Plasma Physics I

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Exercise 1

a.) Consider the magnetic flux through a given surface moving with the plasma,

$$\Phi_B(t) \equiv \int\int_{S(t)} \mathbf{B}(t) \cdot \mathbf{dS}$$

Starting from the ideal MHD equations, show that the magnetic flux is *frozen* in the plasma,

$$\frac{d\Phi_B}{dt} = 0$$

What does this imply on the magnetic topology in such a plasma?

b.) Consider now that the plasma has some finite resistivity. How does this affect the magnetic flux?

c.) Find the diffusion equation for the magnetic field in a resistive plasma. Estimate the diffusion time of the magnetic field in ITER (characteristic length $L = 3\,\mathrm{m}$, electron temperature $T_e = 10\,\mathrm{keV}$).

Exercise 2

a.) Demonstrate that the propagation of a transverse wave along the z axis ($\boldsymbol{k} = k\boldsymbol{e_z}$) in a string with tension S and mass per unit length M is given by:

$$\frac{\partial^2 y}{\partial z^2} = \frac{M}{S}\frac{\partial^2 y}{\partial t^2}$$

b.) Considering the ideal MHD model, demonstrate that the *Alfvén* waves (or *shear* waves), propagating along the magnetic field ($\boldsymbol{k} \parallel \boldsymbol{B_0}$, $\boldsymbol{B_0} = B_0\boldsymbol{e}_z$), can be described with the same equation of a transverse wave in a string. Identify the terms M and S in the equation in a.) for the Alfvén waves.

c.) The tokamak ITER will operate with a plasma D - T at $13\,\mathrm{keV}$ with a uniform electron density $n_e = 10^{20}\,\mathrm{m}^{-3}$ and a magnetic field $B = 6\,\mathrm{T}$. Evaluate the phase velocity of the Alfvén waves for that plasma.

d.) Fusion reactions $D^+ + T^+ \rightarrow He^{++}_{(3.5\mathrm{MeV})} + n_{(14\mathrm{MeV})}$ occur when a plasma is heated with ion beams D^+ with energy of $1\,\mathrm{MeV}$. Which charged particles are resonant with the Alfvén waves (same velocity with wave)?
