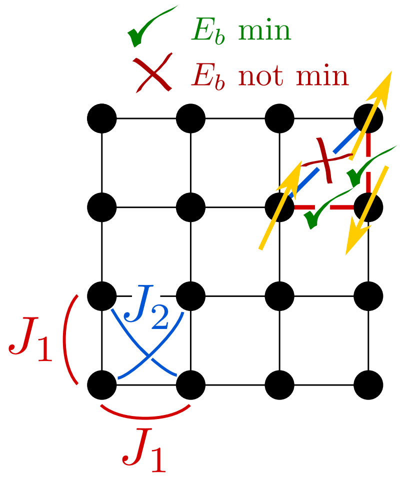
Consider a classical spin model on a square lattice with $N = L^2$ lattices sites. The Hamiltonian is given as

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j \,, \tag{1}$$

with antiferromagnetic couplings $J_{1,2} > 0$. Here, $\langle ij \rangle$ denotes the usual summation over nearest neighbours and $\langle\langle ij \rangle\rangle$ the sum over next-nearest neighbours (along the diagonals). The Hamiltonian describes a competition between Néel order (J_1-term) and stripe order (J_2-term). Each term individually can be minimized, but it is not easily possible to identify a state which minimizes all interactions at the same time. This scenario is known as *frustrated magnetism*. In order to gain some insight into the problem we will consider the classical limit, where we treat the spins as unit vectors and attempt to find classical spin configurations minimizing the energy.



(a) Switch to momentum space to obtain $H(\vec{k}) = \sum_{\vec{k}} J(\vec{k})\, \vec{S}_{\vec{k}} \cdot \vec{S}_{-\vec{k}}$ and the Fourier transformed couplings $J(\vec{k})$.

(b) Minimize $J(\vec{k})$ to find the minimal energy configuration. Show that one way to realize the minimum energy configuration for $\vec{k}_0$ minimizing $J(\vec{k})$ is the choice $\vec{S}_{\vec{k}_0} = (0, -i\, L/2, L/2)$, $\vec{S}_{-\vec{k}_0} = (0, i\, L/2, L/2)$ and $\vec{S}_{\vec{k} \neq \pm \vec{k}_0} = 0$.

(c) Sketch the spin configurations for $J_1 < 2J_2$, and $J_1 > 2J_2$. What happens in the cases $J_1 = 0$, $J_2 = 0$ and $J_1 = 2J_2$?

(d) Stripe order dominates for $J_1/J_2 < 2$. Describe how one can obtain other states from the degenerate ground states by a superposition of orthogonal spin states.