin absorber) is called Rutherford scattering

Many (>20) collisions requires statistical treatment "Molière theory"

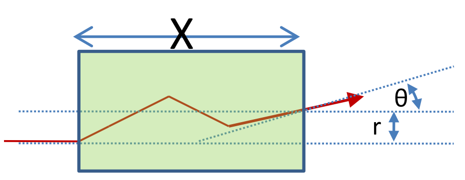
Multiple (Coulomb) scattering “Molière theory”

Obtain the mean deflection angle in a plane by averaging over many collisions and integrating over b:

$$\sqrt{\langle\theta^2(x)\rangle} = \theta_{rms}^{plane} = \frac{13.6\ \mathrm{MeV}}{\beta pc} z\sqrt{\frac{x}{X_0}}\left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

- Material constant X_0: radiation length
- $\propto \sqrt{x} \rightarrow$ use thin detectors
- $\propto 1/\sqrt{X_0} \rightarrow$ use light detectors
- $\propto 1/\beta p \rightarrow$ serious problem at low momenta

In 3 dimensions: $\theta_{rms}^{space} = \sqrt{2}\,\theta_{rms}^{plane}$ $\quad 13.6 \rightarrow 19.2$



Exercise -> π^+(p=180GeV/c), x/X_0=1%, track deviation? (6.2μrad)

Multiple scattering limits the momentum and tracking resolution, particularly at low momenta!

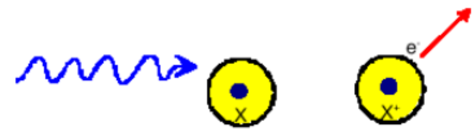
Photons

3 important processes that either create charged particles from photons or/and transfer energy to charged particles allowing photon detection via charged particle detection.

increasing E_γ

- Photoelectric effect

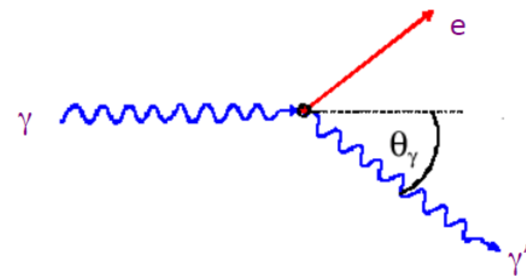
The photon is absorbed by an electron from the atom shell and the transferred energy liberates the electron



$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

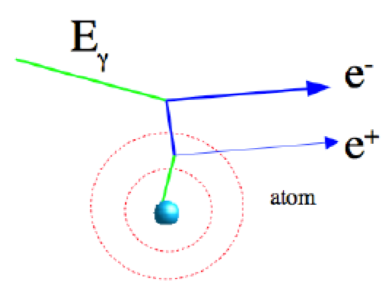
- Compton scattering

Elastic scattering of a photon on a quasi fee electron



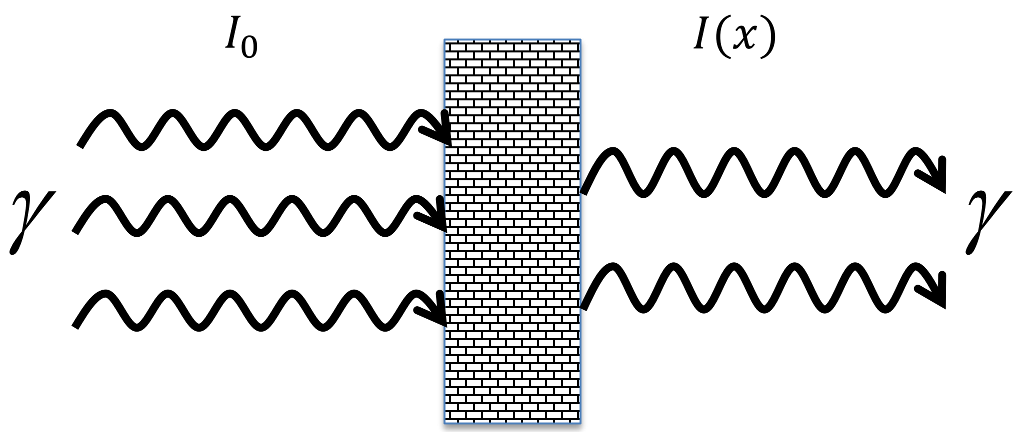
- Pair production

Electron-positron pair production by a photon in the field of a nucleus. The minimum energy for the incident photon is $2m_e$. This effect dominates at high energies.



