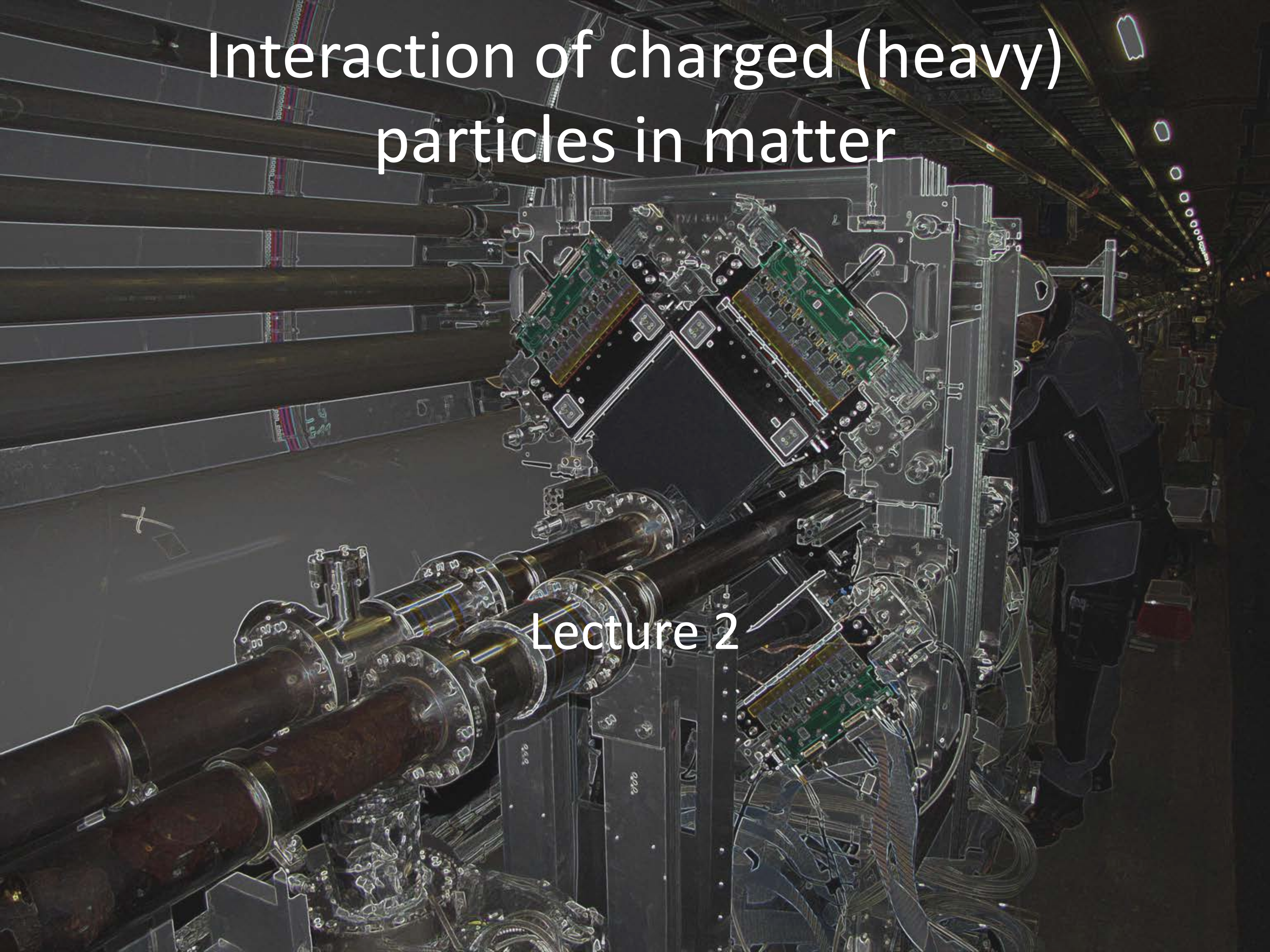
Interaction of charged (heavy) particles in matter

Lecture 2



Principle of particle detection

In order to detect a particle it must interact with the material of the detector.

A transfer of energy between particle and the detector medium is required.

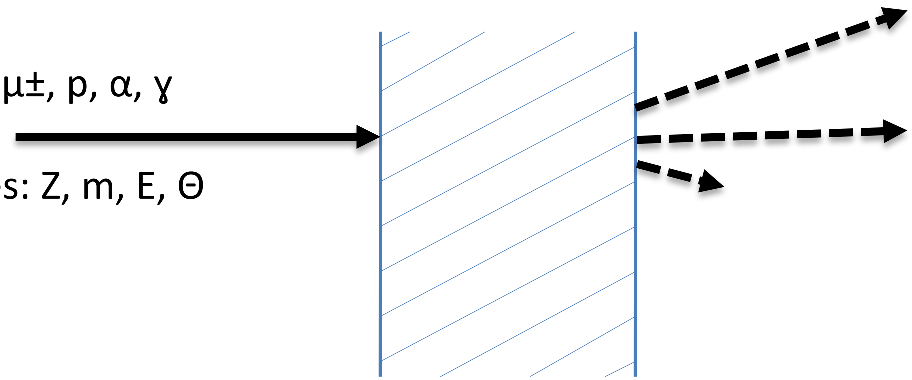
The detection of particles happens via their energy loss!

Performance of particle detectors

Detection efficiency	$\varepsilon_{eff} = N_{detected\ particle} / N_{incident\ particles}$
Position resolution	σ_x
Energy resolution	σ_E/E
Time resolution	σ_t , CTR, SPTR
Particle identification (PID)	Separation power (e, μ, τ, ν, n, K, π)

Particle type: e±, μ±, p, α, ɣ

Particle properties: Z, m, E, Θ



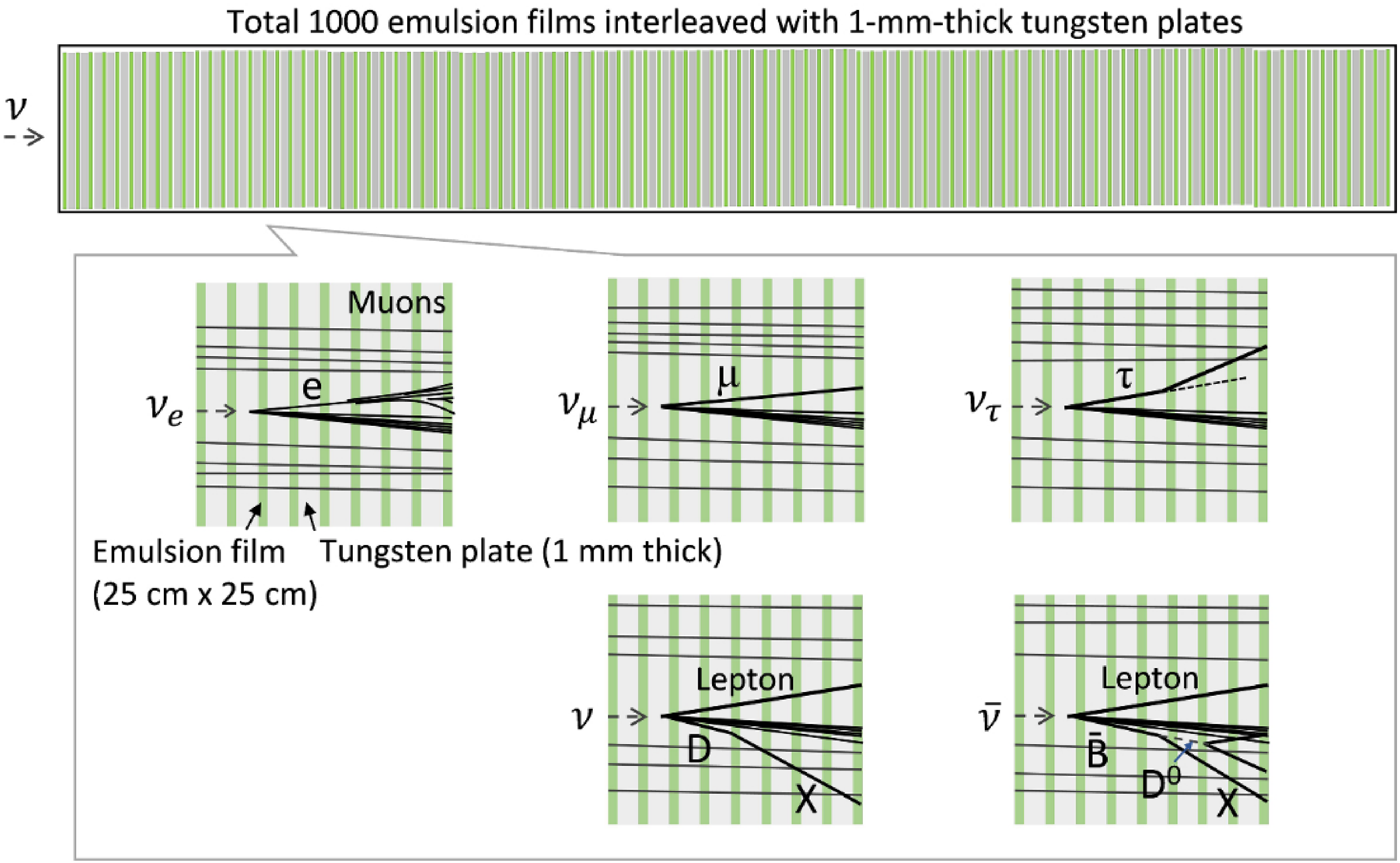
Detector medium characteristics:
Z, A, ρ, d

