# Particle Physics 1 : Exercise 9

## 1) Properties of the chirality operator

Using the properties of the $\gamma$-matrices and the definition of the chirality operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, show that

(a) $(\gamma^5)^2 = 1$
(b) $\gamma^{5\dagger} = \gamma^5$
(c) $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$

## 2) Chiral projection operators

Show that the chiral projection operators $P_R = \frac{1}{2}(1+\gamma^5)$ and $P_L = \frac{1}{2}(1-\gamma^5)$ are indeed projection operators, i.e. they satisfy the following properties :

$$P_R + P_L = 1, \qquad P_R P_R = P_R, \qquad P_L P_L = P_L, \qquad P_R P_L = 0$$

## 3) Differential cross section for $e^-\mu^- \to e^-\mu^-$ scattering

Using helicity amplitudes, calculate the differential cross section for $e^-\mu^- \to e^-\mu^-$ scattering in the following steps

(a) From the Feynman rules of QED, show that the lowest-order QED matrix element for $e^-\mu^- \to e^-\mu^-$ is

$$\mathcal{M}_{fi} = -\frac{e^2}{(p_1 - p_3)^2} g_{\mu\nu} \left[\overline{u}(p_3)\gamma^\mu u(p_1)\right] \left[\overline{u}(p_4)\gamma^\nu u(p_2)\right],$$

where $p_1$ and $p_3$ are the four-momenta of the initial- and final-state $e^-$, and $p_2$ and $p_4$ are the four-momenta of the initial- and final-state $\mu^-$.

(b) Working in the centre-of-mass frame, and writing the four-momenta of the initial and final-state $e^-$ as $p_1^\mu = (E_1, 0, 0, \mathrm{p})$ and $p_3^\mu = (E_1, \mathrm{p}\sin\theta, 0, \mathrm{p}\cos\theta)$ respectively, show that the electron currents for the four possible helicity combinations are

$$\begin{aligned}
\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2(E_1 c, \mathrm{p}s, -i\mathrm{p}s, \mathrm{p}c) \\
\overline{u}_\uparrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2(ms, 0, 0, 0) \\
\overline{u}_\uparrow(p_3)\gamma^\mu u_\uparrow(p_1) &= 2(E_1 c, \mathrm{p}s, i\mathrm{p}s, \mathrm{p}c) \\
\overline{u}_\downarrow(p_3)\gamma^\mu u_\uparrow(p_1) &= -2(ms, 0, 0, 0)
\end{aligned}$$

where $m$ is the electron mass, $s = \sin\theta/2$ and $c = \cos\theta/2$.

(c) Explain why the effect of the parity operator $\hat{P} = \gamma^0$ is

$$\hat{P}u_\uparrow(\mathrm{p}, \theta, \phi) = u_\downarrow(\mathrm{p}, \pi - \theta, \pi + \phi).$$

Hence, or otherwise, show that the muon currents for the four helicity combinations are

$$\begin{aligned}
\overline{u}_\downarrow(p_4)\gamma^\mu u_\downarrow(p_2) &= 2(E_2 c, -\mathrm{p}s, -i\mathrm{p}s, -\mathrm{p}c) \\
\overline{u}_\uparrow(p_4)\gamma^\mu u_\downarrow(p_2) &= -2(Ms, 0, 0, 0) \\
\overline{u}_\uparrow(p_4)\gamma^\mu u_\uparrow(p_2) &= 2(E_2 c, -\mathrm{p}s, i\mathrm{p}s, -\mathrm{p}c) \\
\overline{u}_\downarrow(p_4)\gamma^\mu u_\uparrow(p_2) &= 2(Ms, 0, 0, 0),
\end{aligned}$$

where $M$ is the muon mass and $E_2 = \sqrt{\mathrm{p}^2 + M^2}$ is the muon energy in the centre-of-mass frame.

(d) For the relativistic limit where $E \gg M$, show that the matrix element squared for the case where the incoming $e^-$ and incoming $\mu^-$ are both left-handed is given by

$$|\mathcal{M}_{LL}|^2 = \frac{4e^4 s^2}{(p_1 - p_3)^4},$$

where $s = (p_1+p_2)^2$. Find the corresponding expressions for $|\mathcal{M}_{RL}|^2$, $|\mathcal{M}_{RR}|^2$ and $|\mathcal{M}_{LR}|^2$.

(e) In this relativistic limit, show that the differential cross section for unpolarised $e^-\mu^- \to e^-\mu^-$ scattering in the centre-of-mass frame is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\alpha^2}{s} \cdot \frac{1 + \frac{1}{4}(1+\cos\theta)^2}{(1-\cos\theta)^2}.$$