Plasma Physics I

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Exercise 1

a.) Evaluate the **Landau damping**

$$\gamma = \frac{-\epsilon_i(\omega_r)}{\partial\epsilon_r/\partial\omega_r}$$

for an **electron plasma wave/Langmuir wave** solution of the dispersion relation of the *Vlasov-Maxwell* model:

$$D(\omega, k) = \epsilon(\omega, k) = 1 - \sum_\alpha \frac{e^2}{m_\alpha \epsilon_0 k^2} \int_{\mathcal{L}} \mathrm{d}u \frac{\mathrm{d}F_{\alpha 0}}{\mathrm{d}u} \frac{1}{u - \frac{\omega}{k}} = 0.$$

Suppose to have a maxwellian distribution, and $\omega \gg k v_{the}$.

b.) Rewrite the result as a function of the ratio between the Debye length and the wave length, i.e. of the product $k\lambda_D$. Find the maximum of γ as a function of $k\lambda_D$ and discuss the results.

c.) Verify that $\gamma/\omega_r \ll 1$.

Exercise 2

Evaluate the **Landau damping** for an **ion-acoustic wave** solution of the dispersion relation of the *Vlasov-Poisson* model,

$$D(\omega, k) = \epsilon(\omega, k) = 1 - \sum_\alpha \frac{e^2}{m_\alpha \epsilon_0 k^2} \int_{\mathcal{L}} \mathrm{d}u \frac{\mathrm{d}F_{\alpha 0}}{\mathrm{d}u} \frac{1}{u - \frac{\omega}{k}} = 0.$$

where the integral should now be evaluated using Landau's rule. Suppose to have a maxwellian equilibrium distribution. Assuming that $k v_{thi} \ll \omega \ll k v_{the}$, $T_e \gg T_i$ and $\lambda \gg \lambda_D$, show that the total damping rate of the wave is $\gamma_t = \gamma_e + \gamma_i$, where γ_e and γ_i are respectively the electron and ion contributions,

$$\gamma_e \approx -\sqrt{\frac{\pi}{8}} k c_s \sqrt{\frac{m_e}{m_i}}$$

$$\gamma_i \approx -\sqrt{\frac{\pi}{8}} k c_s \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{2T_i}\right)$$