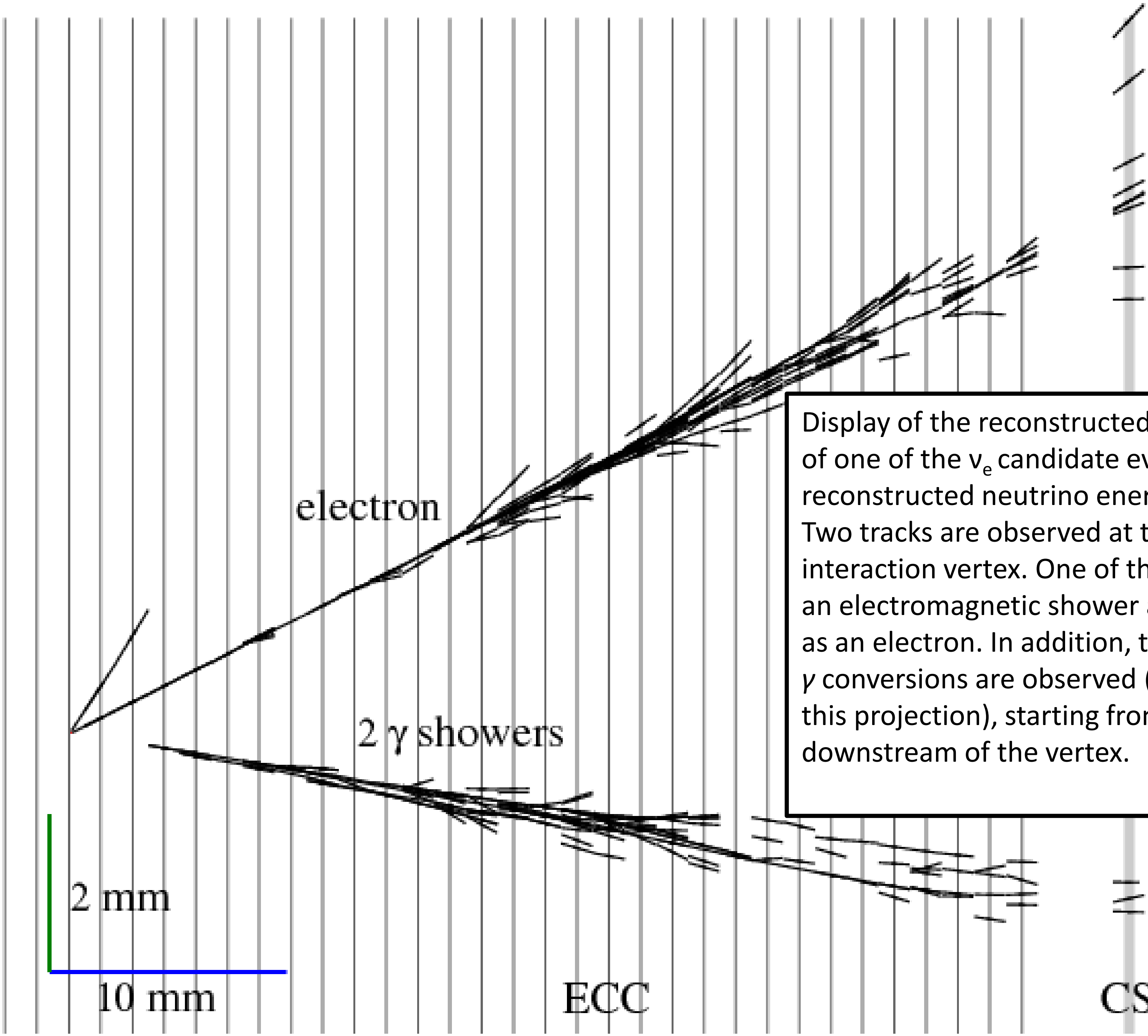
Interaction in matter overview

- Charged particles:
  - Ionisation and excitation of the electrons on the shell of the absorber medium
  - Coulomb scattering in the field of the nucleus
  - Bremsstrahlung as a consequence of the deflection and therefore acceleration
- Photons:
  - Photo electrical effect
  - Compton scattering
  - Pair production
- Hadrons:
  - Inelastic scattering (hadron + nucleus -> new hadrons K, $\pi^{\pm 0}$, n)
- Neutrinos:
  - Scattering on nucleus ($\nu_e + n^0 \rightarrow p^+ + e^-$)

....

Neutrino events in Emulsion Cloud Chamber (ECC)





Display of the reconstructed emulsion tracks of one of the ν_e candidate events. The reconstructed neutrino energy is 32.5 GeV. Two tracks are observed at the neutrino interaction vertex. One of the two generates an electromagnetic shower and is identified as an electron. In addition, two showers from *γ* conversions are observed (overlapping in this projection), starting from 2 and 3 films downstream of the vertex.

Charged heavy particle in matter

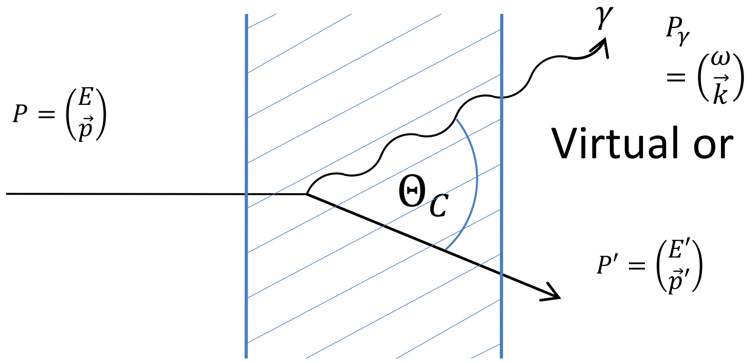
Dominant process for energy loss is the electromagnetic interaction with electrons of atoms.

Interaction with nuclei negligible for energy loss, but they play a role in the angular deflection (multiple scattering).

Particle   M: mass,   v: velocity ($v=\beta c$)

Material   refractive index: n,   dielectric constant: $\varepsilon = \varepsilon_1 + i\,\varepsilon_2$, $\boxed{n^2 = \varepsilon_1}$

dispersion   absorption



Virtual or real photon

Assumption: $\omega << \gamma M = E \Rightarrow \omega << E \Rightarrow |\vec{k}| << |\vec{p}|$

1) Energy-momentum conservation: $\omega = exercise\ldots = vk\cos\Theta_C$

2) Dispersion relation: $\boxed{\omega^2 = \frac{k^2c^2}{\varepsilon}}$ ➡ $\boxed{\sqrt{\varepsilon}\frac{v}{c}\cos\Theta_C = 1}$

With: $c_m = \frac{c}{\sqrt{\varepsilon}} = \frac{c}{n}$ and $\beta_m = \frac{v}{c_m}$ ➡ $\boxed{\cos\Theta_C = 1/\beta_m}$

3 Regions, depending on the photon:

1) Below excitation energy (optical region)

$\varepsilon \; real, \varepsilon > 1$

$\Rightarrow \Theta_C \; real \; for \; \beta_m > 1$

Cherenkov radiation

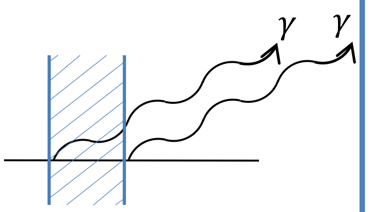
2) Excitation energy (2eV … 5keV) (resonance region)

$\varepsilon = \varepsilon_1 + i\varepsilon_2$ complex with $\varepsilon_2 > 0, \varepsilon_1 \sim 1$ can oscillate

Photons are virtual, no radiation but ionization by exchange of virtual photons.

