Everything you always wanted to know about

Noise

*(and the methods to reduce it)**

Dr. Benjamin Dwir

Laboratory of Physics of Nanostructures (LMSC)

Benjamin.dwir@epfl.ch

PH.D2.397

*But were afraid to ask

Outline:

- Introduction
- Sources of noise
- Sources of interference
- Methods of noise reduction
- Methods of interference reduction
- Summary

Introduction: What is noise?

The name comes from Audio, so let's listen to some noise!

But noise can be also visual:

In this course, we'll talk about noise in the context of (electrical signal) measurements and Data acquisition

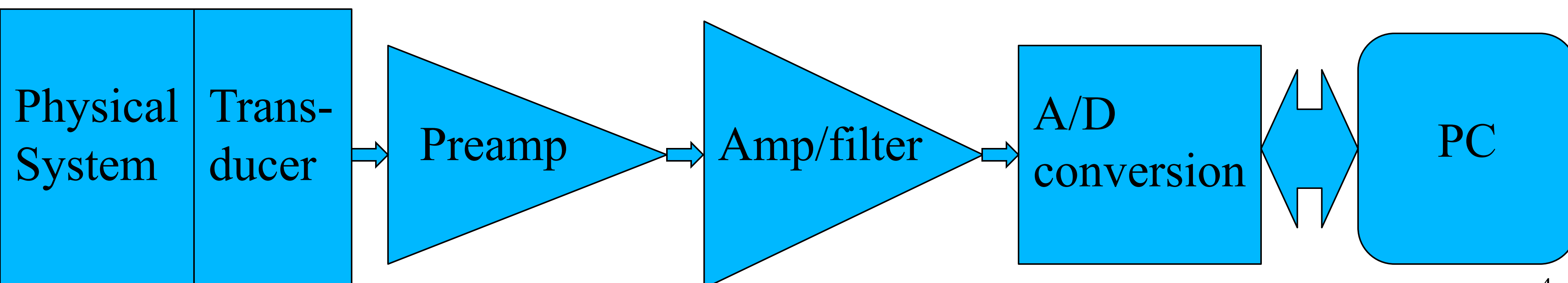
The data acquisition scheme

Most experimental data acquisition systems have the general structure:

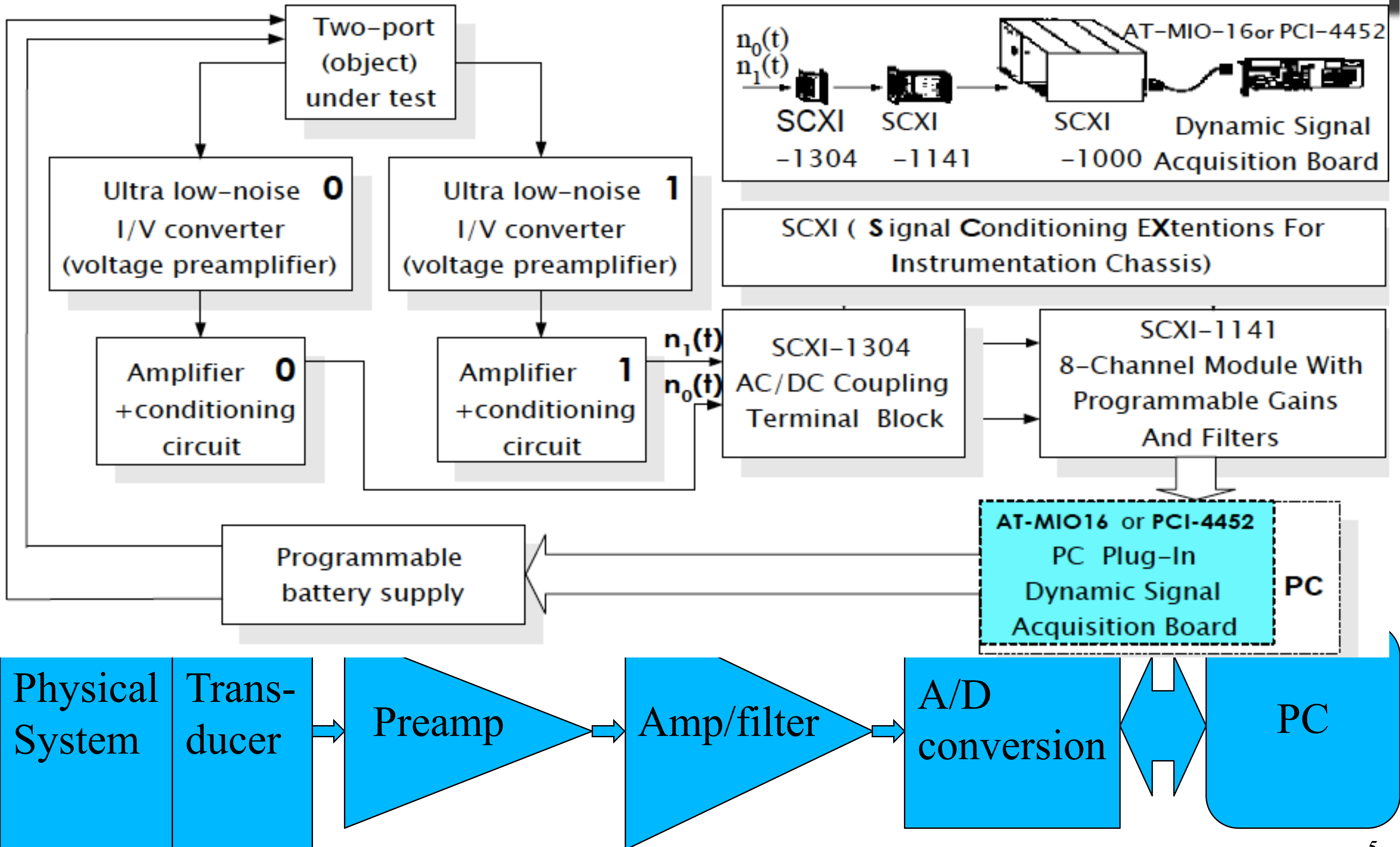
- Physical (chemical) system, with transducers to measure
- Pre-amplifier, Amplifier/filter
- A/D converter linked to a PC

Some systems have additional elements:

- Excitation source(s)
- Auxiliary elements (variable gain, signal shaping, etc.)

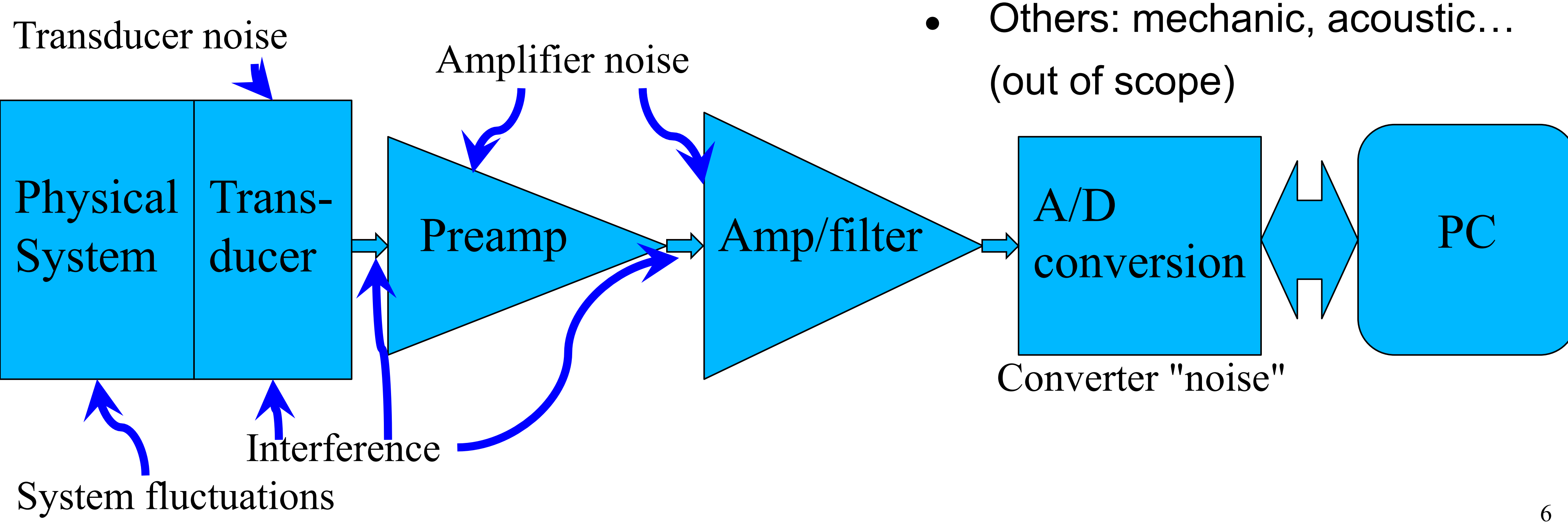


The data acquisition scheme



The sources of noise

- "Internal" noise
  - Thermal noise
  - Shot noise
  - Flicker noise
  - ADC "noise"
- "External" noise (Interference)
  - Power-line
  - RF
  - Thermal, magnetic, and other EMF sources
  - Others: mechanic, acoustic… (out of scope)



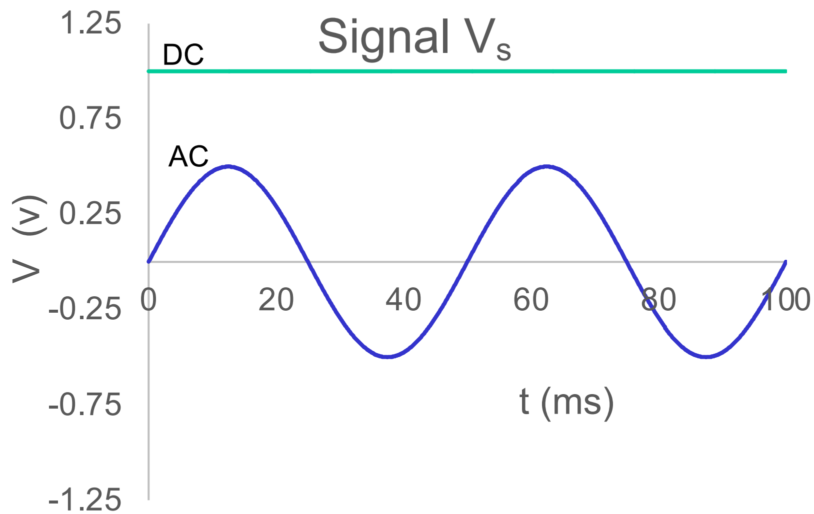
What is noise?

Noise is unwanted addition to the signal $V_s(t)$ that we want to measure:

$V_m(t) = V_s(t) + V_n(t)$, $V_s(t)$ = signal voltage, $V_n(t)$ = noise voltage.

