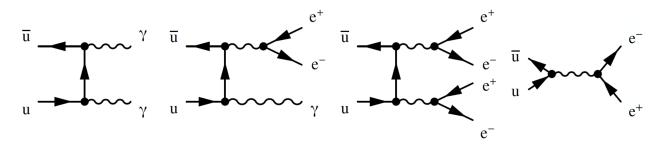
Particle Physics 1: Exercise 3

1) π^0 decays

The observed branching ratios are $BR(\pi^0 \to \gamma \gamma) = 98.8\%$, $BR(\pi^0 \to \gamma e^+ e^-) = 1.2\%$, $BR(\pi^0 \to e^+ e^- e^+ e^-) \sim 3 \times 10^{-5}$ and $BR(\pi^0 \to e^+ e^-) = 6 \times 10^{-8}$. By counting the number of QED vertices it might be expected that the martix elements for the processes are : a) $\mathcal{M} \propto e^2$; b) $\mathcal{M} \propto e^3$; c) $\mathcal{M} \propto e^4$; and d) $\mathcal{M} \propto e^2$. Consequently, one would expect the branching ratios to be proportional to a) $|\mathcal{M}|^2 \propto \alpha^2$; b) $|\mathcal{M}|^2 \propto \alpha^3$; c) $|\mathcal{M}|^2 \propto \alpha^4$; and d) $|\mathcal{M}|^2 \propto \alpha^2$. Relative to the dominant $\pi^0 \to \gamma \gamma$ decay mode it might be expected that $BR(\pi^0 \to \gamma e^+ e^-) \sim \mathcal{O}(10^{-2})$ and $BR(\pi^0 \to e^+ e^- e^+ e^-) \sim \mathcal{O}(10^{-4})$, in reasonable agreement with the observed values.



The observed branching ratio for e^+e^- is much smaller than the one predicted from simple vertex counting (the contribution from this Feynman diagram is heavily helicity suppressed - a concept that you will study later in particle physics courses). This is an important point: simple vertex factor counting only addresses one of the contributions to the matrix element squared, other factors may be just as important.

2) Prohibited processes in the SM

- (a) The process $\mu^- \to e^+ e^- e^+$ would require a change of flavour. There is no problem with producing a particle and its antiparticle (here $e^+ e^-$) and therefore the required flavour change is $\mu^- \to e^-$. Since there is no corresponding Standard Model vertex, the process cannot occur. This used to be referred to as conservation of muon and electron numbers. However, with the discovery of neutrino oscillations this concept is obsolete.
- (b) Two changes of flavour are required $u \to d$ and $\nu_{\tau} \to \mu^{-}$. Whilst the first of these flavour changes can be achieved by a charged-interaction vertex, the second cannot. Leptons only couple to the corresponding weak eigenstate neutrino flavour. In addition, electric charge is not conserved.
- (c) This process requires a vertex that has the effect $\nu_{\tau} \to \tau^{+}$, i.e. turning a particle into an antiparticle. No such vertex exists.
- (d) Here the net number of particles antiparticles changes. This cannot happen because all Standard Model vertices involving fermions are three point interactions with a single boson. As a consequence the net number of particles antiparticles in the Universe is constant.

3) Electromagnetic showers

The number of particles in a shower doubles at every radiation length of material traversed. Hence, the typical particle energy $\langle E_n \rangle$ in a shower after n radiation lengths is

$$\langle E_n \rangle = E/2^n. \tag{1}$$

The shower terminates when this mean energy is equal to the critical energy, hence

$$n\ln 2 = \ln\left(E/E_c\right). \tag{2}$$

For a 500 GeV electromagnetic shower in Tungsten this corresponds to n=16 radiation lengths or 5.6 cm of Tungsten. The above analysis is overly simplistic. In practice, due to fluctuations, the shower extends deeper than the above calculation would suggest, although with ever decreasing number of particles.

4) Čerenkov detectors for CPLEAR

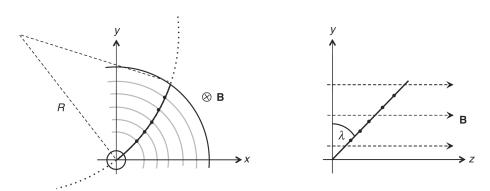
The momentum of the particle can be obtained from

$$p[GeV] \cos \lambda = 0.3 B[T] \cdot R[m]$$
 (3)

where λ is the angle between the particle direction and the plane transverse to the magnetic field (see the figure below), which in this case is zero since the particle is travelling perpendicular to the magnetic field. Using the values of B and R given in the text, the above formula leads to p = 528 MeV. Since the particle produced a Čerenkov signal, $\beta > 1/n$, i.e. $\beta > 0.8$. Using $E = \gamma m$ and $p = \gamma m \beta c$, the particle's velocity is given by

$$\beta = \frac{pc}{E} = \frac{p}{\sqrt{p^2 + m^2}}. (4)$$

For the hypothesis considered here, for p = 528 MeV, $\beta_{\pi} = 0.97$ and $\beta_{K} = 0.73$. Hence the observed particle track could be due to a pion but cannot be a kaon.



5) Particle interactions in detectors

Here is a scheme of the typical interactions of different particles with the detector.

Refer to chapters 1.2 and 1.3 of the book by Mark Thomson, "Modern Particle Physics" (2013), for a general introduction of particle interactions with matter, detector types, and particle signatures at collider experiments.