3) Above excitation energy (X-ray region)

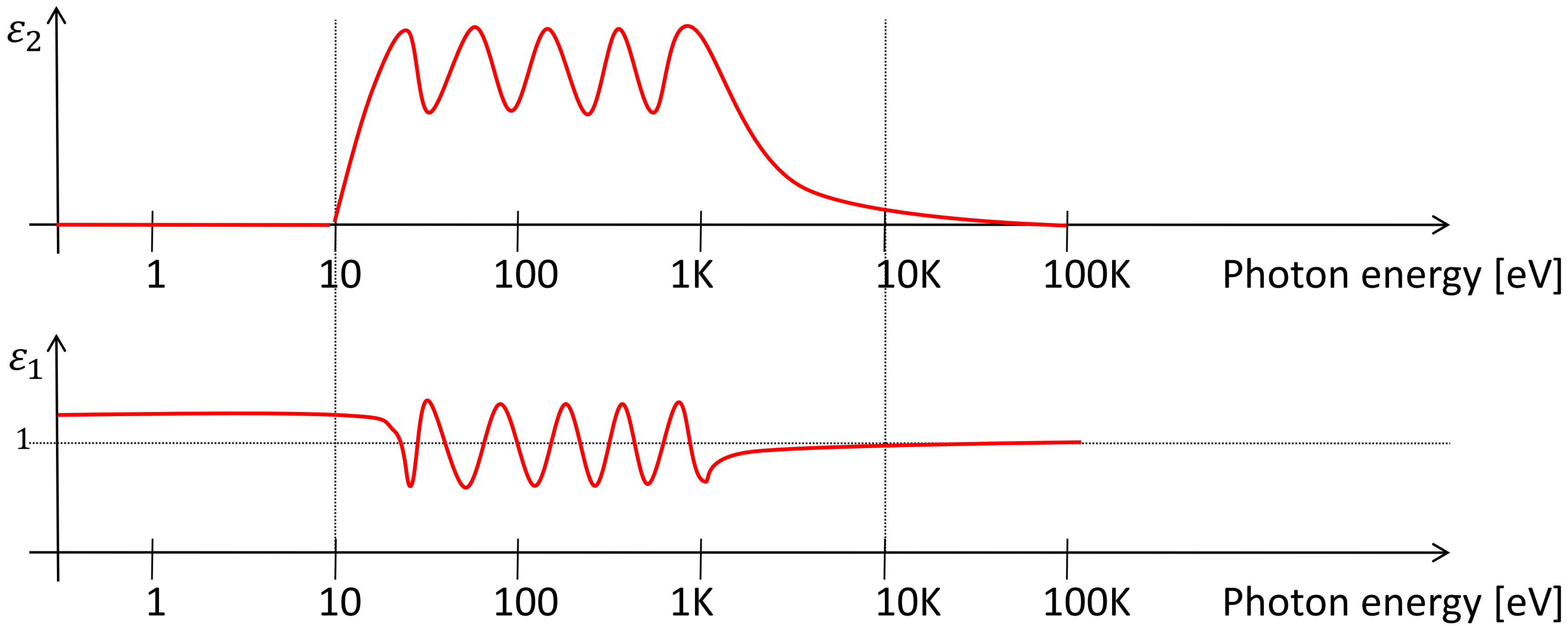
Little absorption: $\varepsilon_2 << 1, \varepsilon_1 < 1$



Photons are virtual, no radiation except at discontinuities

Transition radiation

3 Regions, depending on the photon energy:



Optical	**Resonance**	**X-ray**
Cherenkov radiation	Ionization	Transition radiation
Re(ε)>1, Im(ε)=0	Re(ε)>oscillates, Im(ε)>0	Re(ε)<1, Im(ε) ->0 (vacuum properties, ε=1)

Transverse range of virtual photons

In the resonance region: What is the transverse range of the virtual photons?
How far can they ionize?

Dispersion relation: $$\omega^2 = \frac{k^2 c^2}{\varepsilon}$$

2-dimensionl (z: longitudinal, y=transverse) $k_z = \frac{\omega}{v}$ from: $\omega = vk\cos\Theta_C$

$$k^2 = k_y^2 + k_z^2 = \frac{\omega^2\varepsilon}{c^2} \Rightarrow k_y^2 = \frac{\omega^2\varepsilon}{c^2} - k_z^2 = \frac{\omega^2\varepsilon}{c^2} - \frac{\omega^2}{v^2}$$

$$\Rightarrow k_y = \frac{\omega}{v}\sqrt{\frac{v^2\varepsilon}{c^2} - 1} = \frac{\omega}{v}\sqrt{\beta_m^2 - 1}$$

$$with\ \gamma_m = \frac{1}{\sqrt{1-\beta_m^2}}$$

2 cases:

1) $\beta_m > 1$ (faster than speed of light in medium, water, quartz,...) k_y and k_z real -> a real wave (Cherenkov radiation)

$$\exp\left(i(\vec{k}\cdot\vec{r}-\omega t)\right)$$

2) $\beta_m < 1$ (slower than speed of light in medium, gas) k_y imaginary ➔ damping

$$\exp\left[i(\vec{k}\cdot\vec{r}-\omega t)\right] = \exp\left[i\frac{\omega}{v}(z-vt)\right]\exp\left(-\frac{y}{y_0}\right)$$

Attenuation length:

$$\boxed{y_0 = -\frac{i}{k_y} = \frac{v}{\omega}\frac{1}{\sqrt{1-\beta_m^2}} = \frac{\beta_m\gamma_m}{k} = \frac{c}{\omega}\frac{\beta}{\sqrt{\frac{1}{\gamma^2}+\beta^2(1-\varepsilon)}}}$$

Attenuation length increases linearly with $\beta_m\gamma_m$!

->Expansion of transverse field!

-> relativistic rise in energy loss!

Transverse range of virtual photons, relativistic rise:

2)
$$y_0 = \frac{c}{\omega}\frac{\beta}{\sqrt{\frac{1}{\gamma^2} + \beta^2(1-\varepsilon)}}$$

$\varepsilon_1 > 1$ (optical region) $\varepsilon = \varepsilon_1$ because $\varepsilon_2 = 0$ for the optical region.

If $\beta_m \to 1$ y0 increases until $\beta_m = 1$ where $y_0 = \infty$ Cherenkov radiation sets in

$\varepsilon_1 < 1$ (above ionization threshold)

y0 increases with until β_m up to a maximum:

$$y_0 = \frac{c}{\omega}\frac{\beta}{\sqrt{\frac{1}{\gamma^2} + \beta^2(1-\varepsilon)}} \xrightarrow{\beta\gamma\to\infty} y_0^{\max} = \frac{c}{\omega}\frac{1}{\sqrt{1-\varepsilon}}$$

This is the “relativistic rise” increased transverse range -> interaction with more atoms effect saturates when:

Transverse range of virtual photons, relativistic rise:

$$(\beta\gamma)_{sat} \approx \frac{1}{\sqrt{1-\varepsilon}}$$ Fermi-Plateau

This saturation (described by ε) is classically due to the polarisation of the medium, which will screen the effect of remote atoms.

This effect is larger in denser media $1-\varepsilon \propto \rho$

$$(\beta\gamma)_{sat} \propto \frac{1}{\sqrt{\rho}}$$ “Density effect” -> see density correction in Bethe-Bloch

Symbol	Definition	Value or (usual) units
m_ec^2	electron mass $\times\, c^2$	$0.510\,998\,9461(31)$ MeV
r_e	classical electron radius	
	$e^2/4\pi\epsilon_0 m_ec^2$	$2.817\,940\,3227(19)$ fm
α	fine structure constant	
	$e^2/4\pi\epsilon_0\hbar c$	$1/137.035\,999\,139(31)$
N_A	Avogadro's number	$6.022\,140\,857(74)\times 10^{23}$ mol^{-1}
ρ	density	g cm^{-3}
x	mass per unit area	g cm^{-2}
M	incident particle mass	MeV/c^2
E	incident part. energy γMc^2	MeV
T	kinetic energy, $(\gamma-1)Mc^2$	MeV
W	energy transfer to an electron	MeV
	in a single collision	
k	bremsstrahlung photon energy	MeV
z	charge number of incident particle	
Z	atomic number of absorber	
A	atomic mass of absorber	g mol^{-1}
K	$4\pi N_A r_e^2 m_ec^2$	$0.307\,075$ MeV mol^{-1} cm^2
	(Coefficient for dE/dx)	
I	mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	density effect correction to ionization energy loss	
$\hbar\omega_p$	plasma energy	$\sqrt{\rho\,\langle Z/A\rangle}\times 28.816$ eV
	$\sqrt{4\pi N_e r_e^3}\; m_ec^2/\alpha$	$\hookrightarrow \rho$ in g cm^{-3}
N_e	electron density	(units of r_e)$^{-3}$
w_j	weight fraction of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
X_0	radiation length	g cm^{-2}
E_c	critical energy for electrons	MeV
$E_{\mu c}$	critical energy for muons	GeV
E_s	scale energy $\sqrt{4\pi/\alpha}\; m_ec^2$	21.2052 MeV
R_M	Molière radius	g cm^{-2}

Electronvolt “eV”

Measurement	Unit	SI value of unit
Energy	eV	$1.602176634 \times 10^{-19}$ J
Mass	eV/c^2	1.782662×10^{-36} kg
Momentum	eV/c	5.344286×10^{-28} kg*m/s
Temperature	eV/k_B	$1.160451812 \times 10^{4}$ K
Time	$\hbar/\mathrm{eV}$	6.582119×10^{-16} s
Distance	$\hbar c/\mathrm{eV}$	1.97327×10^{-7} m

(dE/dX) classical derivation (Bohr)

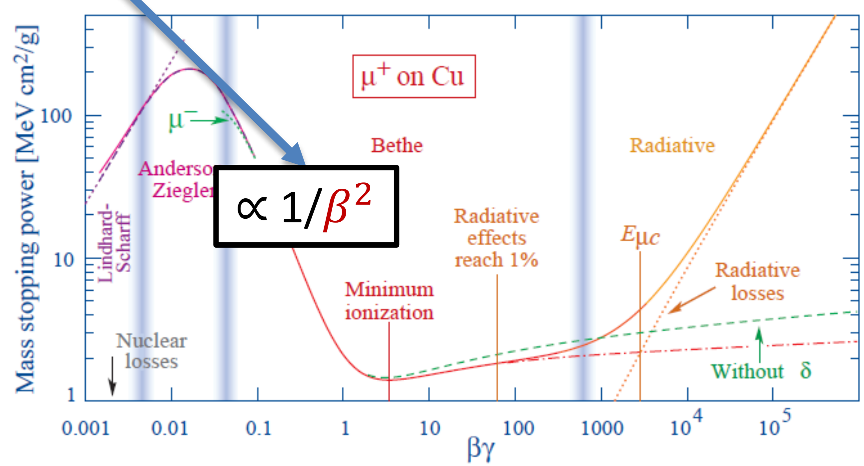
$$\left\langle -\frac{dE}{dX} \right\rangle = K\, z^2\, \frac{Z}{A}\, \frac{1}{\beta^2}\, \frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2}{I}$$

$$\mathrm{K} = \frac{4\,\pi\, e^4}{c^2 m_e}\, N_A = 0.31\ MeV\, cm^2/g$$

- Mean energy loss normalized to the density of the absorber (x -> X)
- Almost independent of the material of the absorber ∝ Z/A (H is an exception) and $\ln\left(\frac{1}{I}\right)$
- Particle “energy dependence” is $\propto 1/\beta^2$

- Units [dE/dx]=MeV/cm and [dE/dX]=MeV cm^2 g^{-1}

Classical effect:
Faster particles have less time to interact and therefore loose less energy!