"Internal" noise:

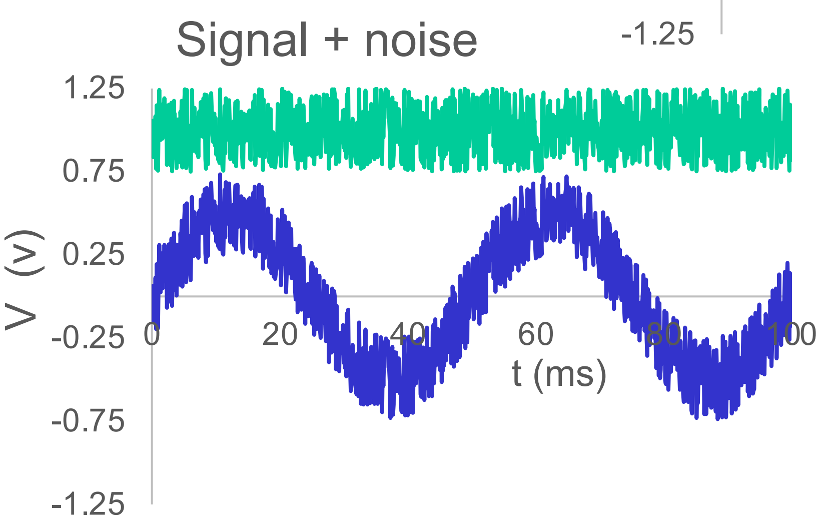
$V_n(t)$ is random.

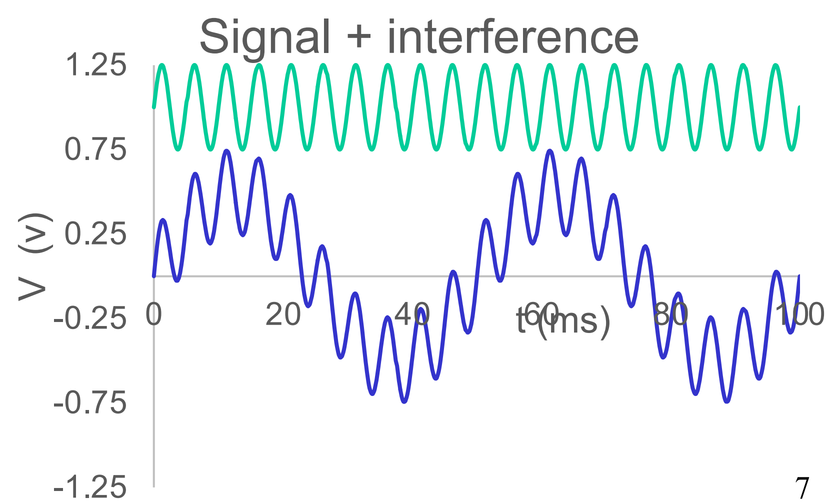


"External" noise (Interference):

$V_n(t) = V_n \sin(\omega t)$.

ω can be low (50 Hz) or high (MHz-GHz).





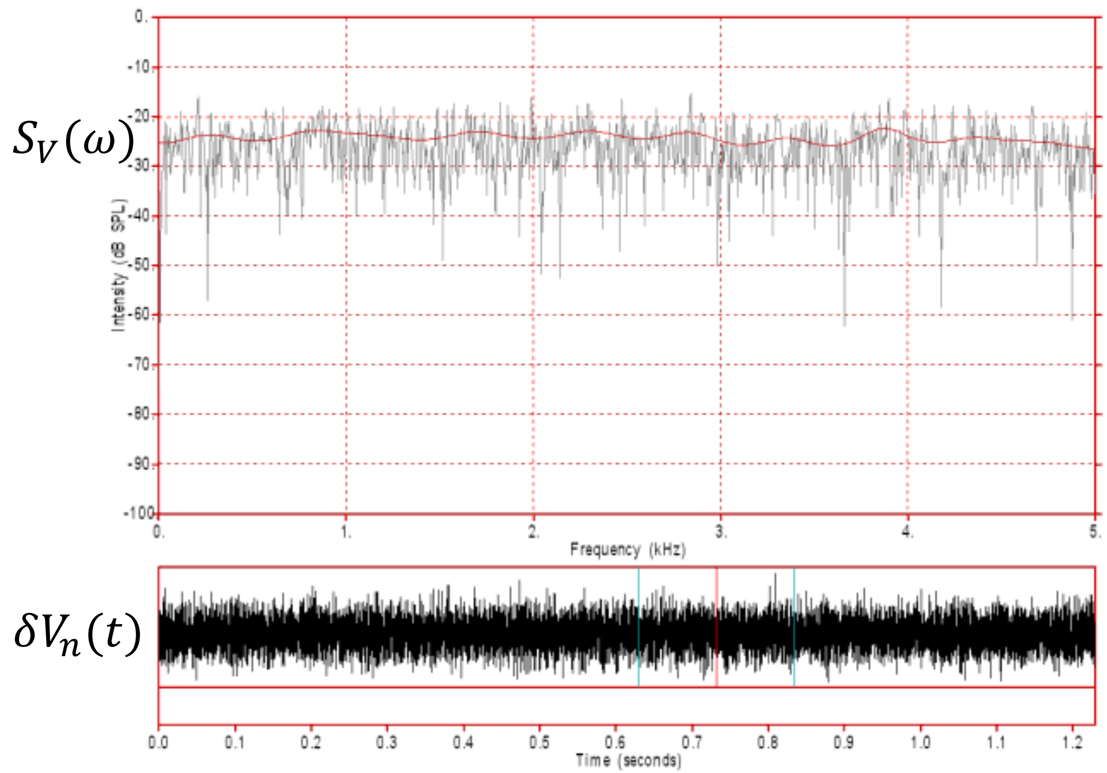
The nature of noise

- Noise is random fluctuations in a signal. Its average is zero $\langle V_n \rangle = 0$, so we have to look at the average of its autocorrelation function:

$$\langle V_n(t) V_n(0) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} V_n(t+\tau) V_n(\tau) d\tau$$

- The Spectral power density $S_V(\omega)$ (power per unit bandwidth) of the noise is the Fourier transform of its autocorrelation function. Since the Fourier transform of a convolution is a product, we get:

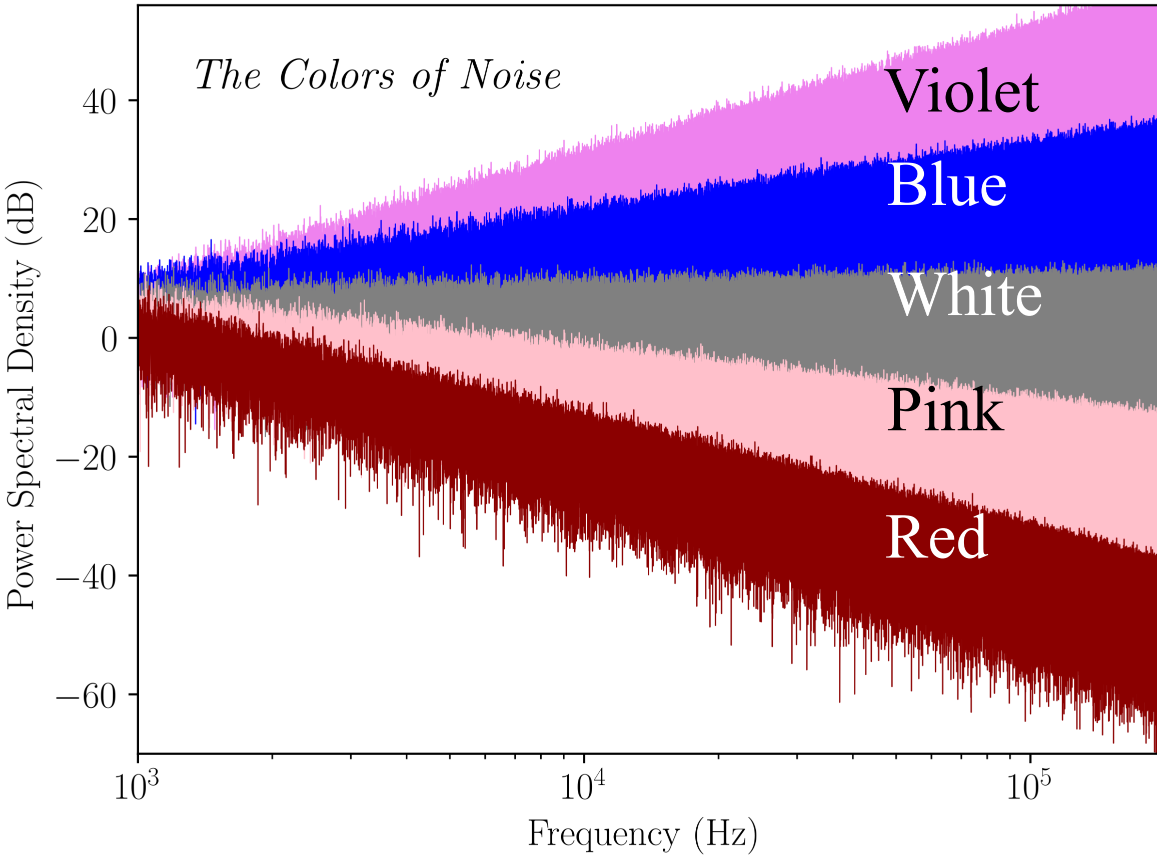
$$S_V(\omega) = \lim_{T\to\infty} \frac{1}{T} \int_{\infty}^{\infty} V_n(\omega) V_n^*(\omega) d\omega$$



LABORATORY OF nanoSTRUCTURES

The nature of noise

- The frequency dependence of the power density is related to the physical nature of the noise source. It's usual to make the analogy with the visible color spectrum:
  - If $S(\omega)$ is independent of frequency, it's called "White noise"
  - If $S(\omega)$ increases for lower frequencies ($\alpha 1/f$) , it's called "Pink noise"
  - If $S(\omega)$ increases for higher frequencies (αf) , it's called "Blue noise"
  - Etc.



