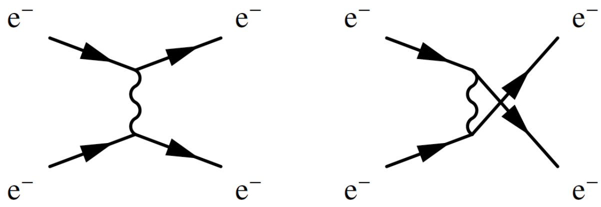
Particle Physics 1 : Exercise 2

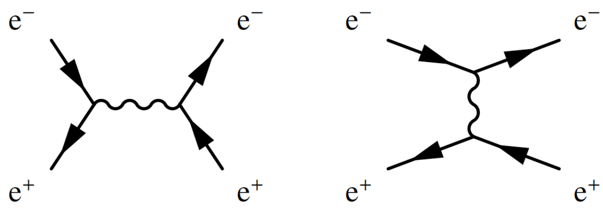
1) Scattering and annihilation Feynman diagrams

(a) For this scattering process there are two diagrams, the u-channel diagram has to be included since there are identical particles in the final state (the exchanged gauge boson could also be a Z or even the H) :

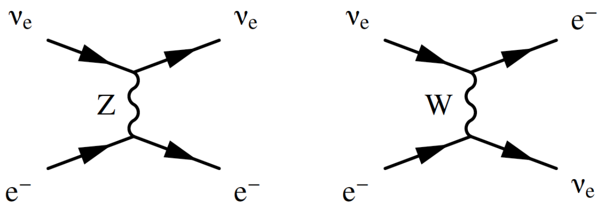


(b) Here there is just the s-channel annihilation diagram.

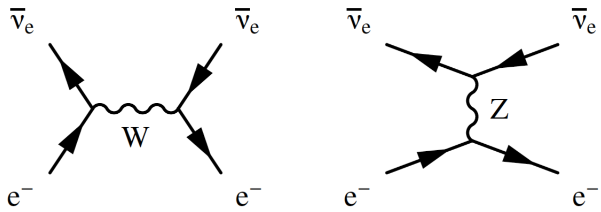
(c) Here there are both s-channel and t-channel diagrams :



(d) Here there are two t-channel diagrams (involving either an exchanged Z or W) :



(e) Here there are s- and t-channel diagrams :



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2) Centre-of-mass energy

Taking the protons to be at rest, the squared centre-of-mass energy s is given by

$$\begin{aligned} s &= (E_\pi + m_p)^2 - p_\pi^2 \\ &= E_\pi^2 - p_\pi^2 + 2E_\pi m_p + m_p^2 \\ &= m_\pi^2 + m_p^2 + 2E_\pi m_p \\ &= m_\pi^2 + m_p^2 + 2m_p\sqrt{p_\pi^2 + m_\pi^2} \\ &= 1.52\ \text{GeV}^2 \end{aligned}$$

Therefore the mass of the Δ is

$$m_\Delta = \sqrt{s} = 1.23\ \text{GeV}$$

3) Rutherford cross section

The differential Rutherford cross section derived in the lecture is

$$\sigma(\theta) = \left(\frac{zZ\alpha}{4E_{\text{kin}}}\right)^2 \frac{1}{\sin^4\theta/2} \tag{1}$$

where θ is the scattering angle with respect to the direction of the incident beam. In the above formula, z is the charge of the incident particle in units of the electron charge e ($z = 2$ for α particles), Z is the atomic number of the target material, α is the electromagnetic constant ($\alpha \approx 1/137$) and E_{kin} is the kinetic energy of the incident particles.

To obtain the full cross section for $\theta > \theta_0$, one needs to integrate the differential cross section of equation (1) from $\theta = \theta_0$ to $\theta = \pi$:

$$\sigma_{\text{full}} = \int_{\theta_0}^{\pi} \mathrm{d}\theta\, 2\pi\sigma(\theta)\sin\theta = 4\pi\left(\frac{zZ\alpha}{4E_{\text{kin}}}\right)^2 \cot^2\frac{\theta}{2} \tag{2}$$

which, given the definition of cross section, corresponds to

$$\sigma_{\text{full}} = \frac{\text{number of incident particles scattered at } \theta > \theta_0 \text{ per unit time per target}}{\text{incident flux}} \tag{3}$$

The fraction of particles scattered at $\theta > \theta_0$ can be measured as the rate of particles R_s scattered at $\theta > \theta_0$, divided by the rate of incident particles R_i. Since the indent flux corresponds to the incident rate per unit area, $F_i = R_i/a$, one can write

$$\frac{R_s}{R_i} = (\sigma_{\text{full}} F_i N_t)\cdot\frac{1}{R_i} = \sigma_{\text{full}}\cdot\frac{R_i}{a}\cdot N_t\cdot\frac{1}{R_i} = \sigma_{\text{full}}\cdot\frac{N_t}{a} \tag{4}$$

In the above formula, N_t/a is the number of target particles per unit area and can be computed as

$$\frac{N_t}{a} = \frac{\rho\delta}{m_{1\text{p}}} = \frac{\rho\delta N_A}{A\cdot 1\ \text{g}} \tag{5}$$

where we used the relation $m_{1\text{p}} = A\cdot 1\ \text{g}/N_A$ between the mass of one target particle $m_{1\text{p}}$ and the atomic mass number A.