(dE/dX) Bethe-Bloch formula (quantum mechanical effects)

The mean energy loss of relativistic charged heavy particles is described by the "Bethe-Bloch equation".

$$W_{Max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} \approx 2m_e c^2 \beta^2 \gamma^2 \qquad M >> 2\gamma m_e$$

(max. kin. energy transferred to a free electron in single collision)

relativistic rise

number of charges of incident particle

density correction

$$\left\langle -\frac{dE}{dX} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Classical without W_{max}

Describes well the mean energy loss in the region of $0.1 < \beta\gamma < 1000$ for intermediate Z materials with an accuracy of a few percent.

Bethe-Bloch formula

Discussion:

- Function of β only independent of the incident particle mass!
- Lower limit valid: βc larger than that of orbital electrons
- Upper limit valid: As long as radiative effects do not dominate
- Dependence on absorber material is small ∝ Z/A (H is an exception)
- I is the mean excitation energy $\boxed{I \approx (10 \pm 1)eV \times Z}$
- Incident particle charge ∝ z^2

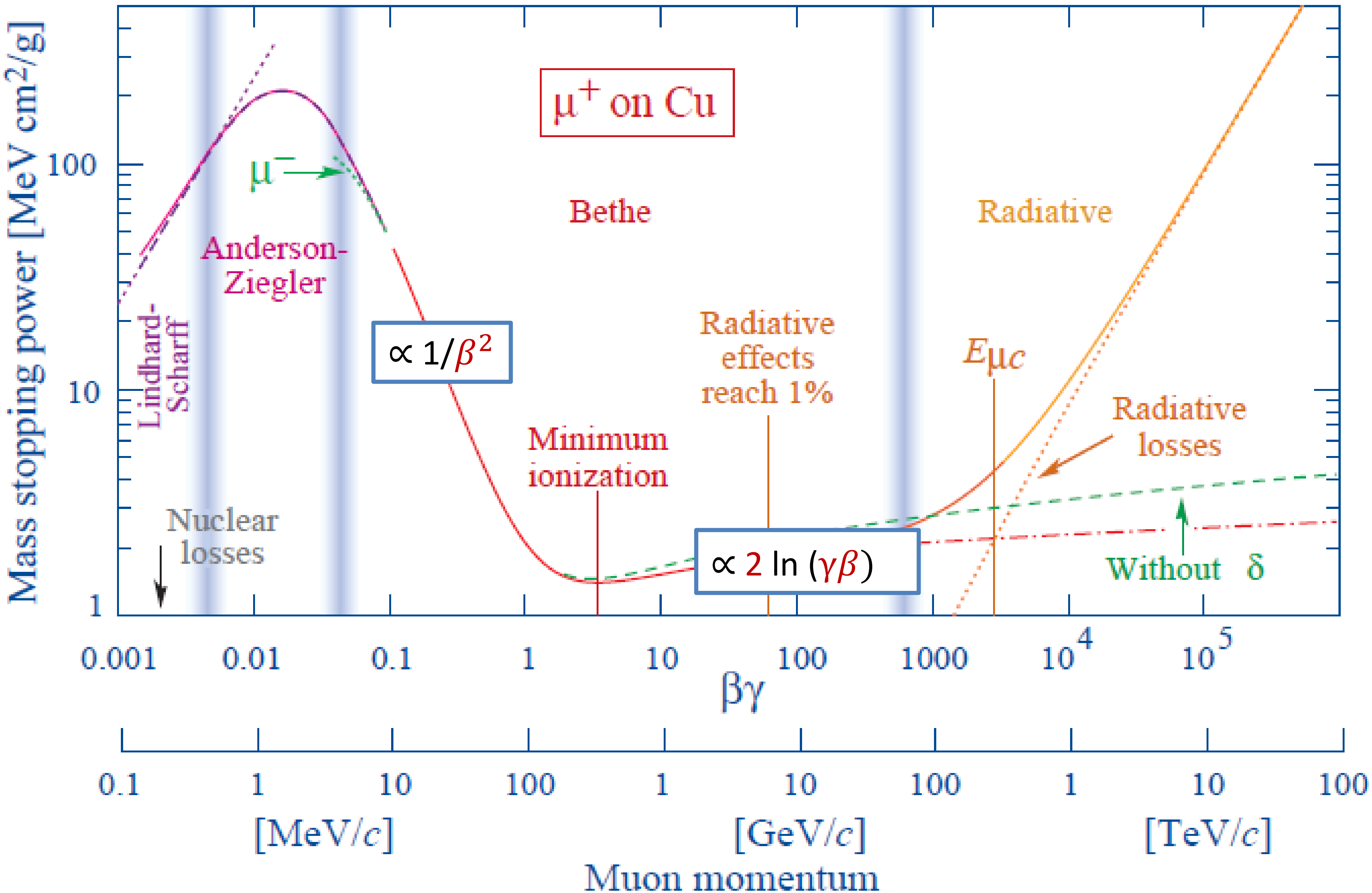
<u>Minimum ionization Particle</u> (MIP)

In typical high energy physics experiments, most particles are MIP

$$\boxed{\beta\gamma \approx 3 - 3.5, \qquad \gamma = \sqrt{1 + (\beta\gamma)^2} \approx 3.2 - 3.6, \beta \approx 0.95}$$

<u>Relativistic rise</u> Transverse extension of ionization is a relativistic effect and increases for higher relativistic particles

<u>Density effect</u> limitation of transverse range through polarization of medium large correction for high density 50%-70% relativistic rise for noble gases much less for liquids and solids.



Mass stopping power [MeV cm²/g]
μ⁺ on Cu
μ⁻
Bethe
Radiative
Anderson-
Ziegler
Lindhard-
Scharff
∝ 1/β²
Radiative
effects
reach 1%
Eμc
Radiative
losses
Minimum
ionization
Nuclear
losses
∝ 2 ln (γβ)
Without δ
βγ
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