Sources of noise: Internal

- Thermal noise
- Shot noise
- Flicker noise
- ADC "noise"

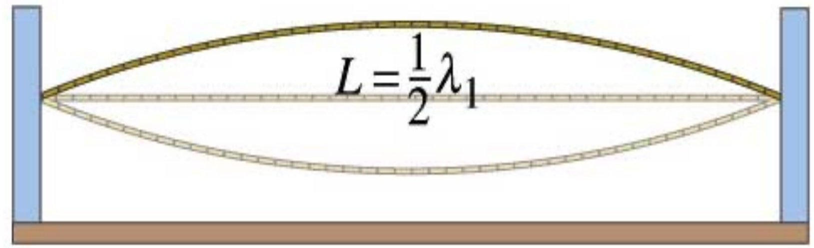
The physics behind thermal noise

There were many developments of the thermal noise equations, starting with Einstein's explanation of the Brownian motion (1905), followed by developments in the 1920s by De Haas-Lorentz, Zernike, and Johnson and Nyquist (1928). We'll see here two derivations:

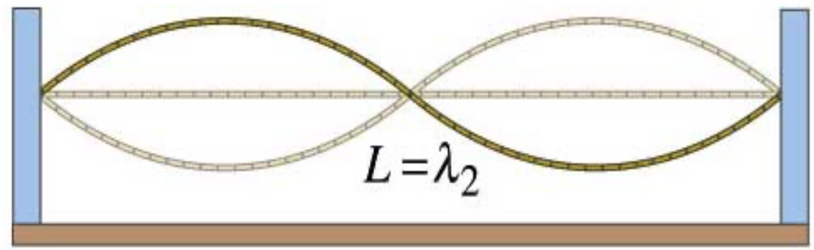
- The Nyquist Gedankenexperiment
- The classical Fluctuation-Dissipation Theorem.

Reminder: waveguide/cavity modes

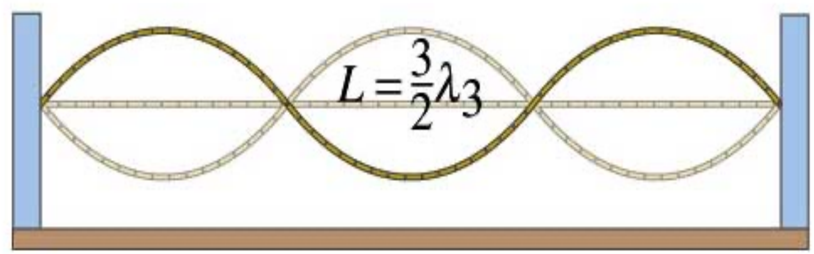
- A cavity of length L can support many standing waves, or **cavity modes**, as elastic waves in a string. They correspond to the conditions: $E(0,t)=0$ and $E(L,t)=0$. These condition can be translated to: $L=m\lambda/2$. Since $\lambda=v/f$ (v = wave velocity), the mode frequencies are: $f=mv/2L$. In most cases, L is macroscopic, so m is big. The frequency difference between adjacent modes is: $\Delta f=v/2L$.
- In a waveguide, the same modes exist, as sound waves in an open tube. The frequency difference between adjacent modes is the same: $\Delta f=v/2L$.



Fundamental or first harmonic, f_1



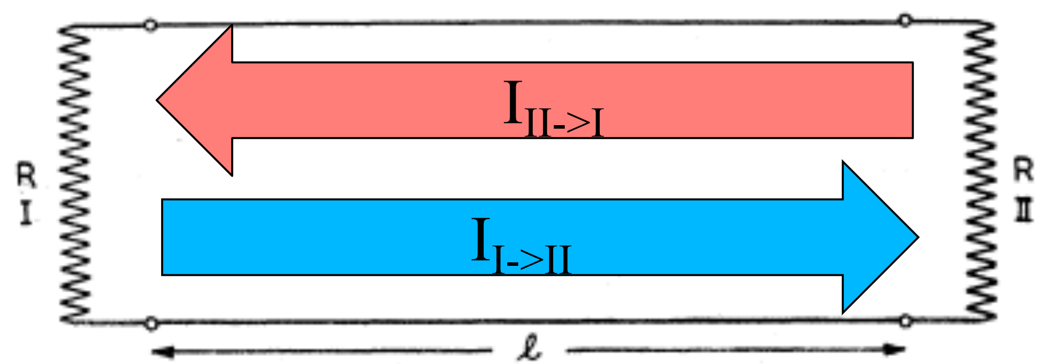
First overtone or second harmonic, $f_2 = 2f_1$



Second overtone or third harmonic, $f_3 = 3f_1$

The Nyquist Gedankenexperiment

- We look at a system with two identical resistors *R*, connected by a transmission line with the same impedance *R*, all in thermal equilibrium at a temperature T>0.
- Thermally excited electron form fluctuating identical current in both directions.
- We short the transmission line at its two ends, keeping the thermal energy in its resonance modes. The number of modes in a frequency interval $\delta f \equiv B$ (Bandwidth) is: $2\ell B/v$, and the thermodynamic equipartiton law associates an energy $k_B T$ to each mode ($k_B T/2$ for *E*-field, $k_B T/2$ for *B*-field), so: $\delta E = 2\ell k_B T B/v$.
- This energy was transferred from the two resistors during a time: $\delta t = \ell/v$, so the power from each resistor is: $\mathrm{P} = \delta E/\delta t = k_B T B$.
- The current is: $\mathrm{I} = \mathrm{V}/2\mathrm{R}$, and the power is: $P = I^2 R$, so the noise voltage is: $V_n^2 = 4R^2 I_n^2 = 4RP = 4Rk_B TB$



The fluctuation-dissipation theorem

- Suppose x(t) is an observable of a dynamic system, which has a response to an external field f(t) given by a "susceptibility" function χ(t), according to:
  $x(t) = \int_{-\infty}^{t} f(\tau)\chi(t-\tau)d\tau$, or in the frequency domain: $\hat{x}(\omega) = \hat{\chi}(\omega)\ \hat{f}(\omega)$, where $\hat{x}(\omega)$ is the Fourier transform of x(t).

- The fluctuations in x(t) can then be described by their power spectrum:

  $$S_x(\omega) = \hat{x}(\omega)\ \hat{x}^*(\omega)$$

- The classical Fluctuation-Dissipation Theorem (FDT) links the fluctuations in $\hat{x}(\omega)$ to the system dissipation $\hat{\chi}(\omega)$:

  $$S_x(\omega) = \frac{2k_B T}{\omega} \operatorname{Im} \hat{\chi}(\omega)$$

- In a mechanical system, the Brownian motion (fluctuations of the position x of a particle in a medium) is related to the viscous drag force (dissipation) χ in the medium.

Thermal noise from the FDT:

The fluctuation-dissipation theorem in an electrical system:

- We use the system variable *x*=Q, related to *f*=*V* by a coefficient $\chi=\alpha$:

  Q(ω)=V(ω)α(ω). Since: $Q(t) = \int I(t)dt = \int \frac{V(t)}{Z}dt$, (using Ohm's law, *Z* is the system impedance), we get: $Q(\omega) = \frac{V(\omega)}{i\omega Z(\omega)}$, so: $\alpha(\omega) = \frac{-i}{\omega Z(\omega)}$.

- The FDT gives (for Q): $S_Q(\omega) \; = \hat{Q}(\omega)\;\hat{Q}^*(\omega) = \frac{2k_BT}{\omega}Im(\alpha(\omega)) = \frac{2k_BT}{\omega}Im\left(\frac{-i}{\omega Z(\omega)}\right)$.

- For V: $S_V(\omega) = S_Q(\omega)/\alpha(\omega)^2 = (i\omega Z(\omega))^2 S_Q(\omega) =$

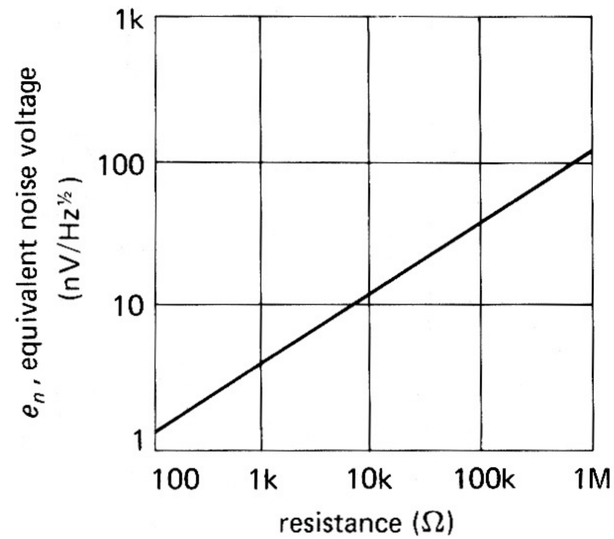
  $-\,\omega^2 Z(\omega)Z^*(\omega)\frac{2k_BT}{\omega}Im\left(\frac{-i}{\omega Z(\omega)}\right) = 2k_BTIm(iZ^*(\omega)) = 2k_BTRe(Z(\omega))$

- The measured voltage noise density is double (positive and negative frequencies in the Fourier space): $V_n^2(\omega) = 2S_V(\omega) = 4k_BTRe(Z(\omega))$

- The real part of *Z* is the electrical resistance *R*

Thermal (Johnson, white) noise 1

For any resistance R, the thermal (Johnson) noise voltage density (per unit frequency) is: $V_n^2 = 4Rk_BT$ or, for a system bandwidth B: $V_n^2 = 4Rk_BTB$

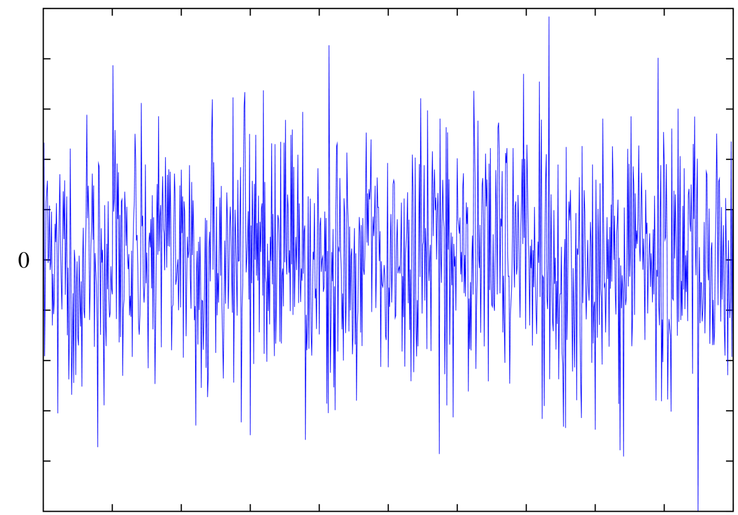


The noise current is: $I_n^2 = 4k_BTB/R$

For a system bandwidth B, the noise power is: $P_n = 2k_BTB$

This noise has the following properties:

- The noise is "internal", or "inherent" to the resistive nature of the system: it is already generated in the sample (except in superconductors), as well as in the resistive parts of the (pre-)amplifier.
- The average of the noise is zero