Photon absorption coefficient and mean free path

Cross section: $\sigma = \sigma(E, Z, A)$ Cross section: $[\sigma] = barn = 10^{-24} cm^2$



$$I(x) = I_0 e^{-\mu x}$$

$$\boxed{\mu = N\sigma = \frac{N_A}{M}\rho\sigma \equiv \frac{1}{\lambda}}$$

μ: absorption coefficient
N: atom density
M: molar mass
λ: mean free path of absorption length

λ is the average distance travelled by a particle between two successive collisions.

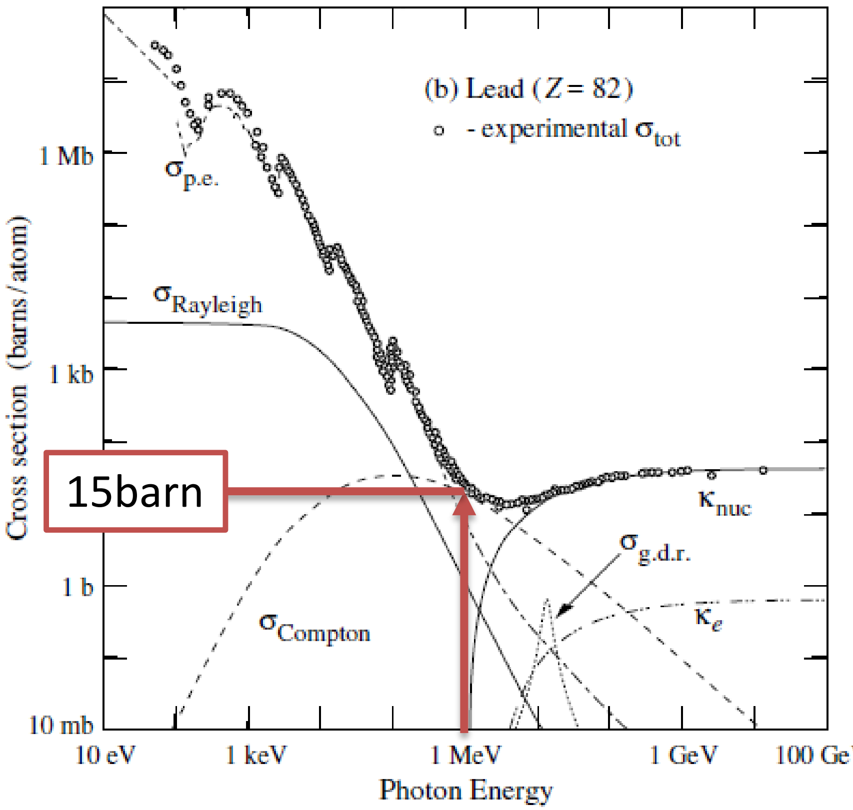
Question: What is the value for λ for a γ of 1MeV energy in Pb?

