# Plasma Physics I

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**Exercise 1**

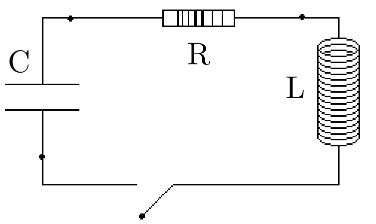
In order to refresh your knowledge of the Laplace transform and complex analysis, solve the equations of a series $RLC$ circuit using the Laplace transform method. The capacitor has charge $Q_0$ and we close the circuit at $t = 0$ (see figure below).

a.) Write, using the Kirchhoff law, the equation describing the temporal evolution of the charge $q$.

b.) Find the Laplace transform of $q(t)$, $\tilde{q}(s)$, with the initial conditions $q(t = 0) = Q_0$ and $I(t = 0) = 0$. Suggestion: define $R/L = 2\delta$; $1/(LC) = \omega_{LC}^2$, and consider only the case $\omega_{LC}^2 - \delta^2 > 0$.

c.) Evaluate the temporal evolution of the charge $q(t)$ by inverting Laplace transform. Comment on the integration path in the complex plane $s$.

d.) Verify the initial conditions.




Laplace transform of a function $f(t)$:

$$\mathrm{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = \tilde{f}(s)$$

Inverse Laplace transform:

$$f(t) = \mathrm{L}^{-1}\{\tilde{f}(s)\} = \frac{1}{2\pi i}\int_{p_0-i\infty}^{p_0+i\infty} \tilde{f}(s)e^{st}ds$$

where $\Re\{p_0\} > \max(\Re\{\text{poles of } \tilde{f}(s)\})$

Laplace transform of derivatives:

$$\mathrm{L}\left\{\frac{df(t)}{dt}\right\} = s\mathrm{L}\{f(t)\} - f(t=0)$$

$$\mathrm{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s\mathrm{L}\left\{\frac{df(t)}{dt}\right\} - f'(t=0) = s^2\mathrm{L}\{f(t)\} - sf(t=0) - f'(t=0)$$

**Exercise 2**

Using the general dispersion relation from the *Vlasov-Maxwell* model:

$$D(\omega,k) = 1 + \sum_\alpha \frac{e^2}{m_\alpha \epsilon_0 k} \int_{-\infty}^{+\infty} du \frac{dF_{0\alpha}}{du} \frac{1}{\omega - ku} = 0$$

evaluate the dispersion relation of the ion-acoustic waves in the limit $kv_{thi} \ll \omega \ll kv_{the}$, $T_e \gg T_i$, and assuming $\lambda \sim 1/k \gg \lambda_D$. Consider $F_{0e}$ and $F_{0i}$ as maxwellian distribution functions.

*Notice that in the case of waves with low frequency (e.g. the ion-acoustic waves), both species have to be considered (electrons and ions).*