Particle Physics 1 : Exercise 11

1) Spin and structure functions

If quarks were spin-0 particles, there would be no magnetic contribution to this QED scattering process. Consequently, $F_1^{ep}(x)$, which is associated with the $\sin^2\theta/2$ angular dependence, would be zero.

2) Proton parton distribution functions

Writing both $u(x)$ and $\overline{u}(x)$ in terms of sea and valence contributions, $u(x) = u_V(x) + u_S(x)$ and $\overline{u}(x) = \overline{u}_S(x)$. Making the reasonable assumption that the sea contributions for quarks and antiquarks are the same, i.e. $u_S(x) = \overline{u}_S$,

$$\int_0^1 (u(x) - \overline{u}(x))\mathrm{d}x = \int_0^1 u_V(x)\mathrm{d}x = 2.$$

3) Experimental measurements of the structure functions

The significance of this question is that it related to the actual early measurements of structure funcions where the target was usually either liquid Hydrogen or liquid Deuterium.

The structure functions $F_2^{ep}(x)$ and $F_2^{en}(x)$ for the elctron-proton and electron-neutron scattering respectively can be written as

$$\begin{aligned} F_2^{ep}(x) &= x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\overline{u}^p(x) + \frac{1}{9}\overline{d}^p(x)\right) \\ F_2^{en}(x) &= x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\overline{u}^n(x) + \frac{1}{9}\overline{d}^n(x)\right). \end{aligned}$$

Using isospin symmetry one can make the following assumptions :

$$\begin{aligned} d^n(x) &= u^p(x) = u(x) \\ u^n(x) &= d^p(x) = d(x) \\ \overline{d}^n(x) &= \overline{u}^p(x) = \overline{u}(x) \\ \overline{u}^n(x) &= \overline{d}^p(x) = \overline{d}(x) \end{aligned}$$

so that the structure functions become

$$\begin{aligned} F_2^{ep}(x) &= x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right) \\ F_2^{en}(x) &= x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right). \end{aligned}$$

Therefore, the structure function of the electron-deuterium scattering, where deuterium is composed of one proton and one neutron, is given by

$$F_2^{eD}(x) = \frac{1}{2}\left(F_2^{ep}(x) + F_2^{en}(x)\right) = \frac{5}{18}x\left(u(x) + d(x) + \overline{u}(x) + \overline{d}(x)\right).$$

The integrals of the two measured distributions are thus given by

$$\int_0^1 F_2^{ep}(x)\mathrm{d}x = \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{and} \quad \int_0^1 F_2^{eD}(x)\mathrm{d}x = \frac{5}{18}(f_u + f_d)$$

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where f_u and f_d are defined as

$$f_u = \int_0^1 \left(xu(x) + x\overline{u}(x)\right) \quad \text{and} \quad f_d = \int_0^1 \left(xd(x) + x\overline{d}(x)\right).$$

Hence,

$$\frac{\int_0^1 F_2^{eD}(x)\mathrm{d}x}{\int_0^1 F_2^{ep}(x)\mathrm{d}x} = 0.84 = \frac{5}{2}\frac{f_u + f_d}{4f_u + f_d}$$

and so

$$\frac{f_u + f_d}{4f_u + f_d} = 0.336.$$

From this relation it is straightforward to show that

$$f_d/f_u \simeq 0.52,$$

which is consistent with the result quoted in the lecture.

4) Parton distribution functions in terms of valence and sea quarks

(a) Since strange quarks have charge -1/3, they couple to the photon in the QED deep inelastic scattering process in the same way as down quarks and therefore

$$F_2^{ep}(x) = \frac{4}{9}x\left[u(x) + \overline{u}(x)\right] + \frac{1}{9}x\left[d(x) + \overline{d}(x) + s(x) + \overline{s}(x)\right].$$

(b) Remembering that the PDFs in the above expression refer to the PDFs for the proton,

$$F_2^{en}(x) = \frac{4}{9}x\left[u^n(x) + \overline{u}^n(x)\right] + \frac{1}{9}x\left[d^n(x) + \overline{d}^n(x) + s^n(x) + \overline{s}^n(x)\right].$$

