

PARTICLE PHYSICS 1 : EXERCISE 5

1) Cross section units

The total $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ annihilation cross section is $\sigma = 4\pi\alpha^2/(3s)$, where $\alpha \sim 1/137$. Calculate the cross section at $\sqrt{s} = 50$ GeV, expressing the answer in both natural units and barns ($1 \text{ b} = 10^{-28} \text{ m}^2$). Compare this to the total proton-proton interaction cross section at $\sqrt{s} = 50$ GeV, which is approximately 40 mb, and comment on the result.

2) Neutrino interaction probability

A 1 GeV muon neutrino is fired at a 1 m thick block of iron ($^{56}_{26}\text{Fe}$) with density $\rho = 7.874 \times 10^3 \text{ Kg m}^{-3}$. If the average neutrino-nucleon interaction cross section is $\sigma = 8 \times 10^{-39} \text{ cm}^2$, calculate the (small) probability that the neutrino interacts in the block.

3) Lorentz-invariant flux

For the process $a + b \rightarrow 1 + 2$ the Lorentz-invariant flux term is

$$F = 4 \left[(p_a \cdot p_b)^2 - m_a^2 m_b^2 \right]^{\frac{1}{2}}.$$

In the non-relativistic limit, $\beta_a \ll 1$ and $\beta_b \ll 1$, show that

$$F \approx 4m_a m_b |\vec{v}_a - \vec{v}_b|$$

where $\vec{v}_a$ and $\vec{v}_b$ are the (non-relativistic) velocities of the two particles.

4) Instantaneous luminosity

At the LEP e^+e^- collider, which had a circumference of 27 km, the electron and positron beams consisted of four equally spaced bunches in the accelerator. Each bunch corresponded to a beam current of 1.0 mA. The beams collided head-on at the interaction point, where the beam spot had an rms profile of $\sigma_x \approx 250 \text{ } \mu\text{m}$ and $\sigma_y \approx 4 \text{ } \mu\text{m}$, giving an effective area of $1.0 \times 10^3 \text{ } \mu\text{m}^2$. Calculate the instantaneous luminosity and estimate the event rate for the process $e^+e^- \rightarrow Z$, which has a cross section of about 40 nb.

5) Integrated luminosity

Consider a future e^+e^- linear collider operating as a Higgs factory at a centre-of-mass energy of $\sqrt{s} = 250$ GeV. The Higgs production process in this case is $e^+e^- \rightarrow HZ$, which has a cross section of 250 fb. If the collider has an instantaneous luminosity of $2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ and is operational for 50% of the time, how many Higgs bosons will be produced in five years of running?