# Particle Physics 1: Exercise 1

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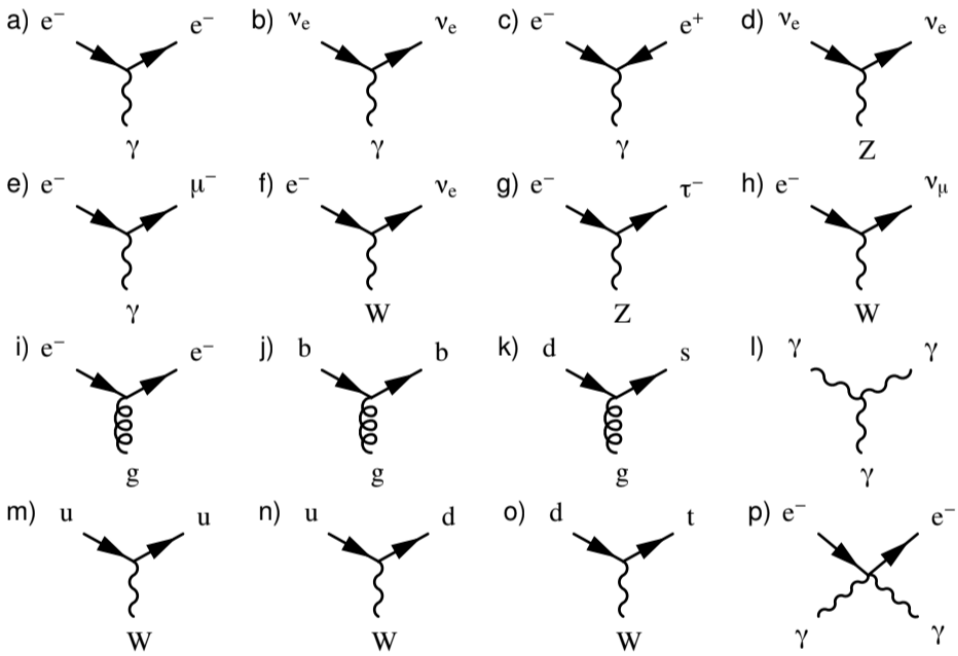
## Exercise 1 - Feynman diagrams




1) State and explain your reasoning for whether each of the diagrams below represents a valid Standard Model vertex
2) Draw the Feynman diagram for $\tau^- \to \pi^- \nu_\tau$ (the $\pi^-$ is the lightest of $\bar{u}d$ meson)
3) Draw the Feynman diagrams for the decays:
    a) $\Delta(\mathrm{uud}) \to n(\mathrm{udd})\pi^+(\mathrm{u\bar{d}})$,
    b) $\Sigma^0(\mathrm{uds}) \to \Lambda(\mathrm{uds})\gamma$,
    c) $\pi^+(\mathrm{u\bar{d}}) \to \mu^+\nu_\mu$

    and place them in order of increasing lifetime

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## Exercise 2 - Lorentz transformation

1) In a collider experiment, $\Lambda$ baryons can be identified from the decay $\Lambda \to \pi^- p$ that gives rise to a displaced vertex in a tracking detector. In a particular decay, the momenta of the $\pi^-$ and p are measured to be 0.75 GeV and 4.25 GeV respectively, and the opening angle between the tracks is 9°. The masses of the pion and proton are 139.6 MeV and 938.3 MeV.
    a) Calculate the mass of the $\Lambda$ baryon
    b) On average, $\Lambda$ baryon of this energy are observed to decay at a distance of 0.35 m from the point of the production. Calculate the lifetime of the $\Lambda$
2) Find the minimum opening angle between the photons produced in the decay $\pi^0 \to \gamma\gamma$, if the energy of the pion is 10 GeV, given that $m_\pi^0$ = 135 MeV

## Solution 1.1

b) The electron neutrino is neutral and therefore does not couple to the gauge boson of the electromagnetic interaction;

c) This diagram violates both charge conservation and has the effect of turning a particle into an antiparticle (the arrows on the electron lines both point towards the vertex);

e) The electron magnetic interaction does not change flavour, and hence a diagram coupling an electron to a muon is not allowed;

g) The weak neutral current also does not change flavour and hence this diagram is forbidden;

h) The weak charged current does change flavour, but *by definition* only couple together leptons with the corresponding neutrino, hence this diagram which couples together an electron and a muon neutrino is not allowed.

i) The electron does not carry the colour charge of the strong interaction - it is colour neutral - and hence the electron does not participate in the strong interaction;

k) The strong interaction does not change flavour and hence a coupling between a down-quark and a strange-quark is forbidden;

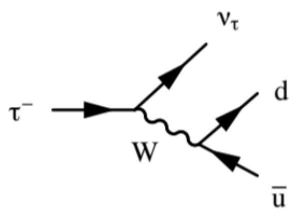
l) In the Standard Model there is no three-photon vertex;

m) Since W bosons are charged, the weak charged current must change flavour;

p) There is no Standard Model vertex couping two fermion lines to two boson lines - all fermion vertices involve a coupling to a single gauge boson.