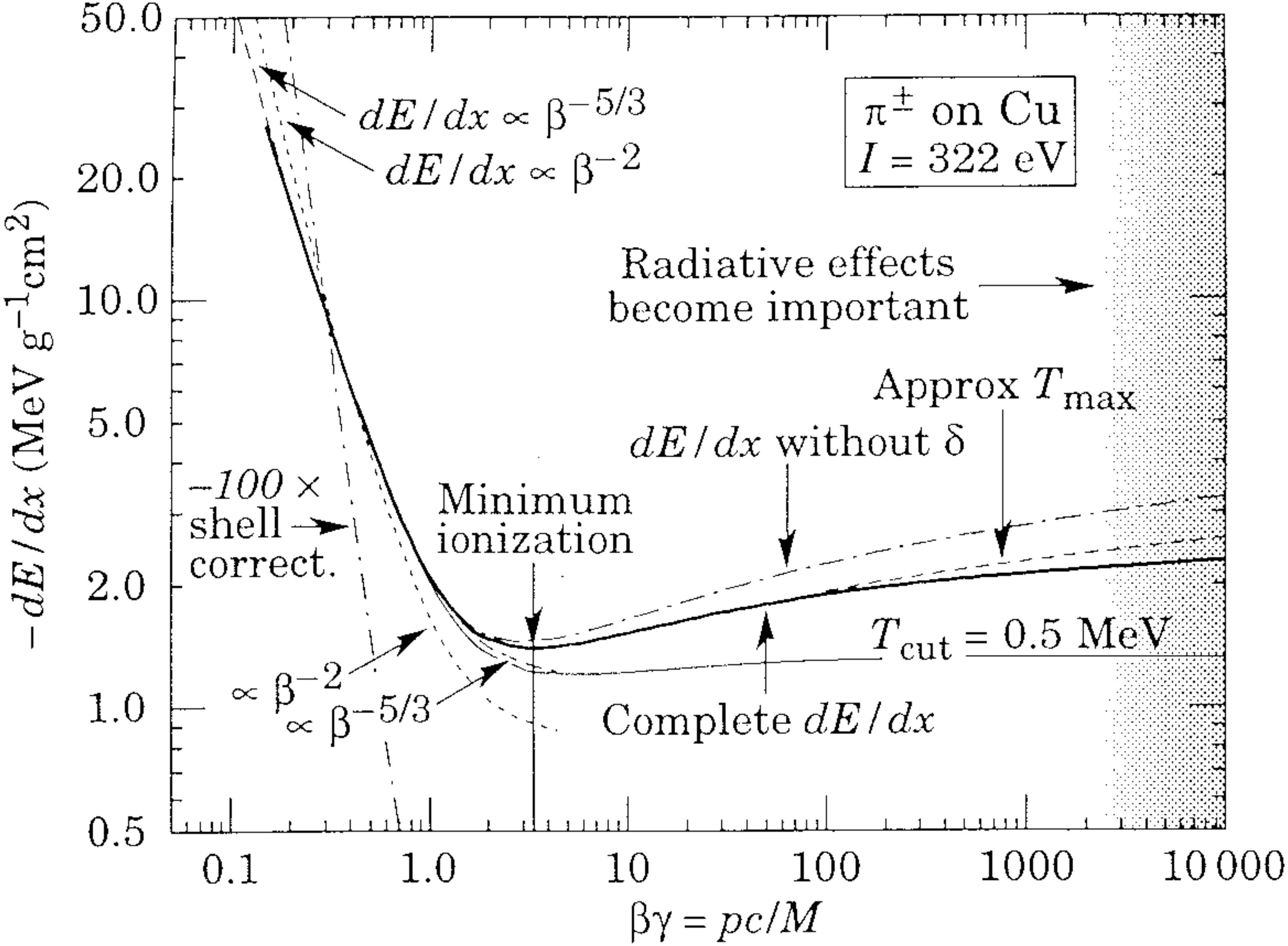


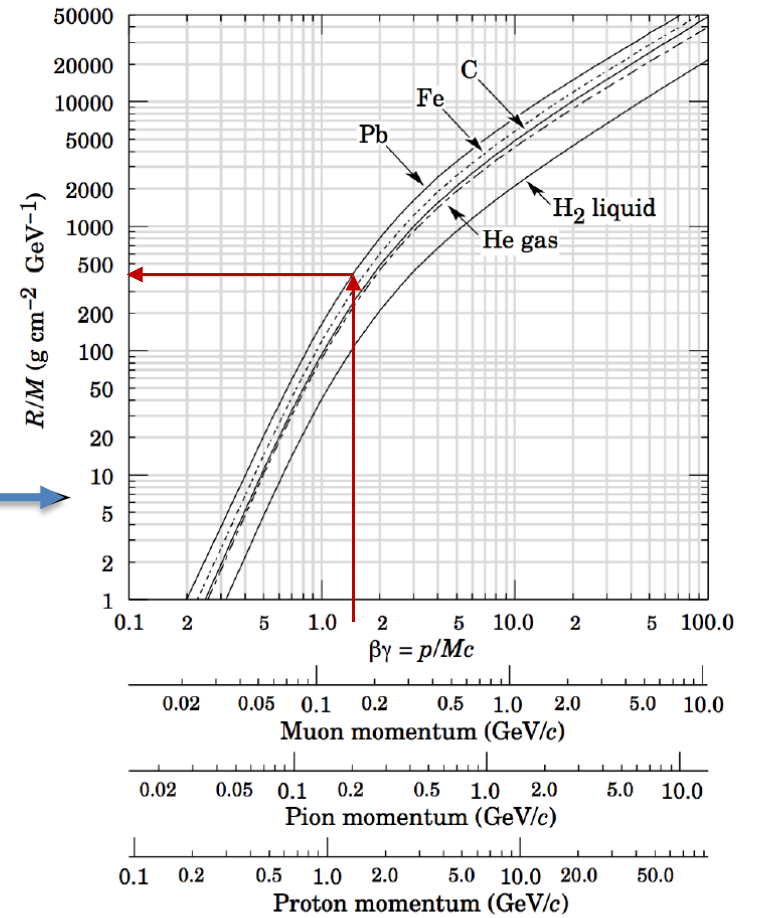
Figure 14.1: Energy loss rate in copper. The function without the density effect correction is also shown, as is the shell correction and two low-energy approximations.

Range of particles

Integration of the energy loss from the initial energy E to zero allows to calculated the range.

$$R = \int_E^0 \frac{dE}{dE/dx}$$

The graph shows the range of heavy charged particles in liquid hydrogen, gaseous helium, carbon, iron, and lead.
R: Range M: particle mass
Example: K+ of p=700MeV/c =>
$\beta\gamma$=1.42, ρ_{Pb}=11.3g/cm^3 R/M=396 =>
R=195gcm^{-2} (r=17cm)



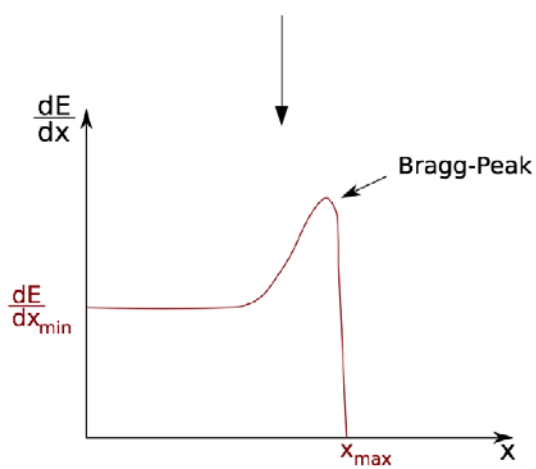
Bragg-Peak

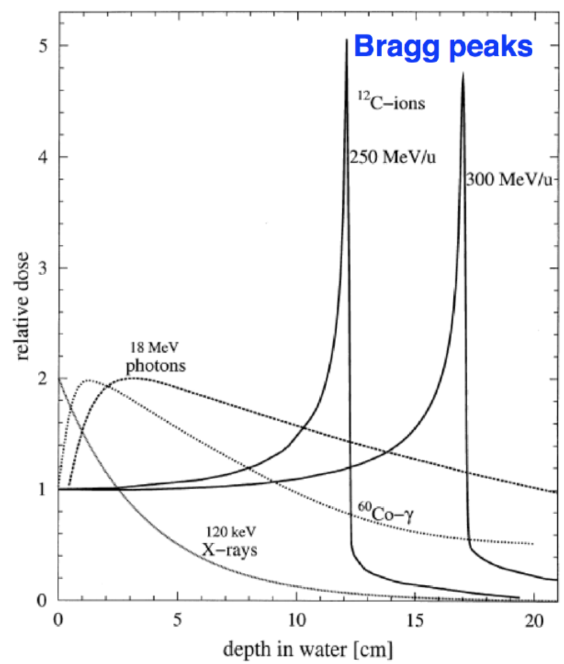
The high specific energy per distance in a material of charged heavy particle produce a "sharp" penetration depth. The large energy deposit at for the "nearly" stopped particle is known by the name Bragg-Peak.

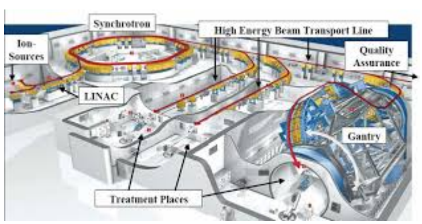
The ion-beam treatment (proton, ^{12}C) is used in medical applications allowing to deposit a large relative dose at a defined depth (water illustrated on the bottom right hand plot). Precise 3D irradiation profiles can be obtained. The large cost of the complex installation of hadron accelerators make the "conventional" gamma irradiation based on electron accelerators (Linacs) the more practical solution.