Using equations (2), (4), and (5) the fractions of α particles scattered at $\theta > \pi/2$ is given by

$$\begin{aligned}
\frac{R_s}{R_i} &= 4\pi \left(\frac{zZ\alpha}{4E_{\text{kin}}}\right)^2 \cot^2 \frac{\pi}{4} \cdot \frac{\rho\delta N_A}{A \cdot 1\ \text{g}} \\
&= 4\pi \left(\frac{2 \times 78}{137 \times 4 \times 6\ \text{MeV}}\right)^2 \cdot \left(1.97 \cdot 10^{-13}\ \frac{\text{m}}{\text{MeV}}\right) \\
&\times \left(\frac{6 \cdot 10^{23} \times 2.15 \cdot 10^4\ \text{kg/m}^3 \times 8 \cdot 10^{-7}\ \text{m}}{0.195\ \text{kg}}\right) = 5.84 \cdot 10^{-5}
\end{aligned}$$

Note that in the above computation the following relation was used to convert the cross section into standard units :

$$1\ \text{eV}^{-1} = 1.97 \cdot 10^{-7}\ \text{m}. \tag{6}$$

4) Particle lifetimes and experiments

The mean decay length of a particle with proper lifetime τ_0 corresponds to

$$L_0 = vt_0 = (\beta c)(\gamma\tau_0) \tag{7}$$

where the time delation relation $t_0 = \gamma\tau_0$ was used. Given the mass m and the momentum p of the particle, $\gamma\beta = p/m$. Therefore, L_0 can be computed as

$$L = \frac{p}{m} c\tau_0. \tag{8}$$

To compute the fraction of particles decaying between a distance x_1 and a distance x_2 from the production point, one has to consider the decay length distribution for the given particle type. The probability density function of the particle lifetime is exponential :

$$p(\tau) = \frac{1}{\tau_0} \exp\left(-\frac{\tau}{\tau_0}\right). \tag{9}$$

Assuming a fixed momentum for all particles, it can be easily shown that also the decay length distribution is exponential

$$p(L) = \frac{1}{L_0} \exp\left(-\frac{L}{L_0}\right). \tag{10}$$

Therefore, the fraction of particles decaying between x_1 and x_2 can be roughly computed as

$$p(x_1 < L < x_2) = \int_{x_1}^{x_2} \mathrm{d}L\, p(L) = \exp\left(-\frac{x_1}{L_0}\right) - \exp\left(-\frac{x_2}{L_0}\right). \tag{11}$$

The mean decay length - given by equation (8) - and the fraction of particles decaying within the LHCb and NA62 decay volumes - given by equation (11) - are reported in the last three rows of Table 1 for each given particle. The LHCb and NA62 decay volumes are considered to be between 0 and 1 m, and between 100 and 170 m respectively.

The LHCb experiment was built to study b- and c-hadrons, like B^0, B^+, D^0 and D^+. These particles are relatively short-lived, with a mean decay length of a few millimeters. Hence, all such particles produced in *pp* collisions decay within 1 m from the production point, and can be studied through the reconstruction of their decay products in the LHCb detector. From the LHCb perspective, K_s is a long-lived particle : most of the produced K_s decay within the experimental decay volume and produce a secondary vertex that can be reconstructed. Most K^+ and π^+ instead trasverse the LHCb detector without decaying. However, there is still a

Particle type	K_s	K^+	π^+	B^0	D^0	B^+	D^+
p [GeV]	20	75	75	40	40	80	50
m [GeV]	0.5	0.5	0.139	5.3	1.9	5.3	1.9
$c\tau$ [m]	0.025	3.7	7.8	$4.6 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$
L [m]	1.0	555	$4.21 \cdot 10^{3}$	$3.5 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	$7.4 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$
Decay fraction in LHCb [%]	63	0.18	$2.3 \cdot 10^{-2}$	100	100	100	100
Decay fraction in NA62 [%]	0	10	1.6	0	0	0	0

TABLE 1

non-negligible fraction of them decaying in the detector and mostly producing a muon-neutrino pair.

Particles like B^0, B^+, D^0, D^+ and K_s have a too short lifetime to be observed and studied by the NA62 experiment, whose detector is located 100 m away from the production point. Instead, NA62 is dedicated to the study of kaon decays, with a relatively large fraction of the produced K^+ ($\sim$ 10%) decaying within its experimental decay volume. Also a significant fraction of the produced π^+ decays within the detector, thus requiring high precision in particle identification.