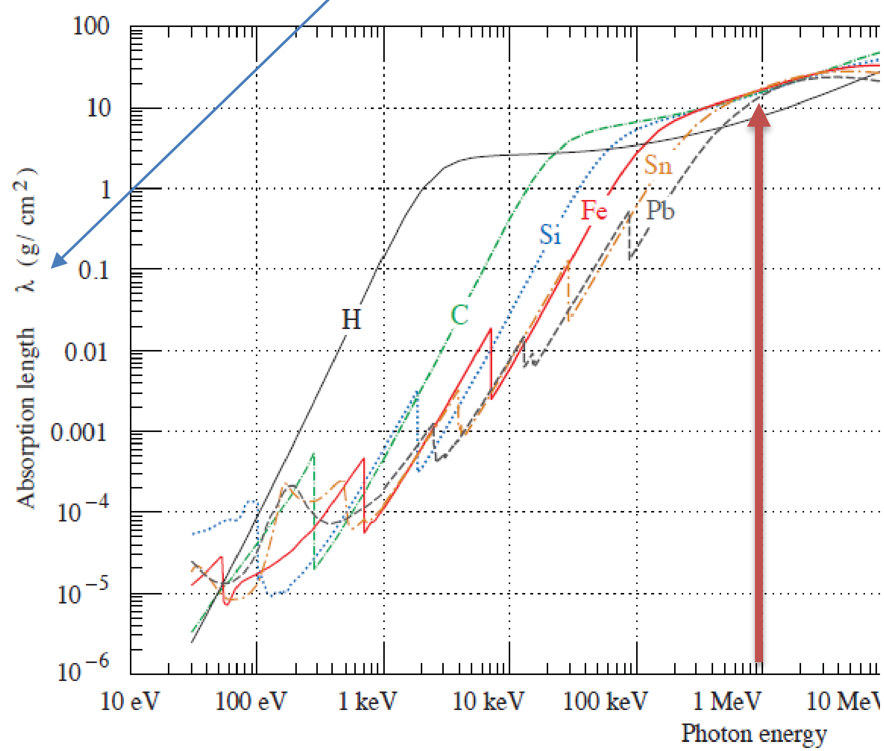
Absorption length of γ

$$\frac{N_A}{M}\rho\sigma = \frac{1}{\lambda}$$

$$\lambda = \frac{M}{N_A\,\rho\sigma} = \frac{207.2g/mol}{6.022\cdot10^{23}1/mol\cdot11.3g/cm^3\cdot15\cdot10^{-24}cm^2} = 2.0\text{cm}$$

Note: The absorption length is scaled with the density





Photoelectric effect

At low energy ($I_0 \ll E_\gamma \ll m_e c^2$):

$$\sigma_{ph} = \alpha \pi a_B Z^5 \left(\frac{I_0}{E_\gamma}\right)^{3.5} \propto \frac{Z^5}{E_\gamma^{\,3.5}}$$

At high energy ($E_\gamma \gg m_e c^2$):

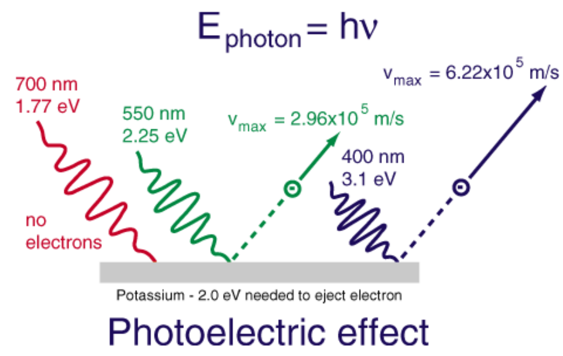
$$\sigma_{ph} = 2\pi r_e^2 \alpha^4 Z^5 \left(\frac{m_e c^2}{E_\gamma}\right) \propto \frac{Z^5}{E_\gamma^{\,1}}$$