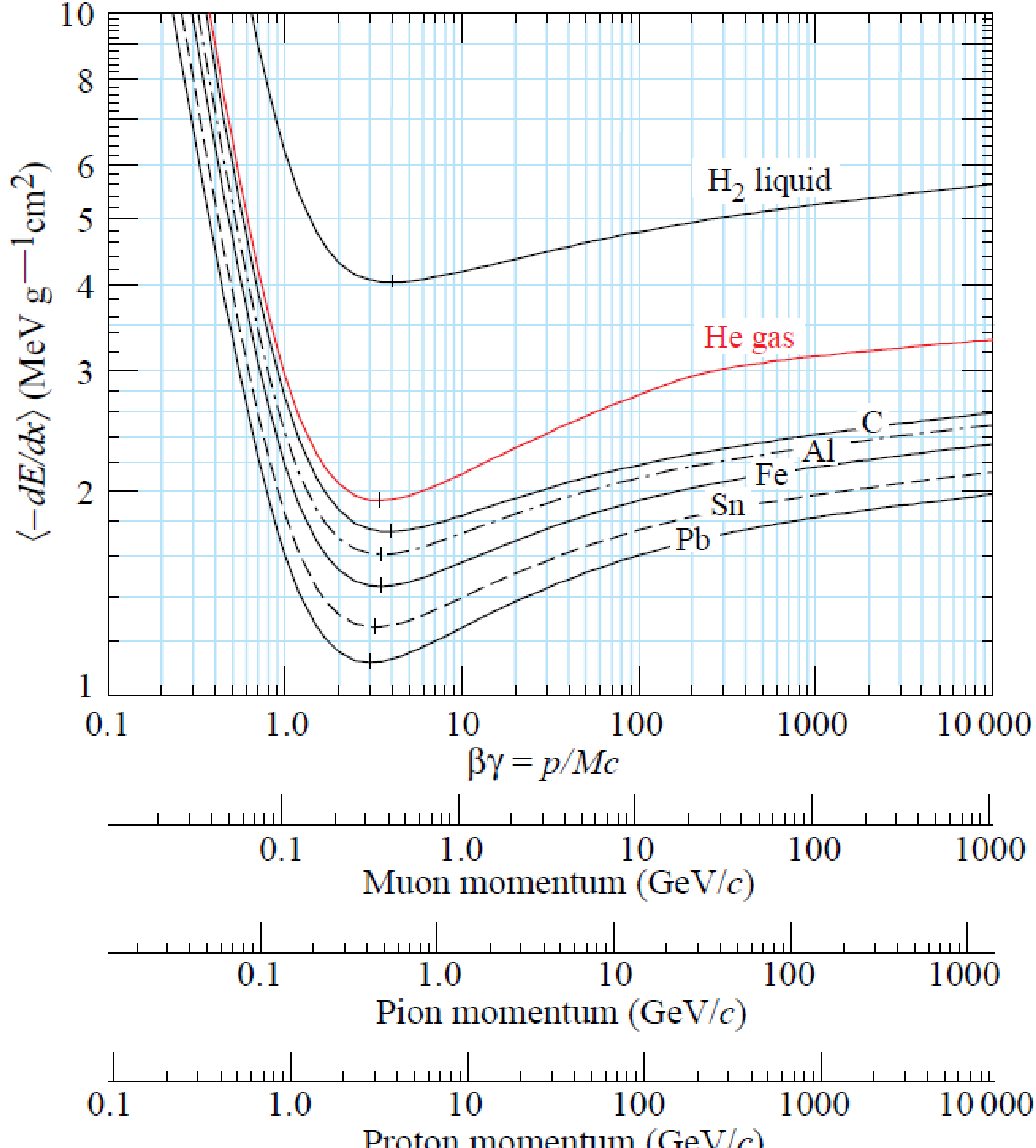
for $\beta\gamma \simeq 3.5$ $\quad \langle \frac{dE}{dx} \rangle \simeq \frac{dE}{dx}_{min}$

for $\beta\gamma \lesssim 3.5$ $\quad$ steep rise $\quad \langle \frac{dE}{dx} \rangle \gg \frac{dE}{dx}_{min}$







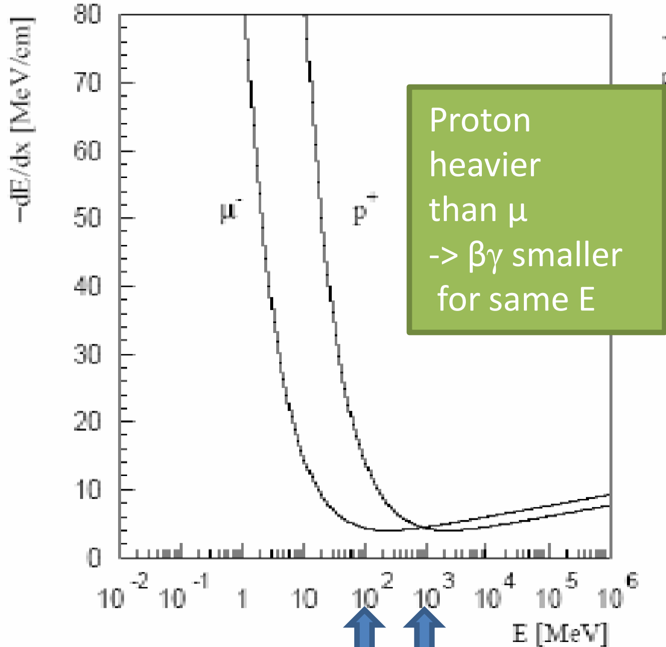


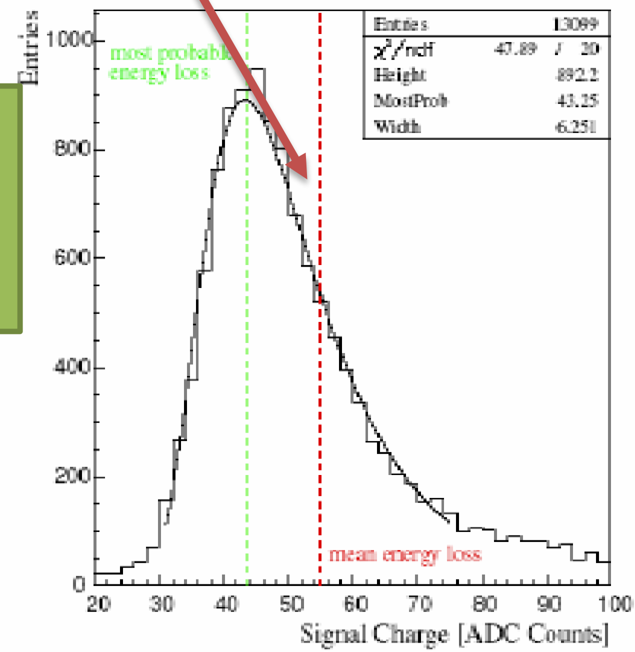
⟨−dE/dx⟩ (MeV g−1cm2)
H2 liquid
He gas
C
Al
Fe
Sn
Pb
βγ = p/Mc
Muon momentum (GeV/c)
Pion momentum (GeV/c)
Proton momentum (GeV/c)

Example: Energy loss in Silicon

$$\left\langle \frac{dE}{dX} \right\rangle = 1.67 MeV\ g^{-1} cm^2$$

For d=300µm silicon (Si) , ρ_{Si}=2.33g/cm^3
$$w_{e-h} = 3.6eV, \Delta E = 117keV \Rightarrow N_{e-h} = 32500$$





Minimal ionizing

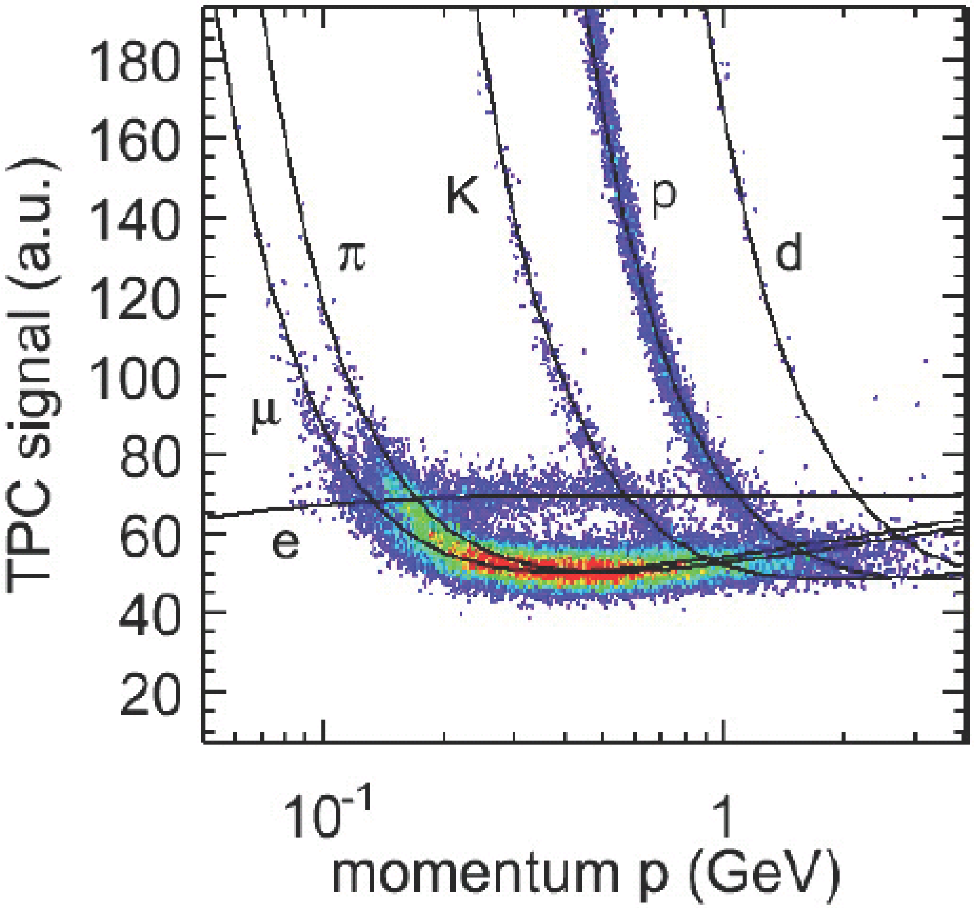
Note that the MPV (median) is smaller than the mean energy deposit!

