Particle Physics 1 : Exercise 4

1) Centre-of-mass momentum

For the decay $a \to 1 + 2$, show that the modulus of the momentum of both daughter particles in the centre-of-mass frame is :

$$|\vec{p}| = \frac{\sqrt{(m_a^2 - (m_1 + m_2)^2)(m_a^2 - (m_1 - m_2)^2)}}{2m_a}$$

2) Lorentz invariant phase space

Assuming that $f(\vec{p})$ is an arbitrary function of the momentum $\vec{p}$, verify the following relation :

$$\int \mathrm{d}^4 p\, f(\vec{p})\delta(p^2 - m^2)\theta(p^0) = \int \frac{\mathrm{d}^3\vec{p}}{2E} f(\vec{p})$$

with $E = \sqrt{\vec{p}^2 + m^2}$. Using this relation, show the invariance of $\frac{\mathrm{d}^3\vec{p}}{2E}$, i.e. that $\frac{\mathrm{d}^3\vec{p}}{2E}$ is a Lorentz scalar.

Here the step function $\theta(p^0)$ is used to select only positive values of p^0 and is defined as

$$\theta(p^0) = \begin{cases} 1, & \text{if } p^0 \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Hint : Integrate the left side of the equation over $\mathrm{d}p^0$ and use the following property of the δ-function : if $x_i\,(i = 1, ..., n)$ are zeroes of the function $g(x)$, i.e. $g(x_i) = 0$, then

$$\delta\left(g(x)\right) = \sum_{i=1}^{n} \frac{\delta(x - x_i)}{|g'(x_i)|}$$

where $g'(x) = \frac{\mathrm{d}g(x)}{\mathrm{d}x}$

3) Branching ratio

Calculate the branching ratio for the decay $K^+ \to \pi^+\pi^0$ given the partial decay width $\Gamma(K^+ \to \pi^+\pi^0) = 1.2 \times 10^{-8}$ eV and the mean kaon lifetime $\tau(K^+) = 1.2 \times 10^{-8}$ s.

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