Thermal (Johnson, white) noise 2

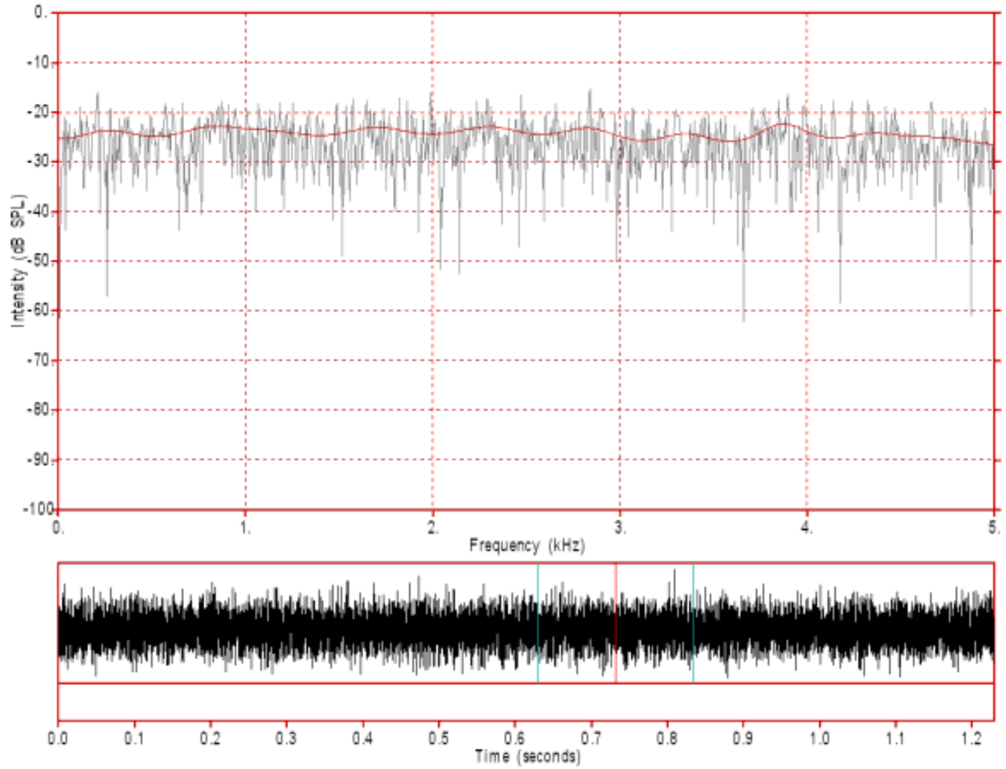
- The spectrum is "flat" (also called "white"): the same voltage density per Hz, for all frequencies

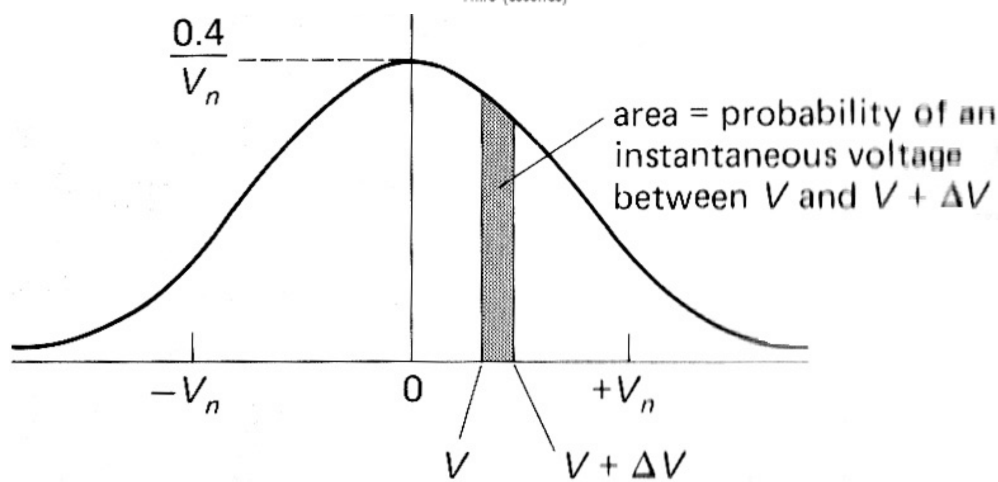
- Noise is random, so noises add up as sum-of-squares:

$$V_{n(tot)} = \sqrt{V_{in}^2 + V_{an}^2}$$

- The probability distribution of voltage is Gaussian:

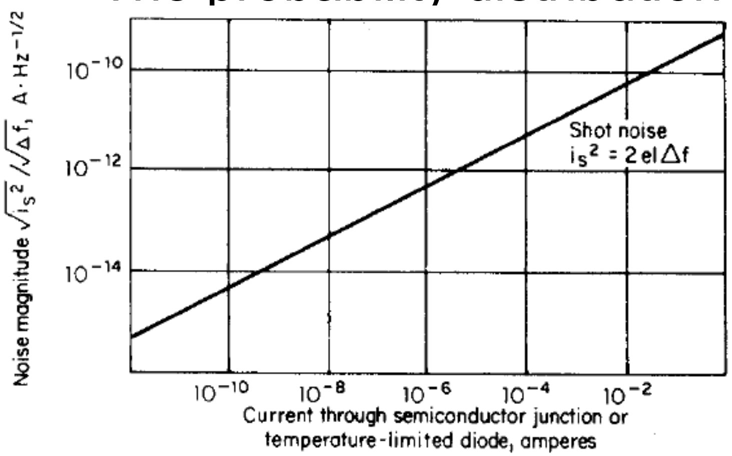
$$p(V) = \frac{1}{\sqrt{2\pi}V_n} e^{-\left(\frac{V^2}{2V_n^2}\right)} dV$$

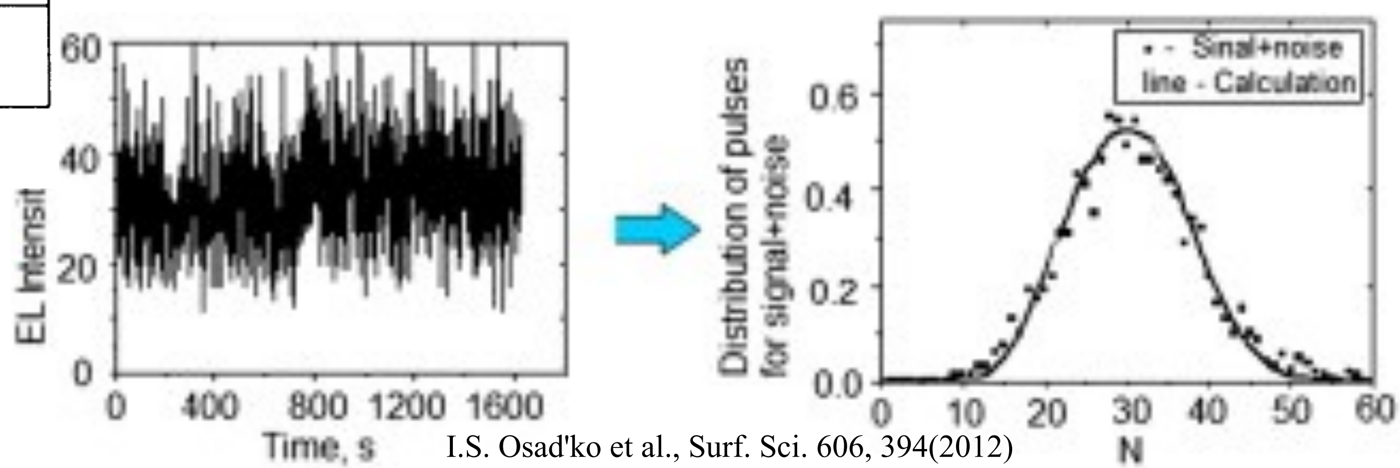




Shot noise

- The discrete nature of electrons gives rise to statistical fluctuations: $\Delta N = \sqrt{N}$
- The result is a noise component (fluctuations) in electrical current, with current density per unit frequency: $I_n^2 = 2eI$ or for a bandwidth B: $I_n^2 = 2eIB$
- The frequency spectrum is also flat (same noise density per unit frequency)
- The probability distribution is Poissonian, but becomes Gaussian for high currents

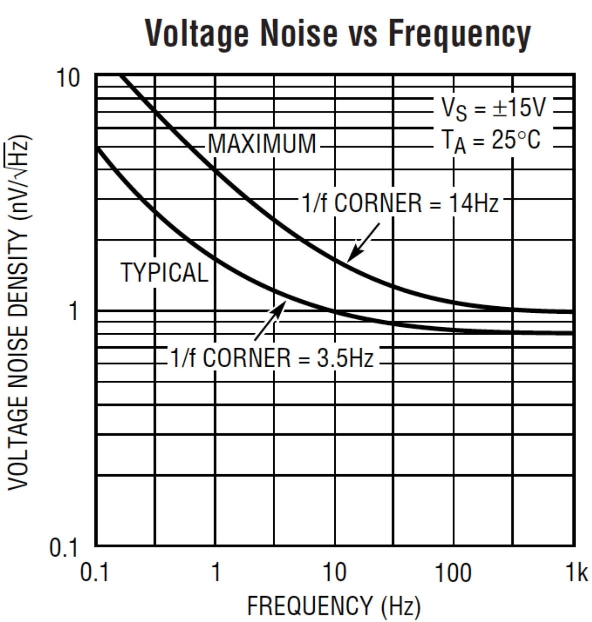


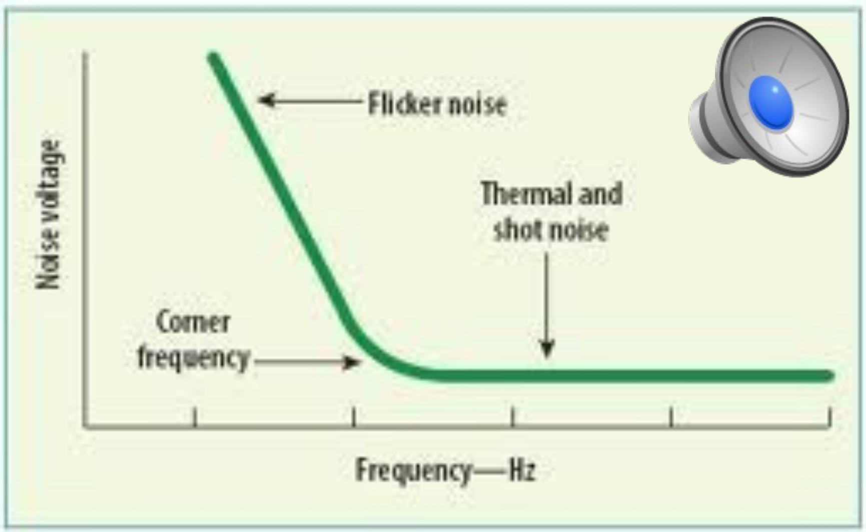


I.S. Osad'ko et al., Surf. Sci. 606, 394(2012)