Comparison with "plastic" Polystyrene as used for SciFi using the same energy deposit:

$$\left\langle \frac{dE}{dX} \right\rangle = 1.89 MeV\ g^{-1} cm^2$$

For d=0.584mm polystyrene (PS) , ρ_{PS}=1.06g/cm^3
$$\Delta E = 117keV$$

dE/dx method



Energy loss measurement in ALICE TPC, 2009

dE/dx is proportional to the particle velocity:

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

Cherenkov radiation detector

A charged particle with velocity β in a medium with refractive index n may emit light along a conical wave front in form of Cherenkov radiation.

The angle (Cherenkov cone half angle) of emission is given by: $\cos\Theta_C = \frac{1}{\beta \cdot n(\lambda)}$

The number of photo electrons detected in a given device is:

$$N_{PE} = L \frac{\alpha^2 z^2}{r_e m_e c^2} \int \varepsilon(E) \cdot \sin^2 \Theta_C(E) \cdot dE$$

ε(E): Efficiency for collection and detection of Cherenkov light
L: Length of the detector radiator
n(λ): Wavelength dependant refractive index
N_o: Detector quality factor

The detection efficiency and the Cherenkov angle depend on the photon energy E. As the typical energy dependent variation of n is small, the integral can be split into the mean value for the $\sin^2\theta$ and the detection efficiency.

$$\boxed{N_{PE} = L N_0 \langle \sin^2 \Theta_C \rangle}$$ with $$N_0 = \frac{\alpha^2 z^2}{r_e m_e c^2} \int \varepsilon \, dE$$

Big Θ_C ensures large number of photons!

Cherenkov angle for different radiators Example 3 RICH detectors at LHCb

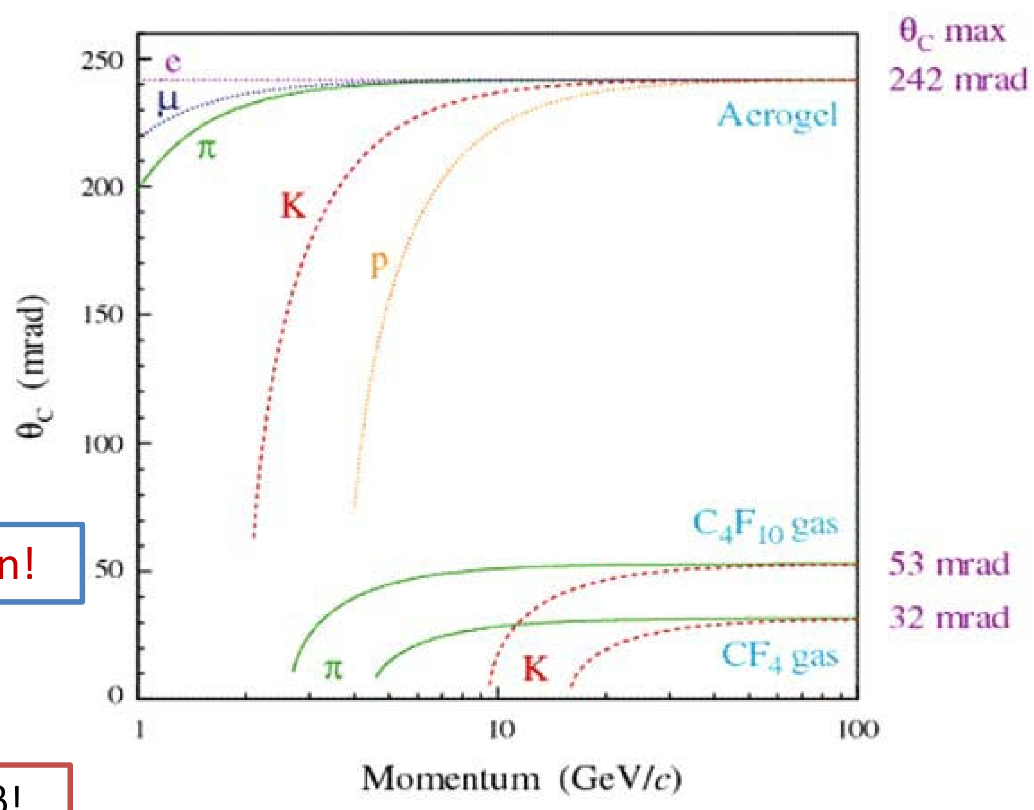
- Each radiator is optimized for a different momentum region
- Particle identification PID is based on the measurement of the Cherenkov angle.

$$\boxed{\frac{\Delta\beta}{\beta} = \tan\Theta_C \cdot \Delta\Theta_C}$$

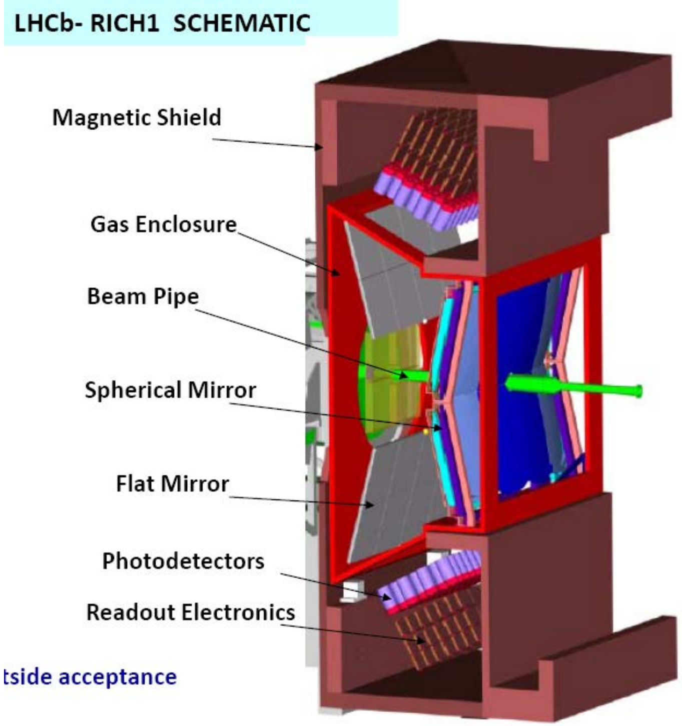
Small θ_C ensures good velocity resolution (i.e. mass) resolution. But, N_{PE} grows with θ_C and $\Delta\theta_C$, so The detector is inefficient if θ_C too small.

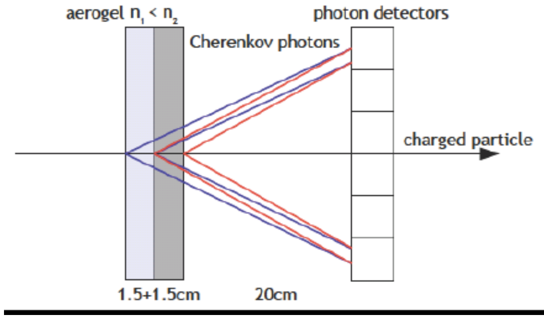
Small Θ_C ensures good velocity resolution!

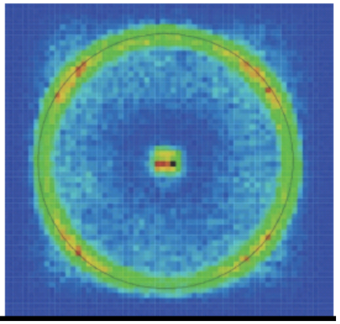
=> Use three different radiators for different β!



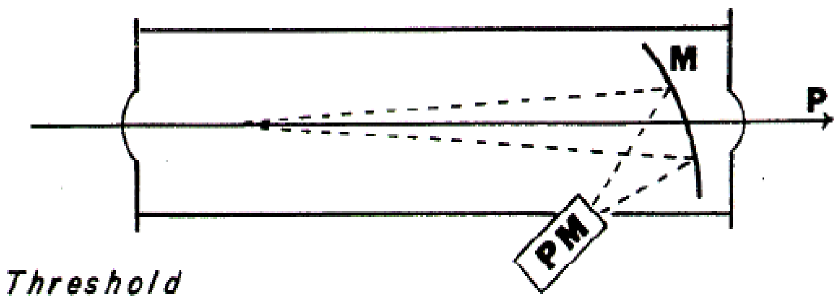
RICH in LHCb

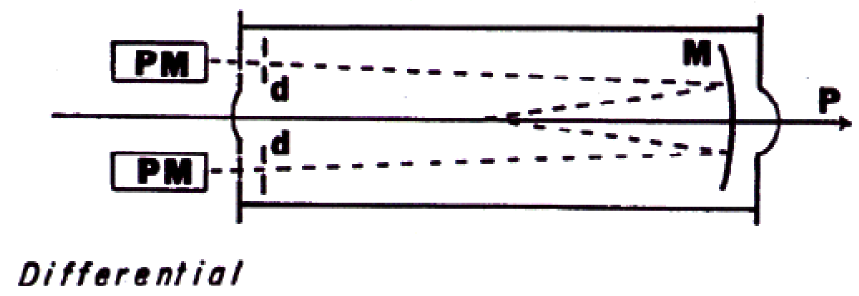

LHCb- RICH1 SCHEMATIC
Magnetic Shield
Gas Enclosure
Beam Pipe
Spherical Mirror
Flat Mirror
Photodetectors
Readout Electronics
tside acceptance