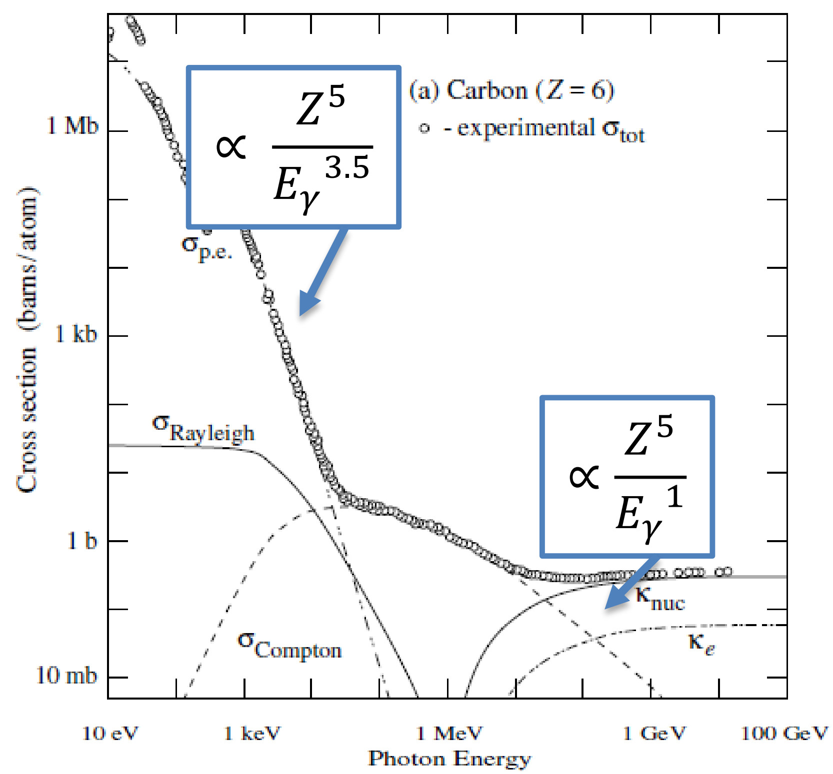
I_0: Ionisation potential

Example: $a_B = 0.53 \cdot 10^{-10}$ m

$I_0 = 13.6$ eV

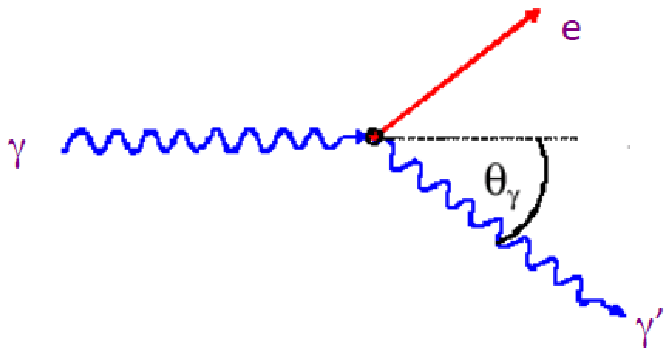
Strong Z dependence of cross section!





Compton scattering

Scattering of γ on quasi-free electrons: $\gamma + e \rightarrow \gamma' + e$



Energy of the outgoing γ:

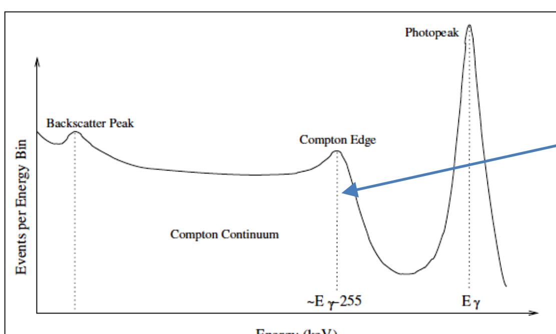
$$\boxed{E_{\gamma'} = \frac{E_\gamma}{1 + (E_\gamma / m_e c^2)(1 - cos\theta_\gamma)}}$$

Kinetic energy of the outgoing e:

$$\boxed{E^e_{kin} = E_\gamma \frac{(1 - cos\theta_\gamma)(E_\gamma / m_e c^2)}{1 + (E_\gamma / m_e c^2)(1 - cos\theta_\gamma)}}$$

Backward scattering $\theta\gamma = \pi$:

$$E^e_{kin\,max} = E_\gamma \frac{2(E_\gamma / m_e c^2)}{1 + 2(E_\gamma / m_e c^2)}$$



Example:
$E_\gamma = 1\ MeV$
$E_{\gamma'\,min} = 0.2\ MeV$
$E^e_{kin\,max} = 0.8$ MeV

The e misses some energy!

Compton cross section

The cross section for the Compton scattering is given by the formula from Klein-Nishina. It is given as the cross section per electron in the atom.

For small photon energy ($E_\gamma \ll m_e c^2$):

$$\sigma^e{}_c = \sigma_{Th}(1\text{-}E_\gamma/m_e c^2)$$

For high photon energy ($E_\gamma \gg m_e c^2$):