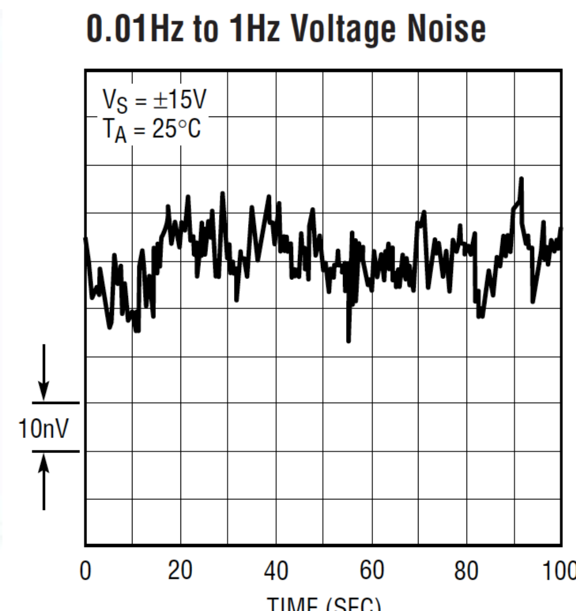
Flicker (pink, 1/f) noise

- Electronic components (e.g. amplifiers) produce internal noise, due to fluctuations in charge carriers (electrons and holes) in the semiconductor
- Especially notorious is the 1/*f*, flicker, or "pink" noise: power density decreases with frequency.
- The combination is characterized by the corner frequency: equilibrium between white and pink noises



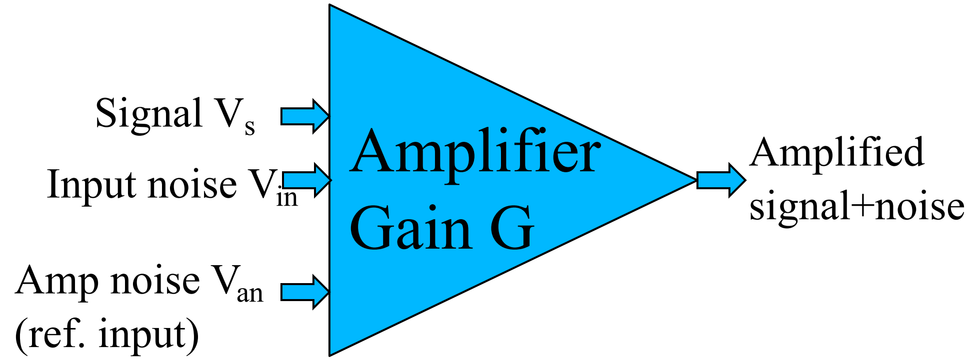


Flicker noise is low-level semiconductor device noise that increases as a function of inverse carrier frequency, or 1/f.



Amplifier noise: configuration

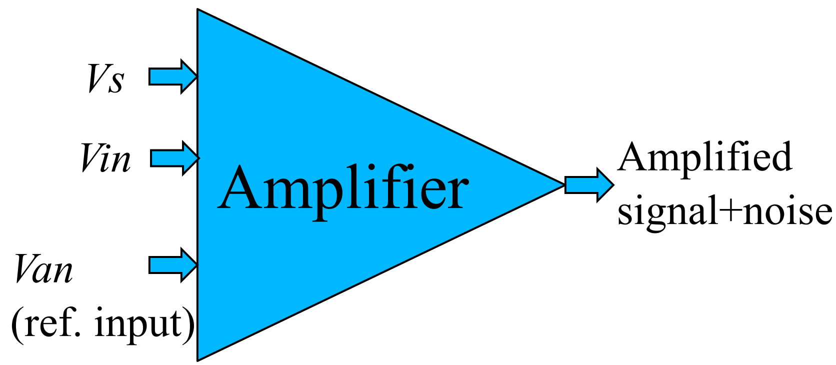
The typical situation:



- Sample of resistance R produces a signal V_s, connected to the input of an amplifier with gain G
- Usually the input signal V_s contains some (usually thermal) noise V_{in}.
- The amplifier adds some noise V_{an} to the signal. This noise is also amplified, giving a component GV_{an} to the output.
- The two random noises sum as square rms values: $V_{n(tot)} = \sqrt{V_{in}^2 + V_{an}^2}$
- At the output, we get the amplified sum of all components: signal + input noise + amplifier noise: $V_{out} = G\left(V_s + \sqrt{V_{in}^2 + V_{an}^2}\right)$

Quantifying noise: SNR

- Low-frequency: $R_{amp} > R_{sample}$
- We measure voltage
- We define: $SNR = \left(\frac{V_s}{V_{n(tot)}}\right)^2$, or: $SNR_{dB} = 10log(SNR) = 20\left(log(V_s) - log\left(V_{n(tot)}\right)\right)$
- The input signal V_s contains some thermal noise V_{in}. The amplifier adds its noise V_{an} , then both are amplified by a gain G. Total noise is: $V_{n(tot)} = \sqrt{V_{in}^2 + V_{an}^2}$
- Thermal noise is: $V_n^2 = 4Rk_BTB$



Quantifying noise: F, NF

- Low-frequency: $R_{amp} > R_{sample}$

- We define the amplifier's noise factor: $F = \frac{SNR_{input}}{SNR_{output}}$

- and noise figure: $NF_{dB} = 10log(F) = SNR_{dB,input} - SNR_{dB,output}$

- The noise factor is then: $F = \frac{V_s/V_{in}}{GV_s/G\sqrt{V_{in}^2+V_{an}^2}} = \frac{\sqrt{V_{in}^2+V_{an}^2}}{V_{in}} = \sqrt{1+(V_{an}/V_{in})^2}$

  and the noise figure: $\mathrm{N}F = 10log(1+(V_{an}/V_{in})^2)$

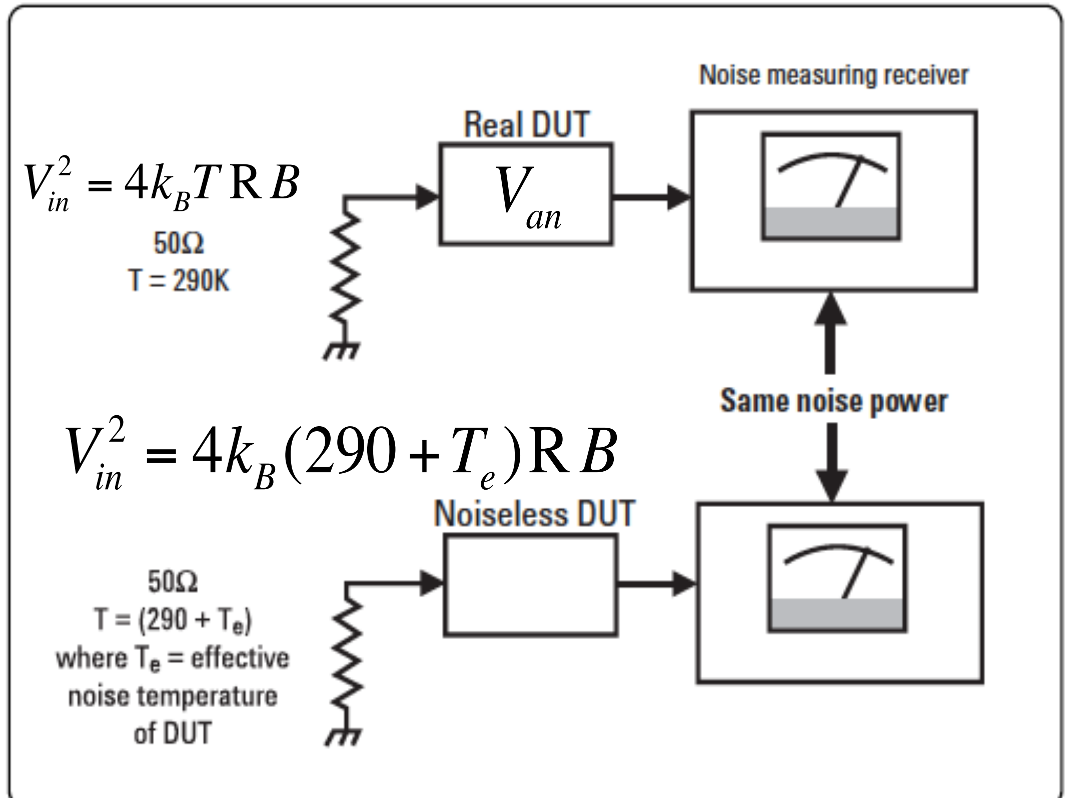
- For thermal noise: $V_{in}^2 = 4Rk_BTB$, we get:

  $F = \sqrt{1+V_{an}^2/4Rk_BTB}$, $\mathrm{N}F = 10log(1+V_{an}^2/4Rk_BTB)$

- Noise figure depends on sample resistance!

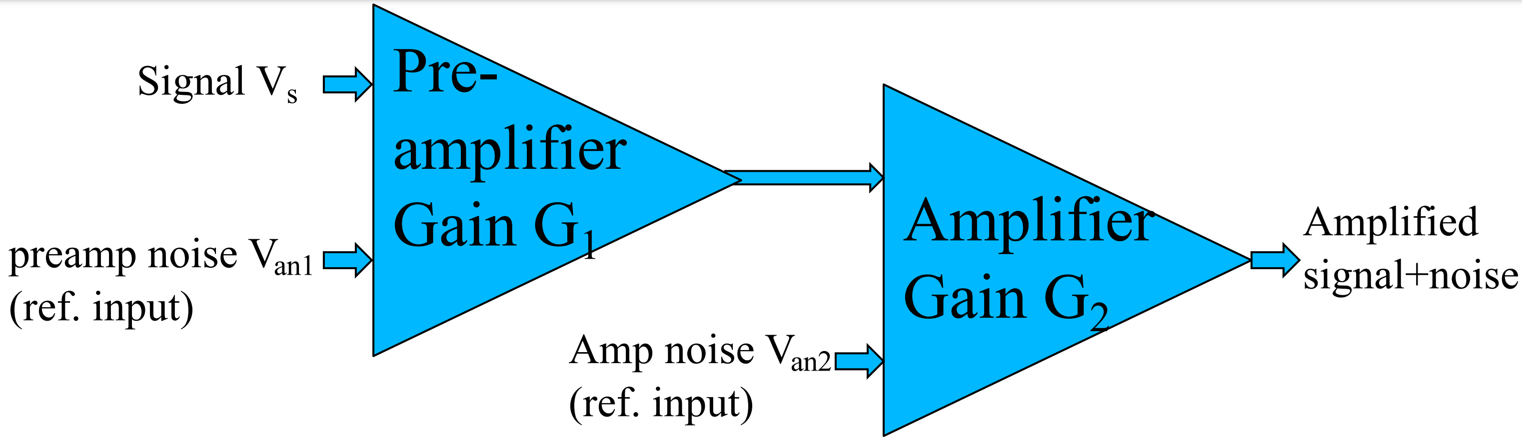
Noise temperature

- Another method of specifying noise: The noise temperature
- Instead of added noise from the amplifier, we suppose that the input resistor is at a higher temperature: $V_{an}^2 = 4Rk_BT_eB$
- Therefore: $F = \sqrt{1 + T_e/290}$, or: $NF = 10log(1 + T_e/290)$
- The "standard" temperature is 290K



LABORATORY OF nanoSTRUCTURES

The role of the preamplifier



The typical situation: pre-amp with gain G_1, noise V_{an1}, then amp with G_2 and V_{an2}.