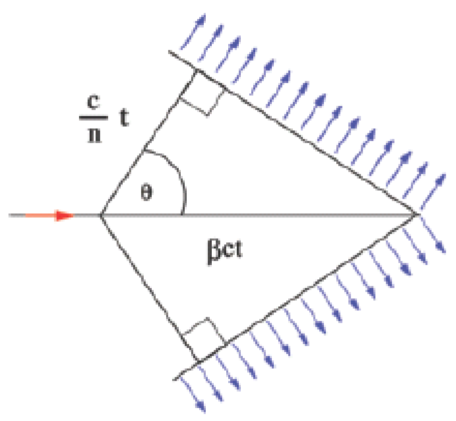

aerogel $n_1 < n_2$
photon detectors
Cherenkov photons
charged particle
1.5+1.5cm
20cm

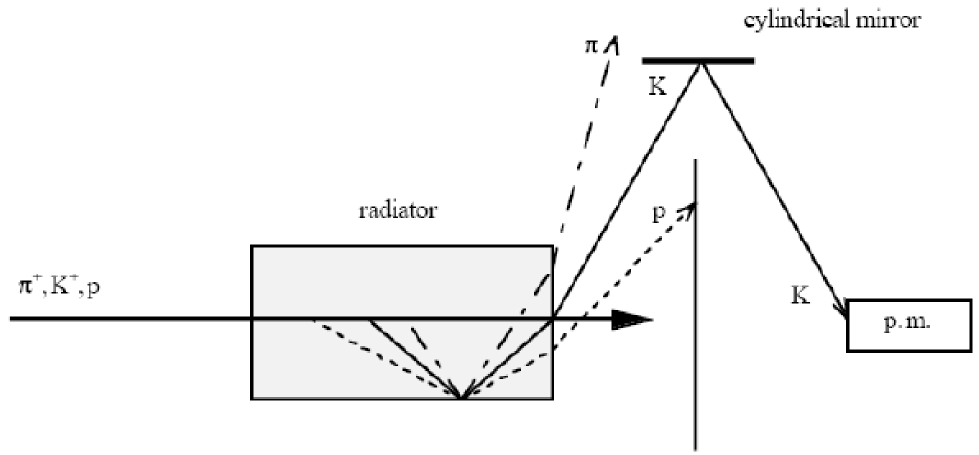


Cherenkov detectors









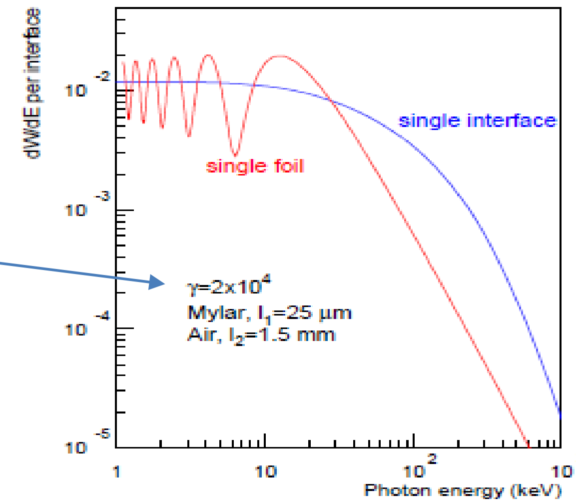
Transition radiation detector TRD

Transition radiation is electromagnetic radiation emitted when a charged particle traverses a medium with a discontinuous refractive index, e.g. the boundaries between vacuum and a dielectric layer.

- The radiation energy per medium to vacuum boundary transition

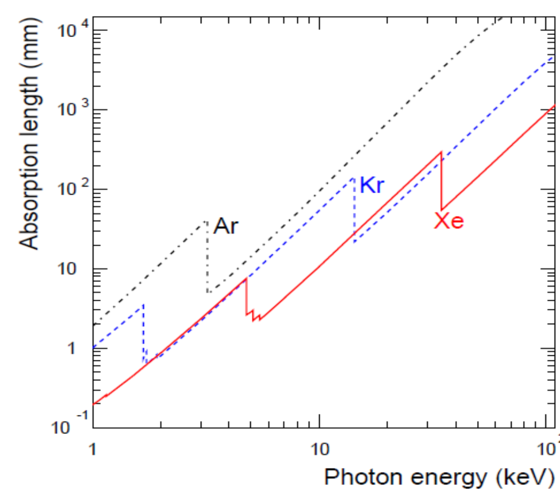
$$\boxed{W = \frac{1}{3}\alpha\hbar\omega_P\gamma}$$

Only high energetic $e^{+/-}$ emit TR of detectable intensity and can be used for PID (high Lorentz factor γ is required)



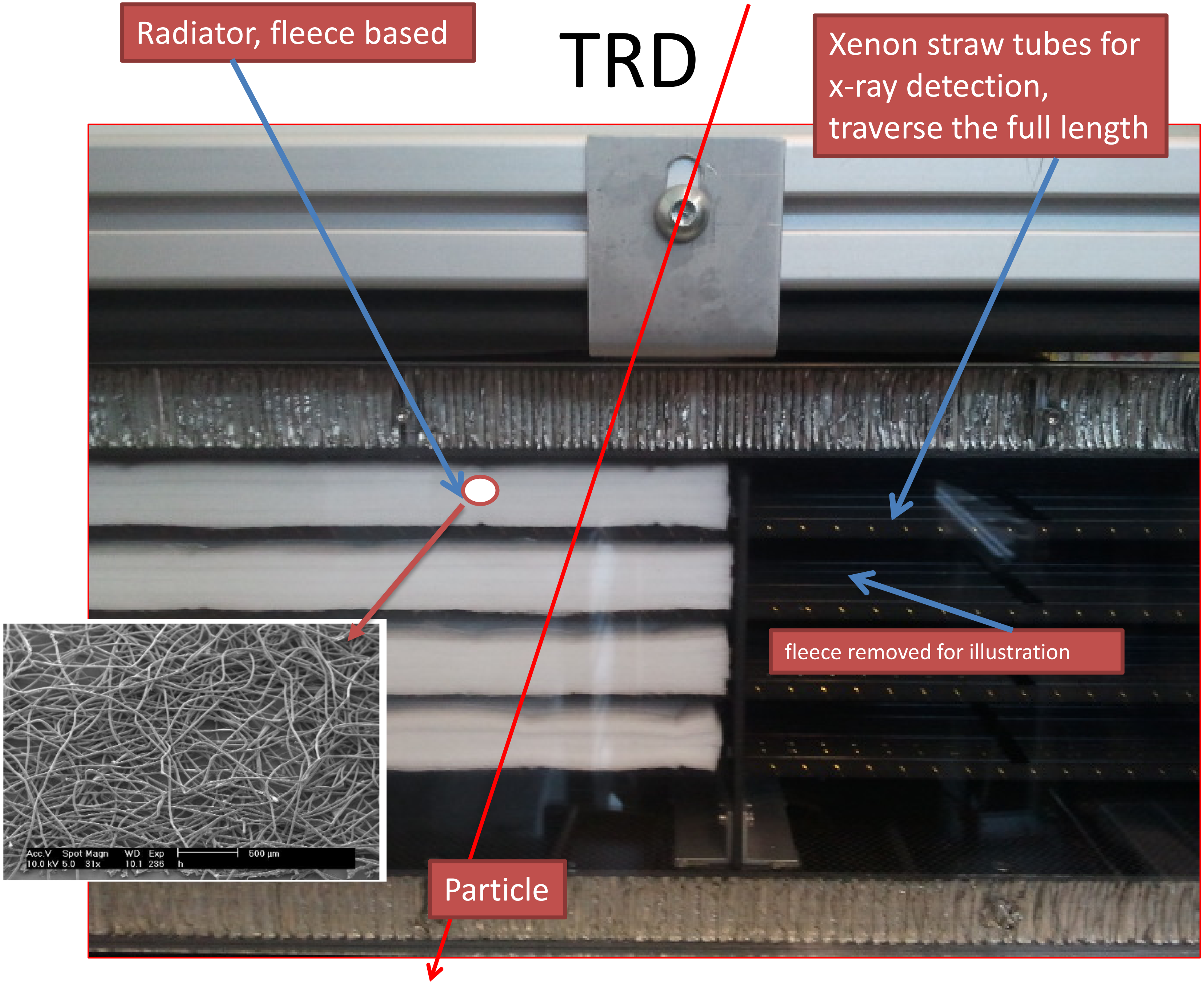
- Plasma frequency $\omega_P = \sqrt{\dfrac{N_e e^2}{\varepsilon_0 m_e}}$ for plastic radiators $\boxed{\hbar\omega_P \approx 20eV}$

- Detector with many transitions (foil stack) to enhance signal are usually build. Typical photon energy is in the x-ray region. Angular emission is very foreword. $\boxed{\Theta \propto 1/\gamma}$



- Particle must traverse a minimum distance, the so-called formation zone Z_f in order to efficiently emit TR , Z_f(air)~mm, $Z_f(CH_2)$~20um, important for detector radiator design.

TRD


Radiator, fleece based
Xenon straw tubes for x-ray detection, traverse the full length
fleece removed for illustration
Particle
Acc.V Spot Magn WD Exp 500 µm
10.0 kV 5.0 31x 10.1 236 h