$$\sigma^e{}_c \propto (\ln E_\gamma)\,/E_\gamma$$

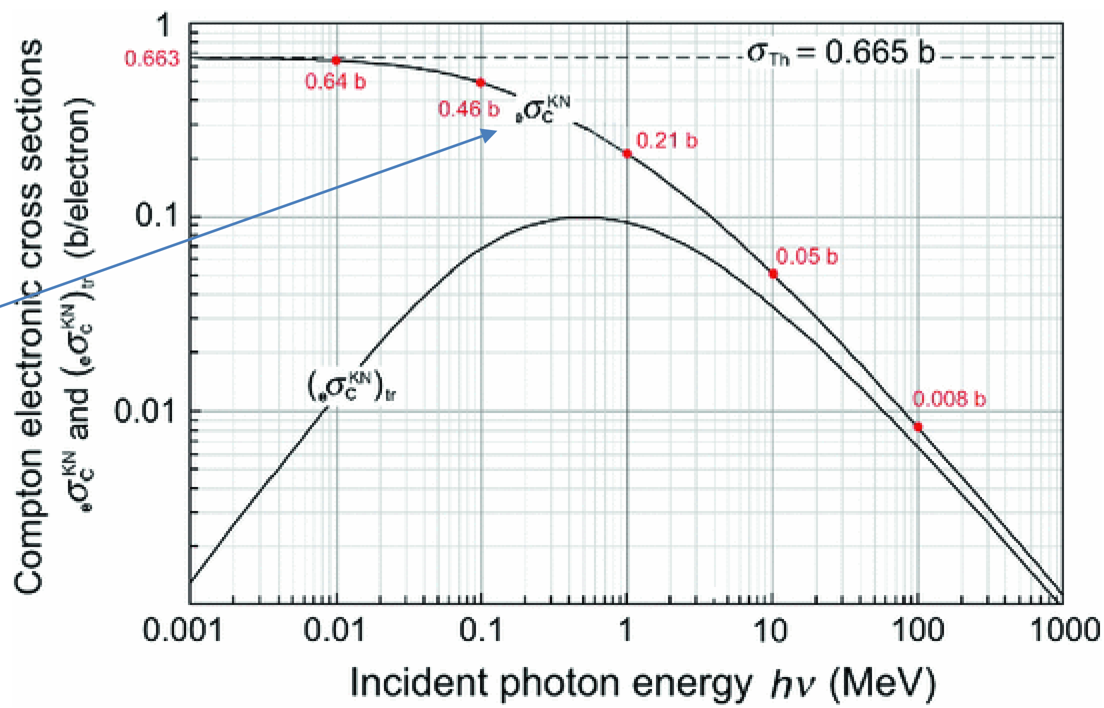
Cross section per atom:

$$\sigma^{atom}{}_c = Z\sigma^e{}_c$$

Thomson cross section:

$$\sigma_{Th} = \frac{8\pi}{3r_e^2} = 0.665\; barn$$

Cross section for Compton scattering



Pair production

For energy-momentum conservation this process can only take place in an EM filed

Pair production in the field of the nucleus:

Threshold process: $E_\gamma > 2m_e c^2(1 + m_e/m_x)$

m_x:Target mass (lighter nuclei means higher threshold), pair production on electrons less likely