- The signal + noise of the preamp are both multiplied by the amp's gain G_2.
- The output signal is then: $V_{an1} = G_2 G_1 V_s + G_2(V_{an2} + G_1 V_{an1})$
- If we look at the input-referenced noise (= output noise / total gain) we get:

$$V_{an(tot)} = V_{an1} + V_{an2}/G_1$$

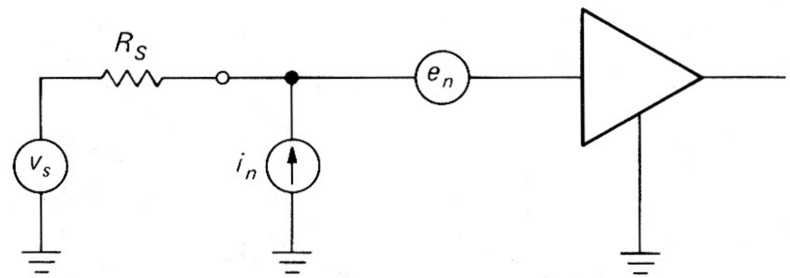
- If the preamp gain is large, the amp's noise is negligible!
- Note: Actually, noise should be added as sum-of-squares, but the result is the same

Electronic amplifier noise specs

In electronic amplifying devices (op-amps, transistors)

we usually find specifications of voltage noise V_n

and current noise I_n



- This model supposes a noiseless amplifier, with noise voltage and current sources added. The current noise flows through the input (sample) resistance.
- The two noises sum as square rms values:

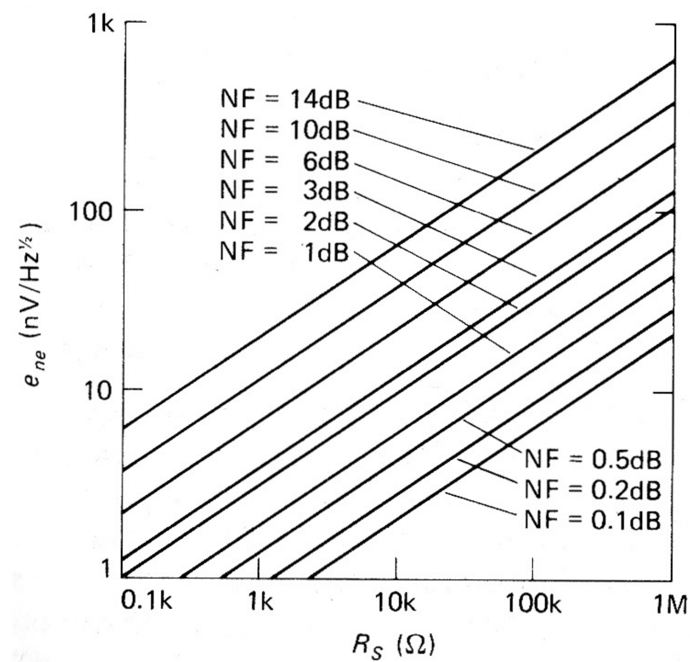
$$V_{an(tot)} = \sqrt{V_n^2 + (I_n R_s)^2}$$

- The total noise (incl. source) is:

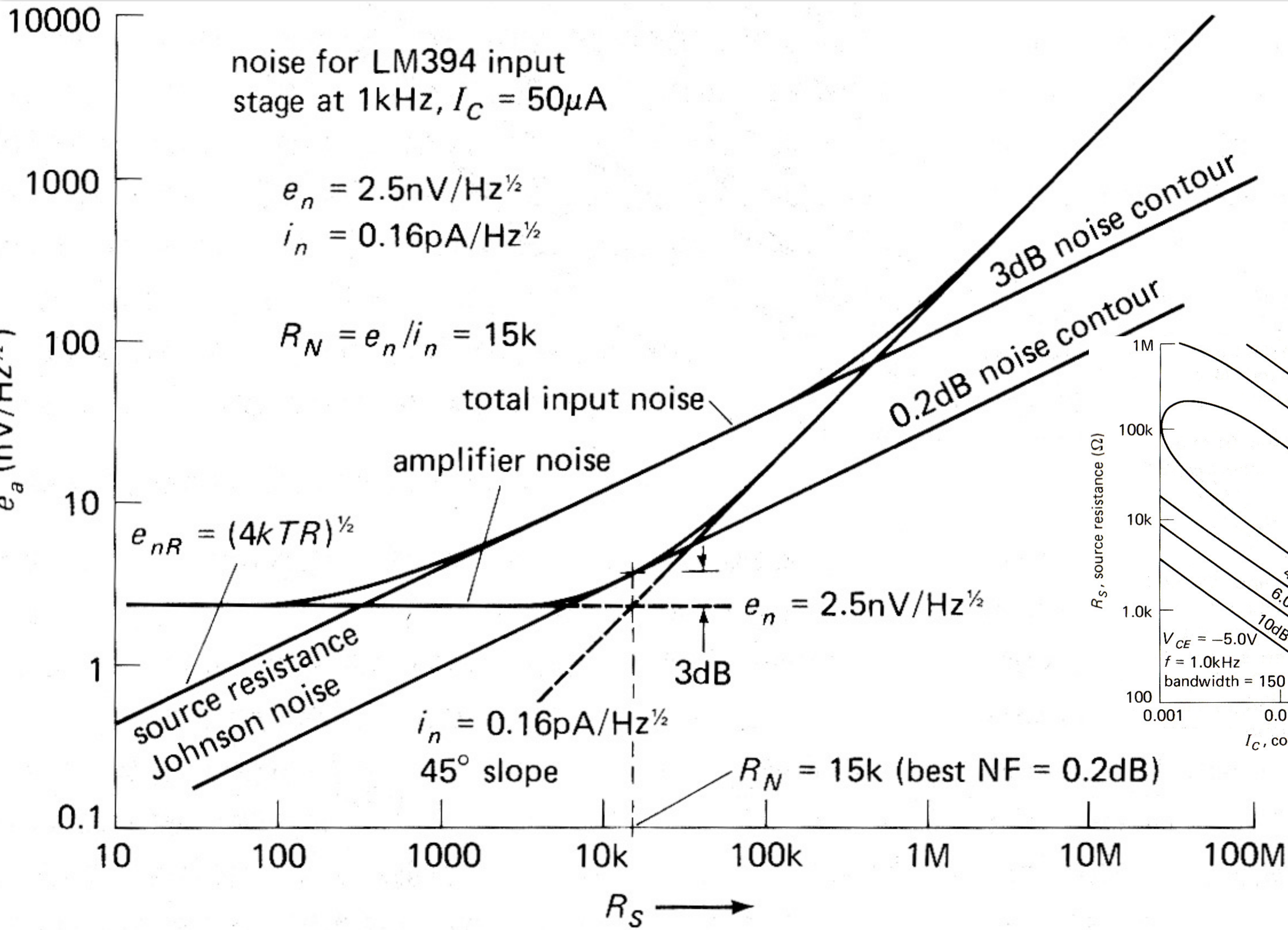
$$V_{an(tot)} = \sqrt{V_n^2 + 4k_T T R_s + (I_n R_s)^2 B}$$

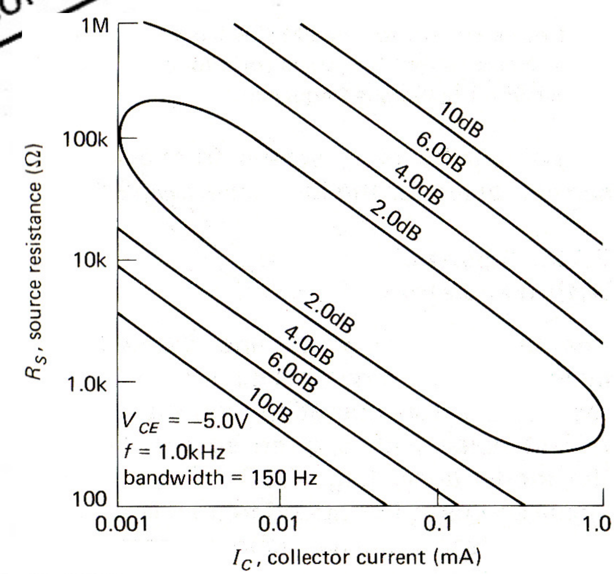
- For given V_n , I_n, we get the lowest noise when:

$$R_s = V_n / I_n$$



Example for amplifier noise





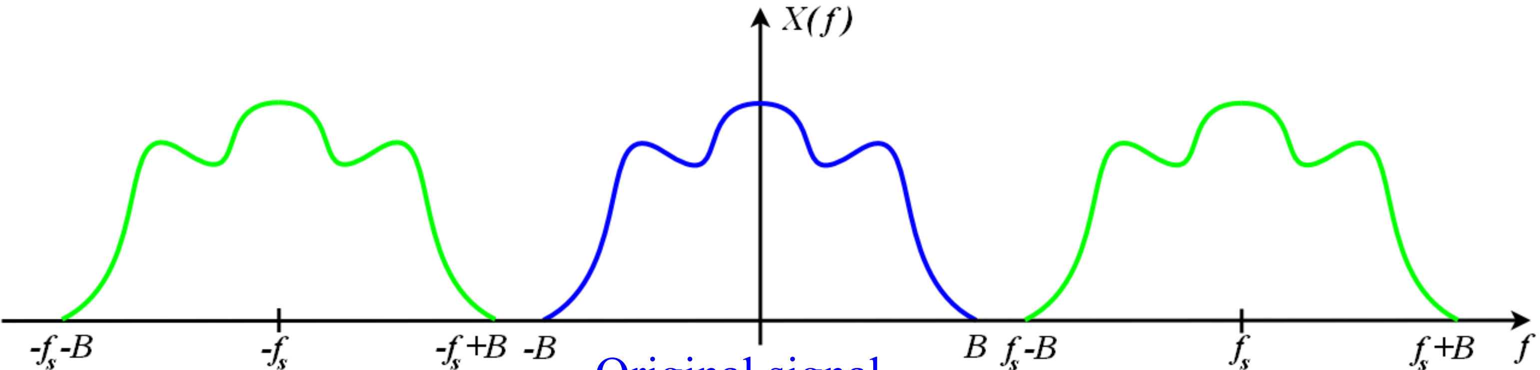
DAC "noise"

The DAC can present two types of distortions, which can contribute to noise:

- Aliasing, or generation of extra frequencies when the input signal bandwidth exceeds half the DAC's sampling frequency
- Quantization noise: each input voltage to the DAC is "forced" to the nearest digital representation, with limited choice of possible values (DAC resolution). A continuous ramp signal would thus end up as a staircase of numbers, the error (difference between signal and representation) is a sawtooth.

