

Quantum Field Theory

Set 1

Exercise 1: System of natural units

Using the *system of natural units* ($\hbar = c = \mu_0 = \epsilon_0 = 1$), express the dimensionality of the following quantities in powers of electronVolts (eV):

- Mass (M)
- Time (T)
- Length (L)
- Velocity (v)
- Force (F)
- Electric charge (Q)
- Electric Field (E)
- Magnetic Field (B)

Recall that $\hbar \sim ML^2/T$ and $c \sim L/T$.

Exercise 2: Dimensional Analysis

Consider a particle with mass m . Work in a system of units where both $\hbar$ and c are different from 1 (for example the cgs system). Construct an object with the units of length using these three quantities. What is its physical interpretation? Compute this length in cm for a proton and an electron whose masses are, **in natural units**, $m_p = 938$ MeV and $m_e = 0.511$ MeV respectively. (Useful relations to remember: $c \simeq 3 \times 10^{10}$ cm/s, $\hbar \cdot c \simeq 197$ MeV · fm).

Some particles are unstable and their mass can only be measured up to some uncertainty Γ . Similarly to the previous exercise construct an object with the units of time using Γ , $\hbar$ and c . What is its physical interpretation? Compute this time in seconds for a Z boson for which $\Gamma_Z = 2.495$ GeV/ c^2 .

Finally, let's consider the case of an Hydrogen atom in the non-relativistic limit. In this case the relevant dimensionful quantities seem to be m_e , m_p , e (electric charge), $\hbar$, c . For simplicity work in the Gaussian system of units, where $4\pi\epsilon_0 = 1$. Identify which of these are relevant for the case of the Hydrogen atom. Build an object with dimensions of length and one with dimension of energy. What are their values in cm and eV, respectively? What are their physical interpretations? (In Gaussian units $e^2 = 1.438 \times 10^{-7}$ eV · cm)

Exercise 3: Relativistic action of a point particle

Recall the infinitesimal proper time of a point particle,

$$d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2. \quad (1)$$

Since $d\tau$ is a Lorentz invariant quantity it is natural to consider the action for a relativistic point particle as

$$S = \alpha \int d\tau, \quad (2)$$

where α is some constant.

- Write the action in the usual form $S = \int L dt$, where the Lagrangian L will depend on the velocity $\vec{v} = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt})$.
- Fix α by taking the non-relativistic limit $c \rightarrow \infty$, and matching with the Newtonian expression. (Recall that a constant can always be added to the action since the equations of motion are not affected.)
- Compute the canonical momentum $\vec{p} = \frac{\partial L}{\partial \vec{v}}$ and the Hamiltonian $H = \vec{p} \cdot \vec{v} - L$. Express the energy $E = H$ in terms of $\vec{p}$ to obtain the relativistic dispersion relation.

Exercise 4: Relativistic Schrödinger equation

Consider the relativistic Schroedinger equation in one spatial dimension

$$i\partial_t \psi(t, x) = \sqrt{-\partial_x^2 + m^2} \psi(t, x)$$

- write how Lorentz transformations act on x , t , ∂_t and ∂_x
- show that if $\psi(t, x)$ satisfies the above equation and if it transforms as $\psi'(t', x') = \psi(t, x)$ under Lorentz transformations, then the transformed ψ satisfies the same equation in the new frame

$$i\partial'_t \psi'(t', x') = \sqrt{-\partial'^2_x + m^2} \psi(t', x')$$