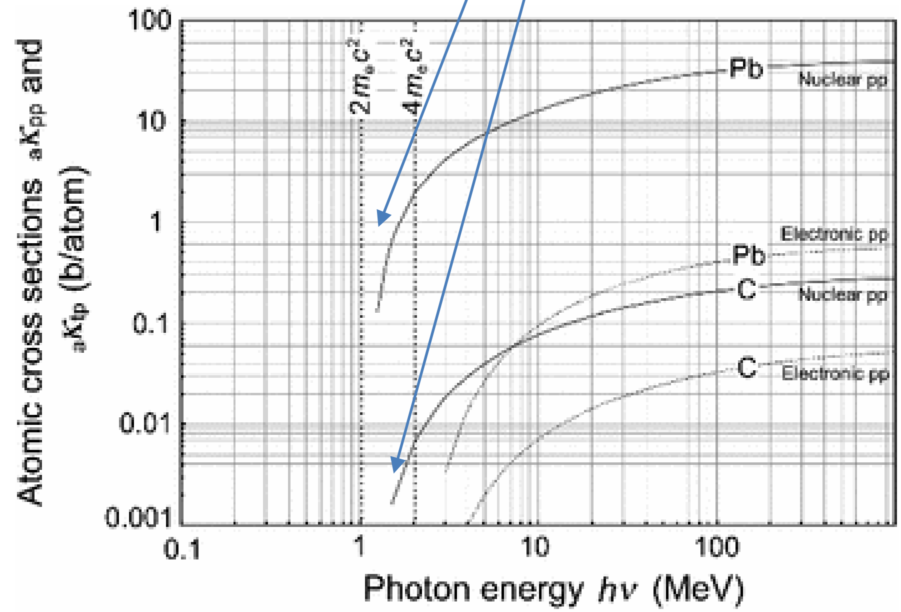
$\varepsilon = E_\gamma/m_e c^2$

$1 \ll \ \varepsilon < 1/\alpha Z^{1/3}$

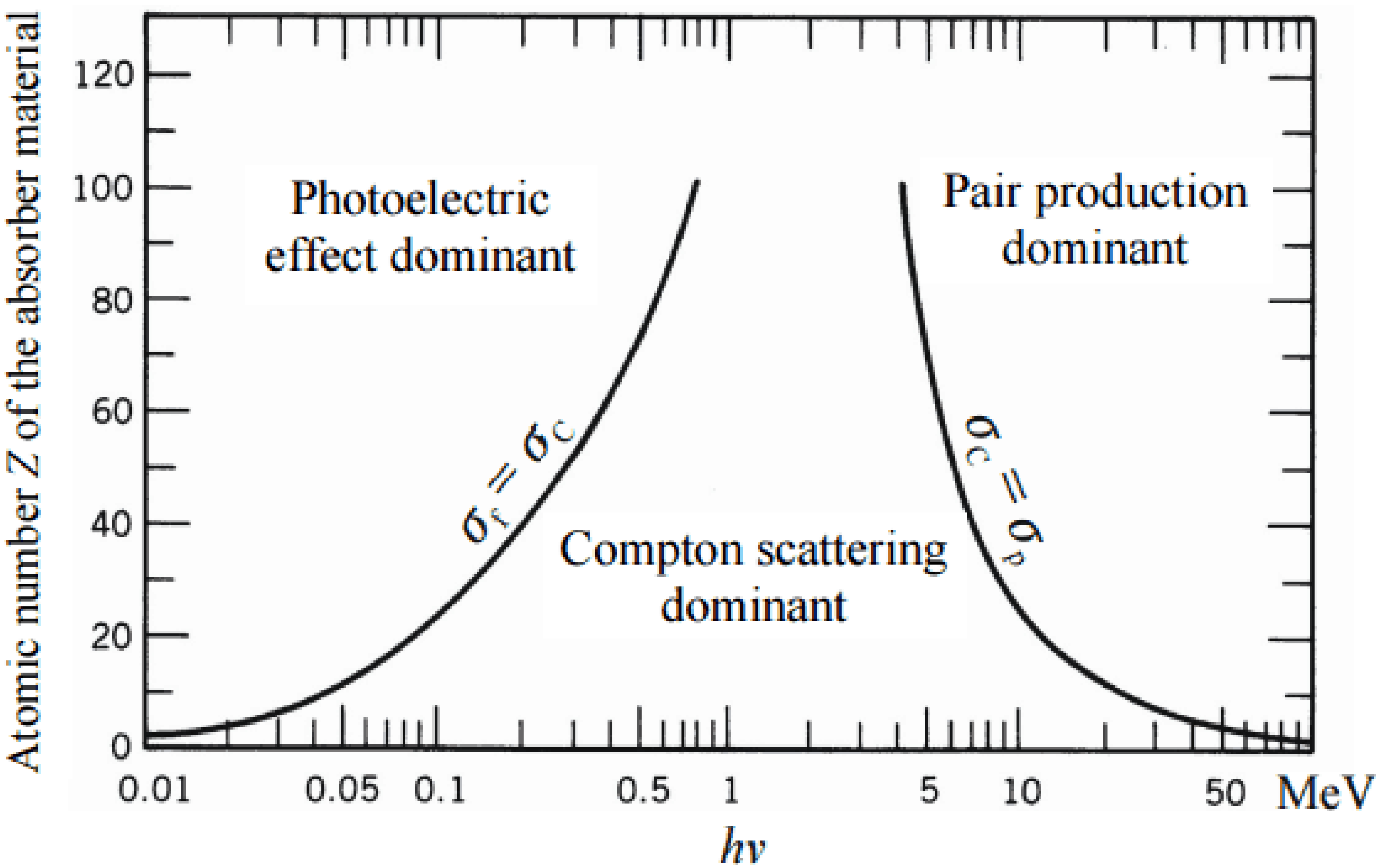
$$\sigma_{pair}^{atom} = 4\alpha r_e^2 Z^2\left(\frac{7}{9}\ln(2\varepsilon) - \frac{109}{54}\right)$$

