

Plasma Physics I

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Exercise 1

Consider the electric resistivity (*Spitzer* formula) of a completely ionized plasma, composed of electrons and ions (single species):

$$\eta = \frac{Ze^{1/2}m_e^{1/2}4\pi\ln\Lambda}{(4\pi\epsilon_0)^23\sqrt{2\pi}T_e^{3/2}}$$

all constants are in SI units, $[T_e] = \text{eV}$.

Consider a cylindrical hydrogen plasma (length $L = 2 \text{ m}$, radius $a = 0.5 \text{ m}$) with an uniform temperature $T_e = 5 \text{ keV}$ and density $n_e = 10^{20} \text{ m}^{-3}$. Evaluate the potential difference necessary to have a current of $I = 10^6 \text{ A}$ along the axis. Compare this value with the voltage necessary to produce the same current in a stainless steel cylinder ($\eta \sim 7 \times 10^{-7} \Omega\text{m}$) with same dimensions.

Exercise 2

Consider the electrons in the "tail" of the distribution function ($v \gg v_{the}$). Show that the collision frequency for these electrons is:

$$\nu = \nu_p^{e/e'} + \nu_p^{e/i} = (2 + Z) \frac{n_e e^4 \ln \Lambda}{4\pi\epsilon_0^2 m_e^2 v^3}$$

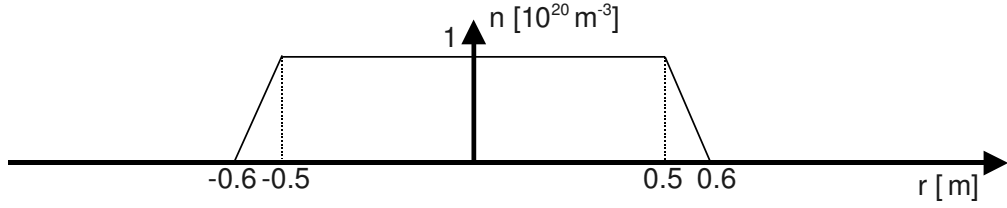
Demonstrate that these energetic electrons can be continuously accelerated (*run-away* regime) if their energy is higher than a critical value corresponding to a critical electric field, the Dreicer electric field, E_D :

$$\frac{1}{2}m_e v^2 > T_e \frac{E_D}{E}$$

Find an expression for E_D and estimate the critical kinetic energy needed for an electron to enter the run-away regime for the electric field found in Exercise 1 using the same parameters.

Exercise 3

Consider a fully ionized hydrogen plasma in a tokamak (major radius R , minor radius a) with a toroidal magnetic field B which is considered to be constant in the plasma (reasonable if $a \ll R$). The particle and heat sources are provided by injection of both neutral beam and electromagnetic waves. The steady state density profile in the radial direction is experimentally measured and is shown in the figure below:



Consider $B = 2 \text{ T}$, $T_e = T_i = 10 \text{ keV}$, $n(r = 0) = 10^{20} \text{ m}^{-3}$, $R = 2 \text{ m}$, $a = 0.6 \text{ m}$.

- a.) Since a source is needed to maintain such steady state, it is obvious that the plasma is flowing out radially despite the magnetic confinement. What process could explain particle transport across the magnetic field? Is it possible to assert that the measured density profile is compatible with a diffusive particle transport?
- b.) The measured particle flux between $r = 0.5 \text{ m}$ and $r = 0.6 \text{ m}$ is $\Gamma_n = 8 \times 10^{20} \text{ m}^{-2} \text{ s}^{-1}$. Calculate the effective diffusion coefficient D_{eff} at this location, and compare it with the classical diffusion coefficient $D_{\perp}$ that you would get considering the main collisional process.
- c.) What can you conclude concerning the particle transport mechanism in this plasma?