Particle Physics 1 : Exercise 7

1) Energy-momentum relation for Dirac spinors

Verify the statement that the Einstein energy-momentum relation is recovered if any of the four orthogonal particle Dirac spinors are substituted into the Dirac equation written in terms of momentum, $(\gamma^\mu p_\mu - m)\, u = 0$.

2) Dirac four-vector current

For a particle with four-momentum $p^\mu = (E, \vec{p})$, the general solution to the free-particle Dirac equation can be written as

$$\psi(p) = [a u_1(p) + b u_2(p)]\, e^{i(\vec{p}\cdot\vec{x} - Et)}.$$

Using the explicit form for u_1 and u_2, show that the four-vector current $j^\mu = (\rho, \vec{j}) = \overline{\psi}\gamma^\mu\psi$ is given by

$$j^\mu = 2p^\mu.$$

Furthermore, show that the resulting probability density and probability current are consistent with a particle moving with velocity $\beta = p/E$.

3) Parity operator

Show that up to an overall phase factor

$$\hat{P} u_\uparrow(\theta, \phi) = u_\downarrow(\pi - \theta, \pi + \phi)$$

where $u_\uparrow$ and $u_\downarrow$ are, respectively, the right-handed and left-handed helicity particle spinors. Comment on the result.

4) Charge conjugation operator

Under the combined operation of parity and charge conjugation ($\hat{C}\hat{P}$) spinors transform as

$$\psi \to \psi^C = \hat{C}\hat{P}\psi = i\gamma^2\gamma^0\psi^*.$$

Show that, up to an overall phase factor,

$$\hat{C}\hat{P} u_\uparrow(\theta, \phi) = v_\downarrow(\pi - \theta, \pi + \phi).$$

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