$\varepsilon \gg 1/\alpha Z^{1/3}$

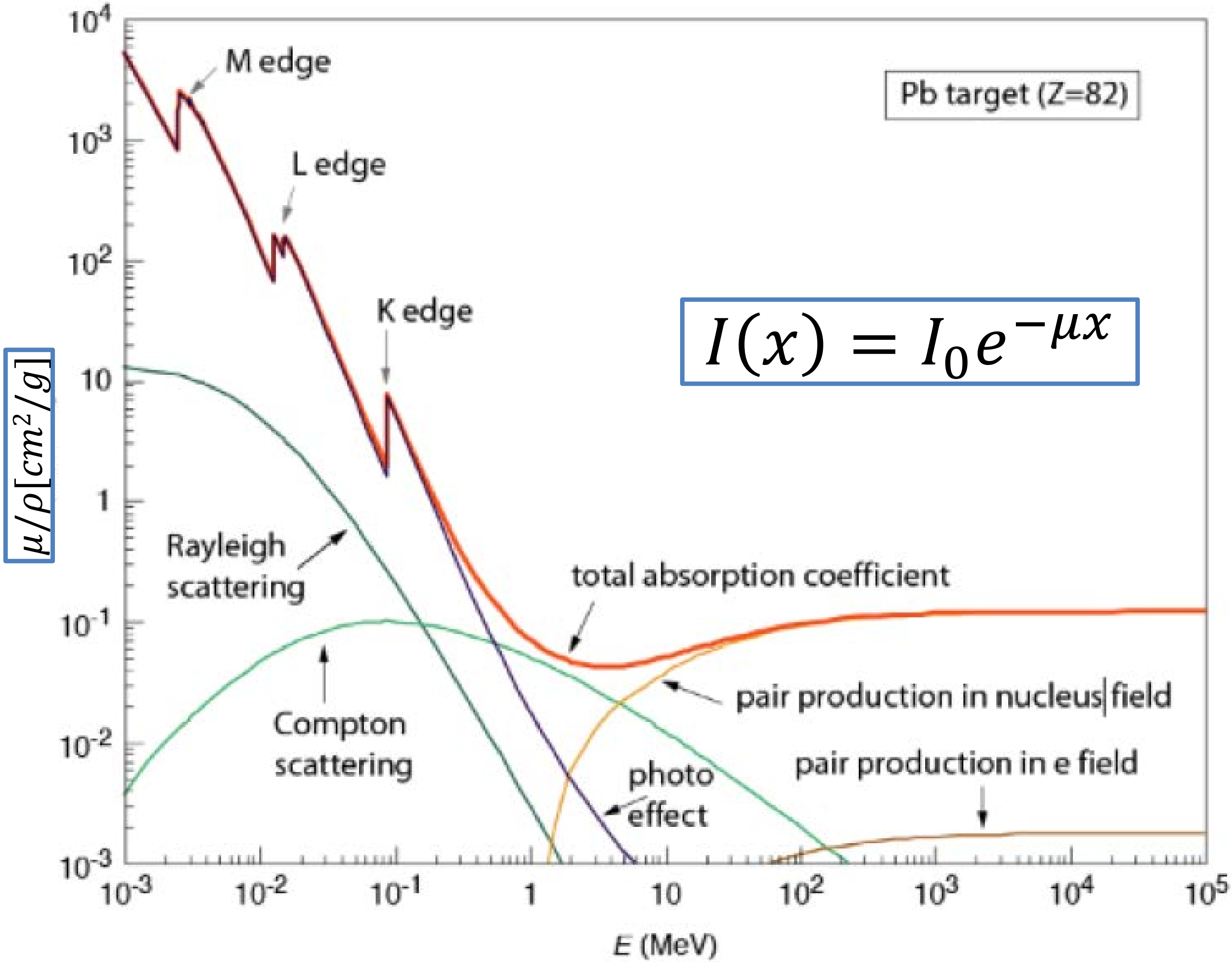
$$\sigma_{pair}^{atom} = 4\alpha r_e^2 Z^2\left(\frac{7}{9}\ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54}\right)$$



Dependence on Z and E



Mass absorption coefficient



$$I(x) = I_0 e^{-\mu x}$$