# Plasma Physics I

Series 13 (December 12, 2024)

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**Exercise 1**

Consider a right-hand circular polarized electromagnetic wave propagating in the direction of the magnetic field lines ($k_y = 0$) in a magnetized plasma described with a kinetic model. During the previous lectures, we have introduced the fields $\vec{E}_R = E_R(\hat{e_x} + i\hat{e_y})$ and $\vec{E}_L = E_L(\hat{e_x} - i\hat{e_y})$. For the right-hand circular polarization ($E_R \neq 0, E_L = 0$), we have found the following dispersion relation:

$$N^2 = \epsilon_R = \epsilon_1 + \epsilon_2.$$

Using the elements of the dielectric tensor $\epsilon$ previously calculated and the kinetic model, show that for a Dirac distribution, $f_\alpha(\mathbf{v}) = n_\alpha \delta(\mathbf{v})$, we find the same dispersion relation previously obtained from the fluid model (see pages 48-51 and p 55 of the Prof. Fasoli's lecture notes):

$$1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega(\omega + \Omega_\alpha)} = N^2$$

*Suggestion*: Use the following relations of the Bessel functions

$$\begin{aligned} J_n(0) &= \delta_{n,0}, \\ \frac{n}{a} J_n(a) &= \frac{1}{2}\left[J_{n-1}(a) + J_{n+1}(a)\right], \\ J_n'(a) &= \frac{1}{2}\left[J_{n-1}(a) - J_{n+1}(a)\right], \end{aligned}$$

to simplify $\underline{\mathbf{T}}_\alpha$.

**Exercise 2**

The CMA (Clemmow-Mullaly-Allis) diagram is useful to assess the accessibility of various methods of EC wave heating in tokamaks. The diagram represents the $x - y$ plane where:

$$X = \frac{\omega_p^2}{\omega^2} = \frac{e^2}{\epsilon_0 m_e \omega^2} n_e \quad \text{and} \quad Y = \frac{\Omega_e^2}{\omega^2} = \frac{e^2}{m_e^2 \omega^2} B^2.$$

As the frequency of the wave is fixed by the source, the CMA diagram can be seen as a plot of $n_e$ vs $B^2$. In this exercise you will draw this diagram and sketch trajectories of EC waves injected perpendicularly in the plasma.

a.) Represent the cutoffs and resonances for X mode injection in terms of $X$ and $Y$ and draw them on the CMA diagram

- Cyclotron resonances: $\omega = n\Omega_e$ where $n = \{1, 2, ...\}$.
- Upper hybrid resonance: $\omega^2 = \omega_p^2 + \Omega_e^2$,
- Cutoff: $(\omega^2 - \omega_R^2)(\omega^2 - \omega_L^2) = 0$ which can be rewritten as $(\omega^2 - \omega_p^2)^2 - (\omega^2 \Omega_e^2) = 0$

b.) Since we typically want to heat the plasma center, the injection frequency is chosen as a multiple of the cyclotron frequency $\Omega_{e0}$ at the center of the plasma. Sketch the propagation of a wave launched from the low-field side ($B < B_0$) across the plasma to the high field side $B > B_0$. Consider harmonics X1: $\omega = \Omega_{e0}$, X2:($\omega = 2\Omega_{e0}$) and X3 ($\omega = 3\Omega_{e0}$). Remember that the density is highest at the plasma center.

c.) On a new diagram, repeat parts a) and b) for O-mode (O1 and O2)

- Cyclotron resonances: Same as X mode
- Cutoff: $\omega = \omega_p$

d.) Based on these CMA diagrams, design two EC heating systems, one for TCV and one for ITER. Take the following constraints into account:

- Toroidal field in ITER: $B = 6$T, in TCV: $B = 1.5$T
- It is technologically complicated (=expensive) to launch from the high field side in most Tokamaks since the central column and ohmic coils are in the way.
- Existing gyrotron sources of $40 - 140$GHz, $\sim 1$MW can be bought "off-the-shelf". Higher frequencies need special development and will be more expensive.