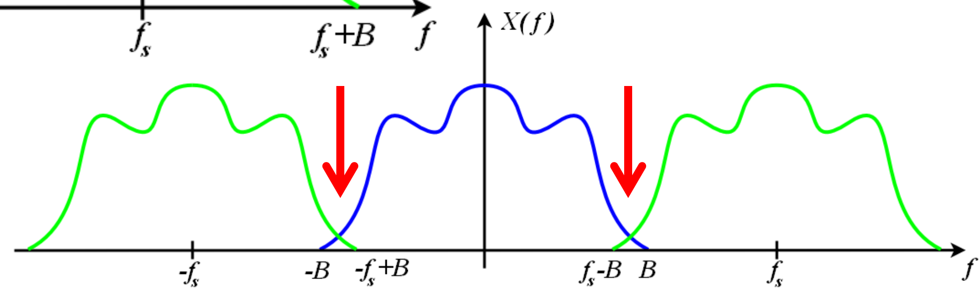
DAC "noise": Aliasing

- According to the Shannon-Nyquist's theorem, a bandwidth-limited signal (max. bandwidth *B*) can be perfectly reconstructed after sampling at a frequency f_s, if the sampling frequency is higher than 2*B*.



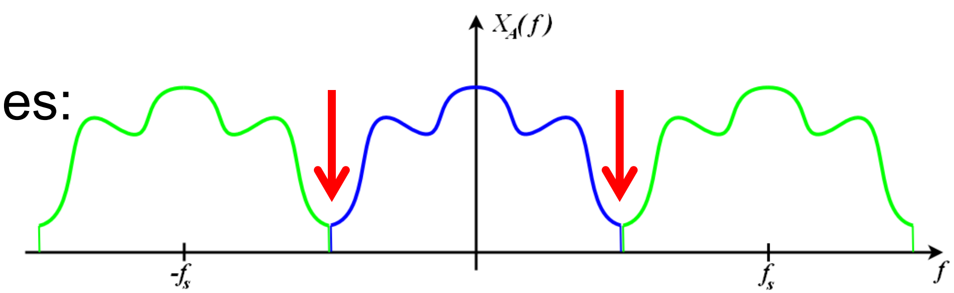
Extra frequencies, generated by sampling: can be filtered out!

Original signal, limited to bandwidth B



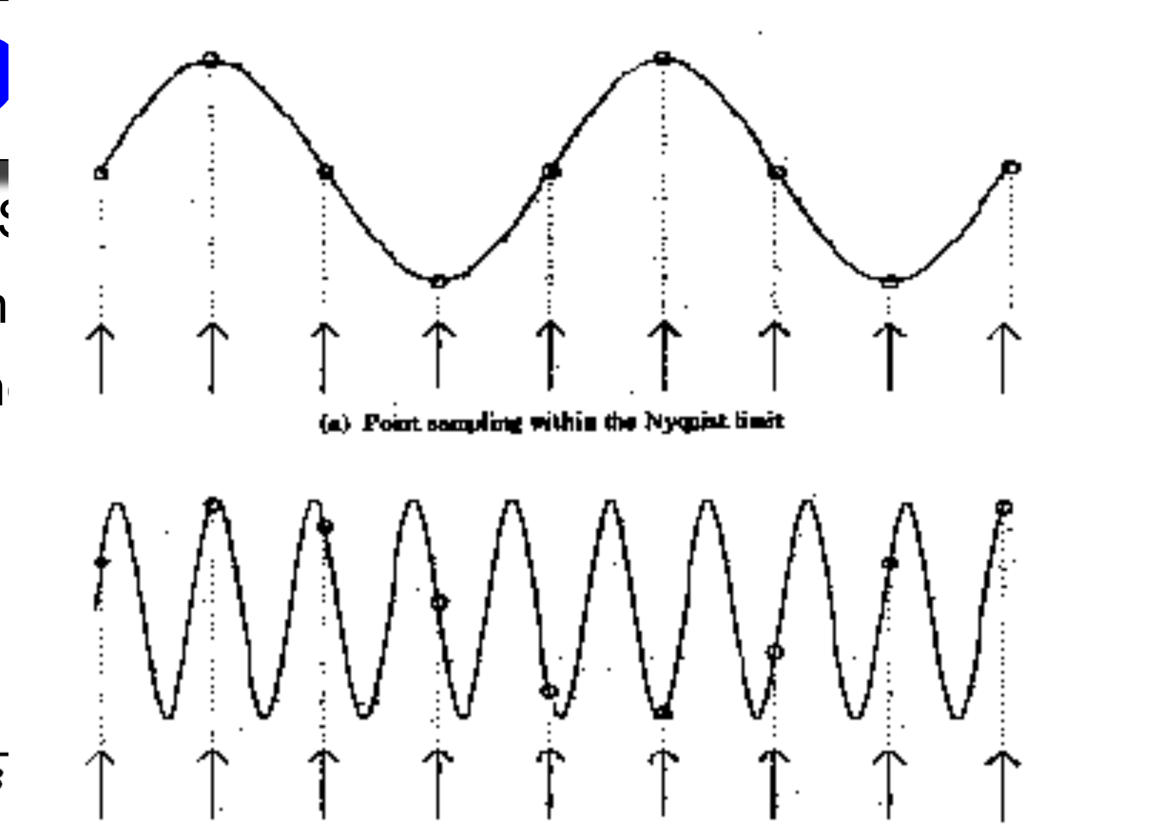
- If this is not the case:

we see generation of extra (aliased) frequencies:

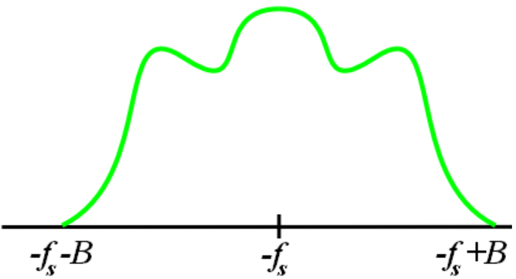


D

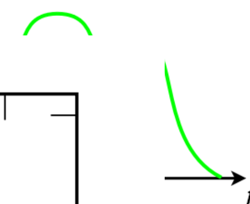
- According to the S ignal (max. bandwidth *B*) can quency f_s, if the sampling frequen


