Particle Physics 1 : Exercise 10

1) Kinematics of e^-p elastic scattering

In a e^-p scattering experiment, the incident electron has energy $E_1 = 529.5$ MeV and the scattered electrons are detected at an angle $\theta = 75°$ relative to the incoming beam.

(a) At this angle, almost all of the scattered electrons are measured to have an energy of $E_3 \approx 373$ MeV. What can be concluded from this observation ?

(b) Find the corresponding value of Q^2.

2) Charge distributions and form factors

For a spherically symmetric charge distribution $\rho(\mathrm{r})$, where

$$\int \rho(\mathrm{r})\,\mathrm{d}^3\mathbf{r} = 1$$

show that the form factor can be expressed as

$$F(\mathbf{q}^2) = \frac{4\pi}{\mathrm{q}} \int_0^\infty \mathrm{r}\sin(\mathrm{qr})\rho(\mathrm{r})\,\mathrm{dr} \sim 1 - \frac{1}{6}\mathrm{q}^2\langle R^2\rangle + ...,$$

where $\langle R^2\rangle$ is the mean square charge radius. Hence show that

$$\langle R^2\rangle = -6\left[\frac{\mathrm{d}F(\mathbf{q}^2)}{\mathrm{dq}^2}\right]_{\mathrm{q}^2=0}$$

3) Proton charge radius

Using the answer to the previous question and the data in Figure 1.a, estimate the root-mean squared charge radius $\langle R^2\rangle$ of the protons.

4) Measurement of form factors

From the slope and intercept of the right plot in Figure 2 obtain the values for $G_M(Q^2)$ and $G_E(Q^2)$ when $Q^2 = 0.292$ GeV^2.

5) Proton charge distribution

The experimental data of Figure 1 can be described by the form factor

$$G(Q^2) = \frac{G(0)}{(1 + Q^2/Q_0^2)^2}$$

with $Q_0^2 = 0.71$ GeV^2. Taking $Q^2 \approx \mathbf{q}^2$, show that this implies that the proton has an exponential charge distribution of the form

$$\rho(\mathbf{r}) = \rho_0 e^{-\mathrm{r}/a},$$

and find the value of a.

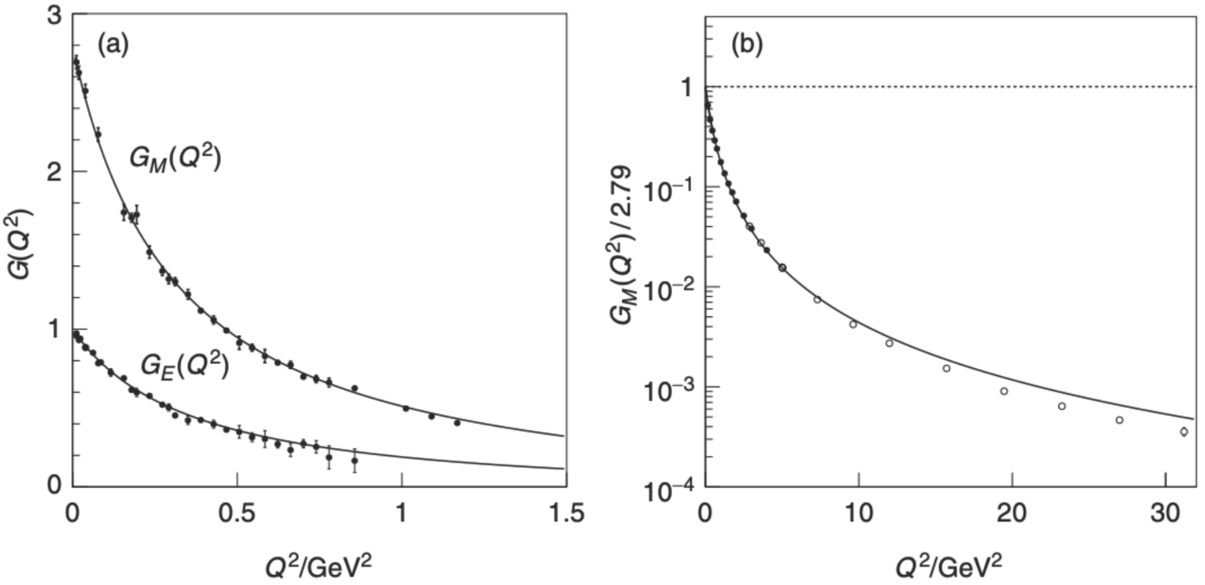

FIGURE 1 – (a) Measurements of $G_E(Q^2)$ and $G_M(Q^2)$ from $e^-p \to e^-p$ elastic scattering data at low Q^2, adapted from Hughes et al. (1965). (b) Measurements of $G_M(Q^2)$ at higher Q^2, data from Walker et al. (1994) (solid circles) and Sill et al. (1993) (open circles). The curves correspond to the dipole function described in the lecture. Figure and caption taken from Thomson (2013).

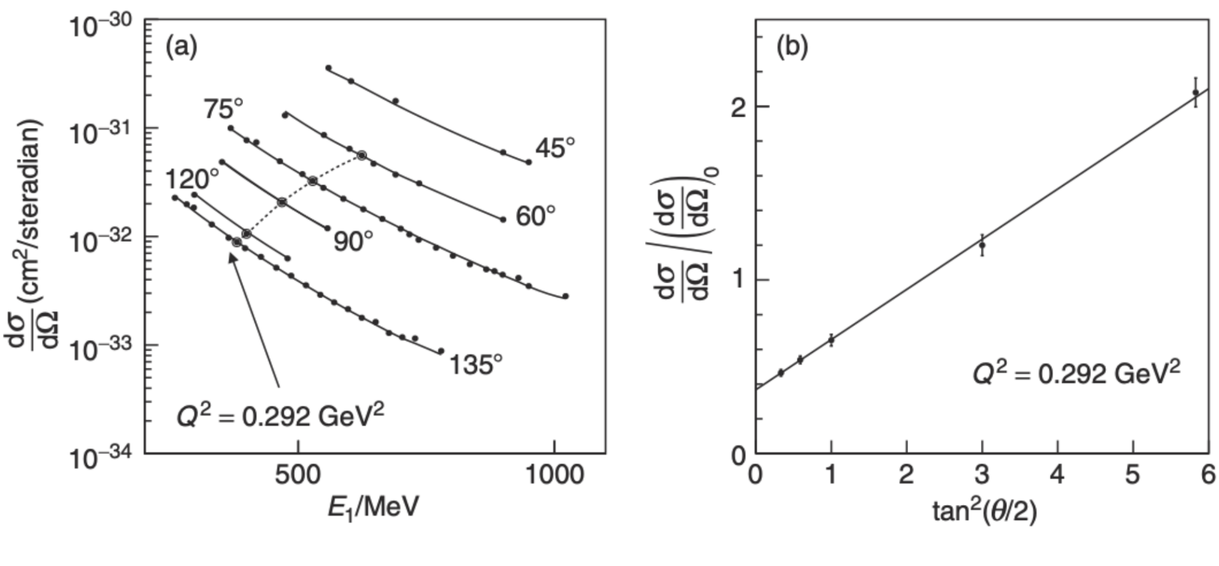

FIGURE 2 – Low energy $e^-p \to e^-p$ elastic scattering data. Data from Hughes et al. (1965). Figure and caption taken from Thomson (2013).