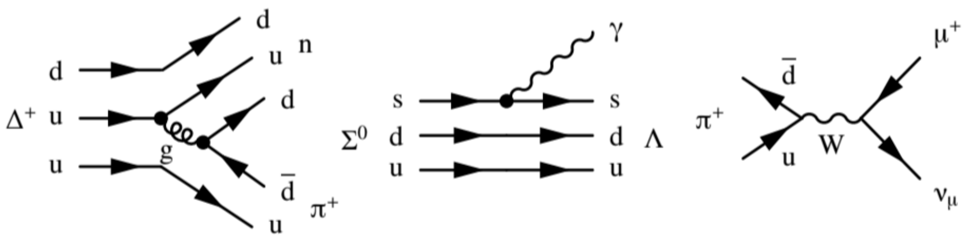
## Solution 1.2

Since the decay involves a change of flavour it can only be a weak charged-current interaction ($W^{\pm}$):




## Solution 1.3

All other things being equal, strong decays will dominate over EM decays, and EM decays will dominate over weak decays. So here the order is a), b), c) with the Feynman diagrams below.



## Solution 2.1

**a)** From $E^2 = \mathrm{p}^2 + m^2$ the energies of the two decay products are

$$E_\pi = 0.763\,\mathrm{GeV} \quad \text{and} \quad E_\mathrm{p} = 4.352\,\mathrm{GeV}\,.$$

The corresponding velocities ($\beta = \mathrm{p}/E$) are

$$\beta_\pi = 0.983 \quad \text{and} \quad \beta_\mathrm{p} = 0.976\,.$$

Since $m_a^2 = E_a^2 - \mathrm{p}_a^2$ and energy and momentum are conserved in the decay

$$\begin{aligned}
m_a^2 &= (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 \\
&= E_1^2 + E_2^2 + 2E_1E_2 - \mathrm{p}_1^2 - \mathrm{p}_2^2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 \\
&= m_1^2 + m_2^2 + 2E_1E_2 - 2\mathrm{p}_1\mathrm{p}_2\cos\theta \\
&= m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\,,
\end{aligned}$$

where the last step follows from $\mathrm{p} = \beta E$.

$$m_\Lambda^2 = m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta) = 1.244\,\mathrm{GeV}^2\,.$$

Hence the mass of the $\Lambda$ obtained from the measurements give is $m_\Lambda = 1.115\,\mathrm{GeV}$.

**b)** Accounting for relativistic time dilation the mean distance travelled will be

$$d = \gamma\beta c\tau\,. \qquad d = v \cdot t = v \cdot \gamma\tau = \beta c \cdot \gamma\tau$$

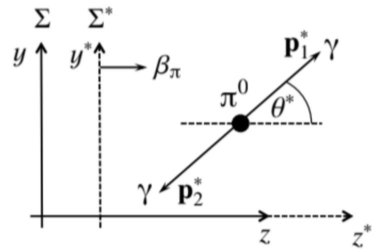
From the relation $\mathrm{p} = \gamma m\beta$, $\gamma\beta = \mathrm{p}/m$. The energy of the $\Lambda$ is simply

$$E_\Lambda = E_\pi + E_\mathrm{p} = 5.115\,\mathrm{GeV}\,,$$

from which $\mathrm{p}_\Lambda = 4.99\,\mathrm{GeV}$ and therefore $\gamma\beta = \mathrm{p}/m = 4.47$. From which

$$\tau = 0.35/4.47c = 2.6 \times 10^{-10}\,\mathrm{s}\,.$$

## Solution 2.2




$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}.$$

In the rest frame of the $\pi^0$, the four-momenta of the photons are $p_1^* = (E, 0, E\sin\theta^*, E\cos\theta^*)$ and $p_1^* = (E, 0, -E\sin\theta^*, -E\cos\theta^*)$, where $E = m_\pi^0/2$. The four-momenta of the photons in the laboratory frame can be found from the (inverse) Lorentz transformation of Equation 2.6 where $p_x = p_x^* = 0$, $p_y = p_y^* = \pm E\sin\theta^*$ and

$$E_1 = \gamma E_1^* + \gamma\beta p_{z1}^* = \gamma E(1 + \beta\cos\theta^*) \quad \text{and} \quad p_{z1} = \gamma p_{z1}^* + \gamma\beta E = \gamma E(\cos\theta^* + \beta)$$
$$E_2 = \gamma E_2^* + \gamma\beta p_{z2}^* = \gamma E(1 - \beta\cos\theta^*) \quad \text{and} \quad p_{z2} = \gamma p_{z2}^* + \gamma\beta E = \gamma E(-\cos\theta^* + \beta)$$

One can either assume that the extreme values occur for the case where $\theta^* = 0$ and $\theta^* = \pi/2$ or one can derive the general expression for the opening angle in the laboratory frame

$$\begin{aligned}
\cos\theta = \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{\mathrm{p}_1 \mathrm{p}_2} &= \frac{p_{x1}p_{x2} + p_{y1}p_{y2} + p_{z1}p_{z2}}{E_1 E_2} \\
&= \frac{-E^2\sin^2\theta^* + \gamma^2E^2(\beta^2 - \cos^2\theta^*)}{\gamma^2E^2(1 - \beta^2\cos^2\theta^*)} \\
&= \frac{-\sin^2\theta^*/\gamma^2 + \beta^2 - \cos^2\theta^*}{1 - \beta^2\cos^2\theta^*} \\
&= \frac{\beta^2(1 + \sin^2\theta^*) - 1}{1 - \beta^2\cos^2\theta^*}
\end{aligned}$$

where the extreme values are $\cos\theta = -1$ and $\cos\theta = 2\beta^2 - 1$.

Here, since $\gamma = E_\pi/m_\pi = 74.1$ and therefore using $\beta^2 = 1 - 1/\gamma^2 = 0.99982$, the minimum opening angle is

$$\cos\theta_{\min} = 2\beta^2 - 1 = 0.9963 \quad \text{or} \quad \theta_{\min} = 0.027\,\text{rad} \equiv 1.5^\circ\,.$$