Plasma Physics I

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Exercise 1

Consider a quasi-neutral electron-proton plasma in which an equilibrium current is flowing. This may be described by a Maxwellian ion distribution at rest and a drifting Maxwellian for the electrons

$$F_i(u) = \frac{n}{\sqrt{2\pi}v_{th,i}} \exp\left[-\frac{u^2}{2v_{th,i}^2}\right] \qquad F_e(u) = \frac{n}{\sqrt{2\pi}v_{th,e}} \exp\left[-\frac{(u-v_d)^2}{2v_{th,e}^2}\right]$$

where $v_{th,i}, c_s \ll v_d \ll v_{th,e}$.

a.) Make a plot of the distribution functions of ions and electrons on the same scale, look in the region, $v_{th,i} < \omega_r/k \ll v_{th,e}$, and show where you expect unstable waves might occur.

b.) Consider an ion-acoustic wave: write an expression for the damping/growth rate, γ, including both electron and ion contributions. Show that the electron contribution introduces a destabilizing term in the expression of γ.

c.) Demonstrate that the condition $T_e \gg T_i$ is generally required for instability and justify the result. Show that $\gamma \sim \sqrt{\frac{\pi}{8}} k v_d (m_e/m_i)^{1/2}$ when $T_e \gg T_i$.

Exercise 2

Consider a uniform plasma with a fixed population of ions and two different electron populations:

- a Maxwellian population with density n_p, temperature T_p and no drift velocity
- a Maxwellian beam with density n_b, temperature T_b and drift velocity $\mathbf{v} = V\mathbf{e_x}$

When the magnitude of the beam density n_b exceeds a certain threshold the two-stream instability can develop. As seen in the lecture, the Landau damping coefficient γ is proportional to the imaginary part of the dielectric function $\epsilon_i(\omega_r)$. Its sign determines wether a given mode can become unstable or not. Supposing that the phase velocity of the instability, v_ϕ, corresponds to a velocity v for which the slope of $f_b(v)$ is maximum and supposing that $V \gg v_{th,b}$, show that the critical density ratio above which there can be an instability is:

$$\frac{n_b}{n_p} = \sqrt{e}\frac{T_b}{T_p}\frac{V}{v_{th,p}}\exp\left(-\frac{V^2}{2v_{th,p}^2}\right).$$