Particle Physics 1 : Exercise 12

1) Isospin in strong interaction decays

This is an interesting application of the use of isospin in strong interactions. We have asserted that the SU(2) flavour symmetry is an exact symmetry of the strong interaction. One consequence is that isospin and the third component of isospin are conserved in strong interactions. Furthermore, from the point of view of the strong interaction Δ^-, Δ^0, Δ^+ and Δ^{++} are indistinguishable. The amplitudes for the above decays can be written as

$$\mathcal{M}(\Delta \to \pi N) \sim \langle \pi N | \mathrm{H}_{\mathrm{strong}} | \Delta \rangle,$$

where N indicates a nucleon - either a proton or a neutron depending on the decay. In the case of an exact SU(2) light quark flavour symmetry the above relation can be rewritten as

$$\mathcal{M}(\Delta \to \pi N) \sim A \langle \Phi(\pi N) | \Phi(\Delta) \rangle$$

where A is the same constant for all the decays of the type $\Delta \to \pi N$ and Φ represents the isospin wavefunction. Here $\langle \Phi(\pi N) | \Phi(\Delta) \rangle$ expresses the conservation of isospin in the interaction. The question therefore boils down to determining the isospin values for the states involved. Consider the decay $\Delta^- \to \pi^- n$, which in terms of isospin states corresponds to

$$\Phi\left(\frac{3}{2}, -\frac{3}{2}\right) \to \Phi(1,-1)\,\Phi\left(\frac{1}{2}, -\frac{1}{2}\right).$$

The decay rate will depend on the isospin of the combined $\pi^- n$ system. Since I_3 is an additive quantum number, the third component of the isospin of the combined $\pi^- n$ system is $-3/2$ and this implies that the total isospin must be *at least* $3/2$. But since the total isospin I lies between $|1-1/2| < I < |1+1/2|$, the isospin of the $\pi^- n$ system is uniquely identified as

$$\Phi(\pi^- n) = \Phi(1,-1)\,\Phi\left(\frac{1}{2}, -\frac{1}{2}\right) = \Phi\left(\frac{3}{2}, -\frac{3}{2}\right).$$

Consequently, the amplitude for the decay is given by

$$\mathcal{M}(\Delta^- \to \pi^- n) \sim A \langle \Phi(\pi^- n) | \Phi(\Delta^-) \rangle = A \langle \Phi\left(\frac{3}{2}, -\frac{3}{2}\right) | \Phi\left(\frac{3}{2}, -\frac{3}{2}\right) \rangle = A.$$

Now consider the decay of the Δ^0 with the isospin state

$$\Phi(\Delta^0) = \Phi\left(\frac{3}{2}, -\frac{1}{2}\right).$$

The decomposition of this state into the equivalent πN system (an isospin-1 state combined with an isospin-1/2 state) can be achieved using isospin ladder operators. Starting from the unique assignment

$$\Phi\left(\frac{3}{2}, -\frac{3}{2}\right) \to \Phi(1,-1)\,\Phi\left(\frac{1}{2}, -\frac{1}{2}\right),$$

The isospin rising operator gives :

$$\begin{aligned}
\hat{T}_+ \Phi\left(\frac{3}{2}, -\frac{3}{2}\right) &= \left(\hat{T}_+ \Phi(1,-1)\right) \Phi\left(\frac{1}{2}, -\frac{1}{2}\right) + \Phi(1,-1)\left(\hat{T}_+ \Phi\left(\frac{1}{2}, -\frac{1}{2}\right)\right) \\
\sqrt{3}\Phi\left(\frac{3}{2}, -\frac{1}{2}\right) &= \sqrt{2}\Phi(1,0)\,\Phi\left(\frac{1}{2}, -\frac{1}{2}\right) + \Phi(1,-1)\,\Phi\left(\frac{1}{2}, \frac{1}{2}\right)
\end{aligned}$$

Updated: 6 décembre 2023

or equivalently

$$\Phi(\Delta^0) = \sqrt{\frac{2}{3}}\Phi(\pi^0 n) + \sqrt{\frac{1}{3}}\Phi(\pi^- p).$$

Thus the decay amplitudes of the Δ^0 can be written

$$\mathcal{M}(\Delta^0 \to \pi^0 n) \sim A\langle\Phi(\pi^0 n)|\sqrt{\frac{2}{3}}\Phi(\pi^0 n) + \sqrt{\frac{1}{3}}\Phi(\pi^- p)\rangle = \sqrt{\frac{2}{3}}A$$

and

$$\mathcal{M}(\Delta^0 \to \pi^- p) \sim A\langle\Phi(\pi^- p)|\sqrt{\frac{2}{3}}\Phi(\pi^0 n) + \sqrt{\frac{1}{3}}\Phi(\pi^- p)\rangle = \sqrt{\frac{1}{3}}A.$$

Since decay rates are proportional to the amplitude squared

$$\Gamma(\Delta^- \to \pi^- n) : \Gamma(\Delta^0 \to \pi^- p) : \Gamma(\Delta^0 \to \pi^0 n) = 3 : 1 : 2.$$

The other ratios follow from the same arguments.

2) Leptonic decays of neutral vector mesons

The underlying process is the QED annihilation process $q\bar{q} \to e^+e^-$, where the matrix element can be expressed as

$$\mathcal{M}(q\bar{q} \to e^+e^-) \sim \langle e^+e^-|\hat{Q}_q|q\bar{q}\rangle = AQ_q,$$

where A is assumed to be a constant and Q_q is the charge of the quark in the annihilating quark-antiquark pair. For the ϕ meson, which is a pure $s\bar{s}$ state, the matrix element

$$\mathcal{M}(\phi \to e^+e^-) \sim \langle e^+e^-|\hat{Q}_q|s\bar{s}\rangle = AQ_s = -\frac{1}{3}A.$$

For the ρ^0 with wavefunction $|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, the phases of the two components are important and the total amplitude depends on the coherent sum of the contributions from the decays of the $u\bar{u}$ and $d\bar{d}$. Here the matrix element can be written as

$$\mathcal{M}(\rho^0 \to e^+e^-) \sim \langle e^+e^-|\hat{Q}_q|\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\rangle = A\frac{1}{\sqrt{2}}(Q_u - Q_d) = A\frac{1}{\sqrt{2}}\left(\frac{2}{3} + \frac{1}{3}\right) = \frac{1}{\sqrt{2}}A.$$

Similarly

$$\mathcal{M}(\omega \to e^+e^-) \sim \langle e^+e^-|\hat{Q}_q|\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\rangle = A\frac{1}{\sqrt{2}}(Q_u + Q_d) = A\frac{1}{\sqrt{2}}\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{3\sqrt{2}}A.$$

Neglecting the small differences in phase space, $\Gamma \propto \mathcal{M}^2$, and so

$$\Gamma(\rho^0 \to e^+e^-) : \Gamma(\omega \to e^+e^-) : \Gamma(\phi \to e^+e^-) \approx 9 : 1 : 2.$$

Updated: 6 décembre 2023