Making the following assumptions

$$u^p(x) = d^n(x), \quad \overline{u}^p(x) = \overline{d}^n(x), \quad d^p(x) = u^n(x), \quad \overline{d}^p(x) = \overline{u}^n(x),$$

the above expression can be rewritten in terms of the proton PDFs as

$$F_2^{en}(x) = \frac{4}{9}x\left[d(x) + \overline{d}(x)\right] + \frac{1}{9}x\left[u(x) + \overline{u}(x) + s(x) + \overline{s}(x)\right],$$

where it has been assumed that the strange quark PDFs from the proton and neutron are identical. Note that this is a reasonable assumption since the strange quark content of the nucleons are from the sea.

Using the above expressions

$$\int_0^1 \frac{\left[F_2^{ep}(x) - F_2^{en}(x)\right]}{x}\mathrm{d}x = \frac{1}{3}\int_0^1 \left[u(x) - d(x) + \overline{u}(x) - \overline{d}(x)\right]\mathrm{d}x$$

Writing the quark PDFs in terms of valence and sea contributions, and assuming that the quark and antiquark sea contributions (which arise from gluon-splitting) are the same, i.e. $u_S(x) = \overline{u}(x)$ and $d_S(x) = \overline{d}(x)$, then

$$\begin{aligned}
\int_0^1 \frac{\left[F_2^{ep}(x) - F_2^{en}(x)\right]}{x}\mathrm{d}x &= \frac{1}{3}\int_0^1 \left[u_V(x) + u_S(x) - d_V(x) - d_S(x) + \overline{u}(x) - \overline{d}(x)\right]\mathrm{d}x \\
&= \frac{1}{3}\int_0^1 \left[u_V(x) - d_V(x) + 2\overline{u}(x) - 2\overline{d}(x)\right]\mathrm{d}x \\
&= \frac{1}{3} + \frac{2}{3}\int_0^1 \left[\overline{u}(x) - \overline{d}(x)\right]\mathrm{d}x,
\end{aligned}$$

where the last step follows from there being two valence up quarks and one valence down quark. The measured value can therefore be interpreted as

$$\int_0^1 \left[\overline{u}(x) - \overline{d}(x)\right] \mathrm{d}x = \frac{3}{2}\left[0.24 - 0.33 \pm 0.03\right] = -0.14 \pm 0.05,$$

demonstrating that there is a deficit of $\overline{u}$ quarks relative to $\overline{d}$ quarks in the proton. This may be explained by a relative suppression of the $g \to u\overline{u}$ process due to the exclusion principle and the larger number of up-quark states which are already occupied.

5) Kinematic variables in $e^- p$ DIS

For $e^- p \to e^- X$ DIS at HERA, the masses of the electron and proton can be neglected and thus the four-momentua can be written :

$$p_1 = (E_1, 0, 0, E_1), \quad p_2 = (E_2, 0, 0, -E_2) \quad \text{and} \quad p_3 = (E_3, E_3 \sin\theta, 0, E_3 \cos\theta).$$

Using these four-momenta

$$q^2 = -2p_1 \cdot p_3 = -2E_1 E_3 (1 - \cos\theta)$$

and

$$p_2 \cdot q = p_2 \cdot p_1 - p_2 \cdot p_3 = 2E_1 E_2 - E_2 E_3 (1 + \cos\theta).$$

Thus, the Bjorken scaling variable x, defined as

$$x = \frac{-q^2}{2p_2 \cdot q},$$

can be written

$$x = \frac{E_3}{E_2}\left[\frac{1 - \cos\theta}{2 - (E_3/E_1)(1 + \cos\theta)}\right].$$

For the event shown in the text, $\theta \approx 150°$. Thus

$$Q^2 = 2E_1 E_3 (1 - \cos\theta) = 2 \cdot 27.5 \cdot 250 (1 - \cos\theta) \simeq 3 \times 10^4 \text{ GeV}^2.$$

Similarly

$$x = \frac{250}{820}\left[\frac{1 - \cos\theta}{2 - (250/27.5)(1 + \cos\theta)}\right] \simeq 0.7.$$

Hence the DIS interaction shown is one of the highest Q^2 DIS interactions observed at HERA and has high x.

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