(a) Point sampling within the Nyquist limit

(b) Point sampling beyond the Nyquist limit

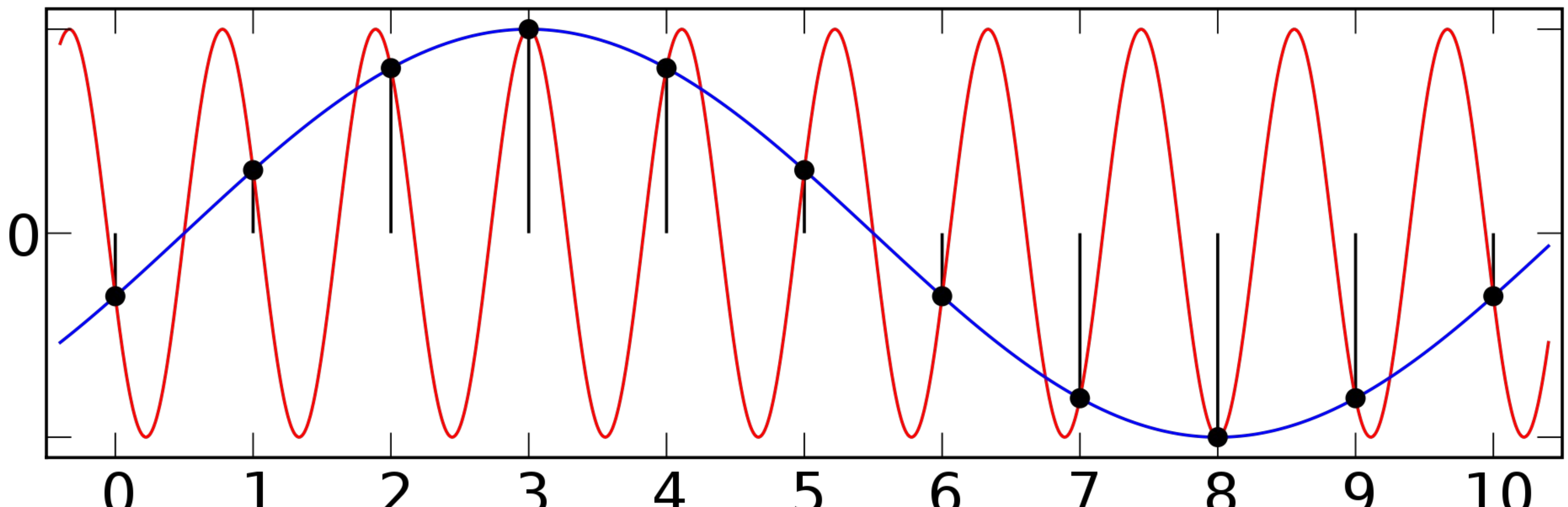


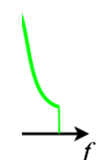
Extra frequencies generated can be filt



- If this

we see

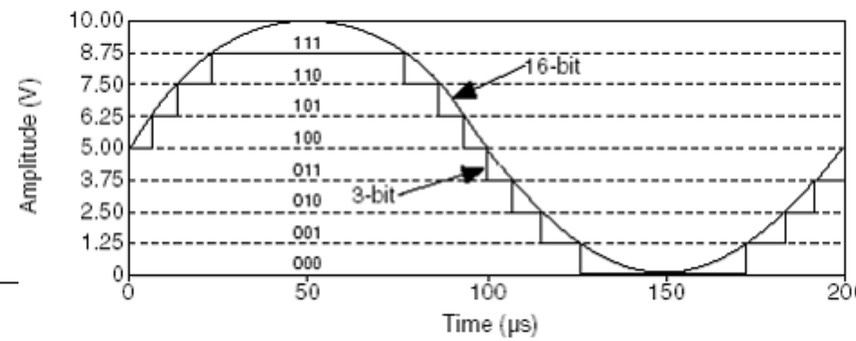


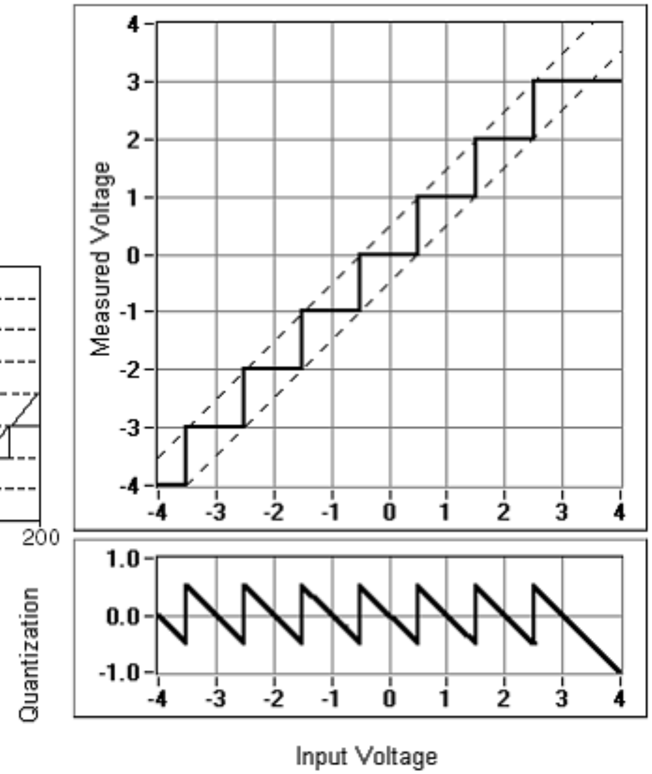


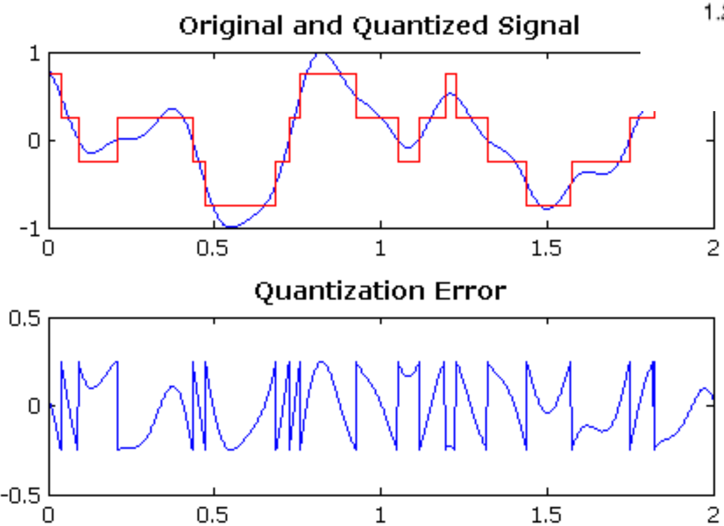
DAC "noise": Quantization noise

The D/A converter has finite voltage resolution, related to the number of bits Q: each bit equals $V_{fs}/2^Q$. The analog signal is forced into this resolution, so the digitized value does not represent exactly the signal. The error is called **Quantization error**

- If the signal has random voltage: SNR (dB) = $6.02 \cdot Q$
- For a sinusoidal signal: SNR (dB) = $1.76 + 6.02 \cdot Q$







Sources of noise: External (interference)

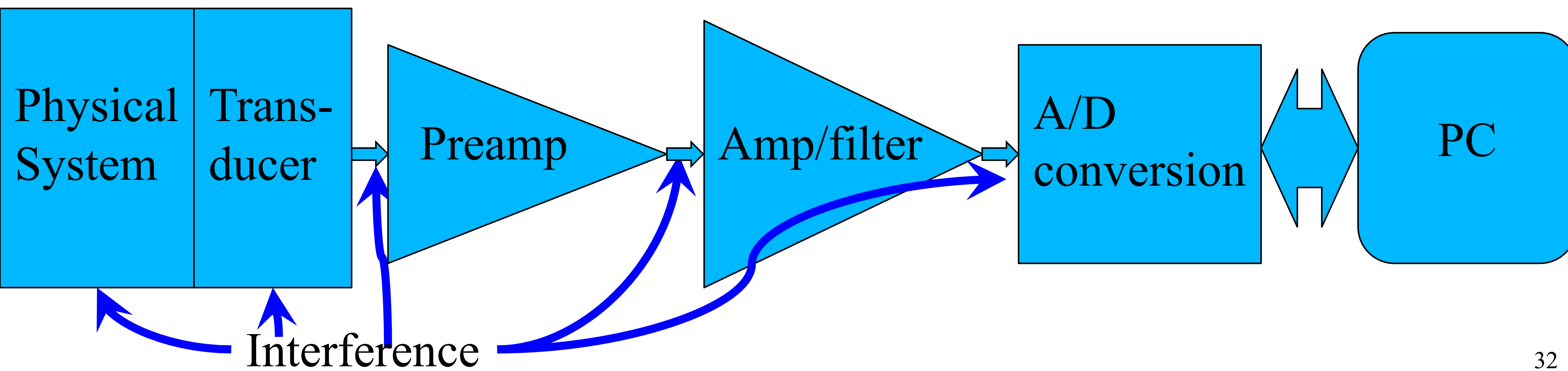
- DC: Thermal, magnetic, and other EMF sources
- AC/RF: capacitive, inductive, ground loop

Interference: general

There are many types of interference, which we need to classify:

- Type of interfering signal: DC, AC, RF, pulse ?
- Place of interference: At the sample/transducer, input cable, amplifier, ….
- Quantity of interference: Amplitude, frequency/pulse width, …

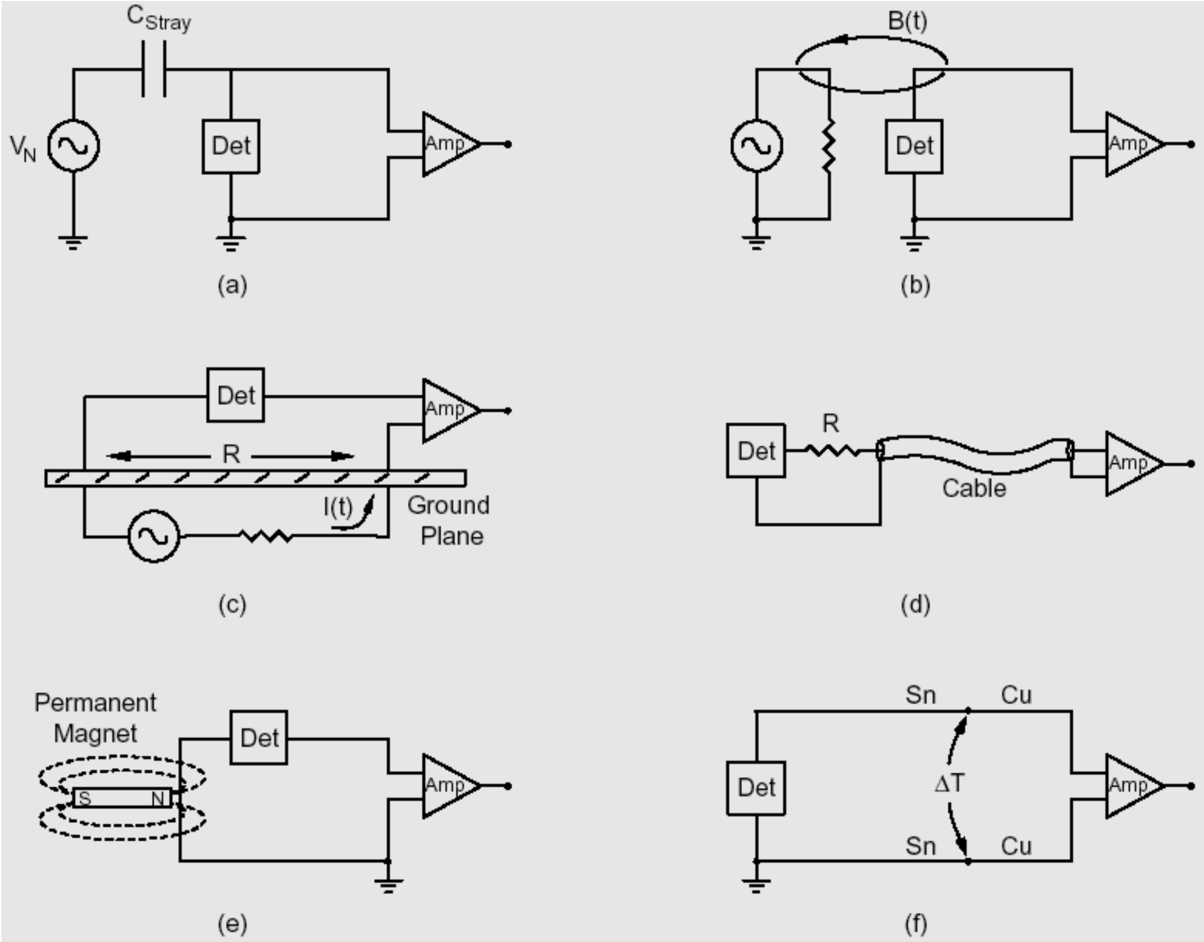
- Most common: 50Hz “hum”, RF interference, thermal emf
- The most sensitive parts: the sample, input cable, preamp



Interference: DC - types

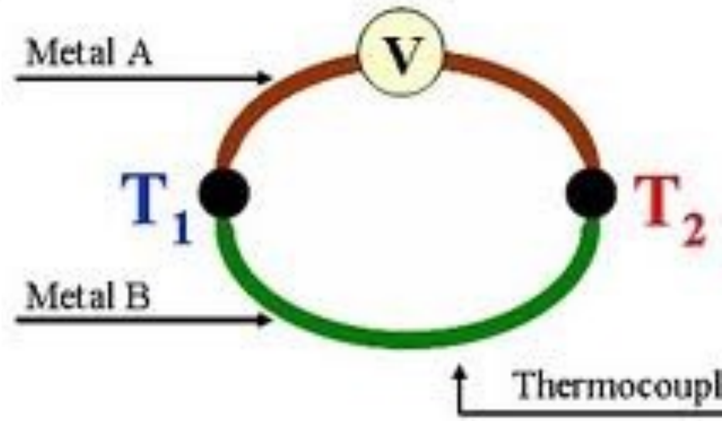
Unwanted DC voltage/current can arrive at the sample or to to the input of the preamp:

- Thermal EMF: thermoelectric (Seebeck) effect, between different metals
- Chemical EMF: produced by humidity, different metals
- Triboelectric EMF (not DC, but very low frequency): produced by friction, cable insulation
- Magnetic EMF (not DC, but very low frequency): produced by moving iron structures, strong magnets
- Ground loops



Interference: DC - thermal

- The thermoelectric (Seebeck) effect: if two different metals form a loop, and their junctions are at two different temperatures, they generate a voltage:



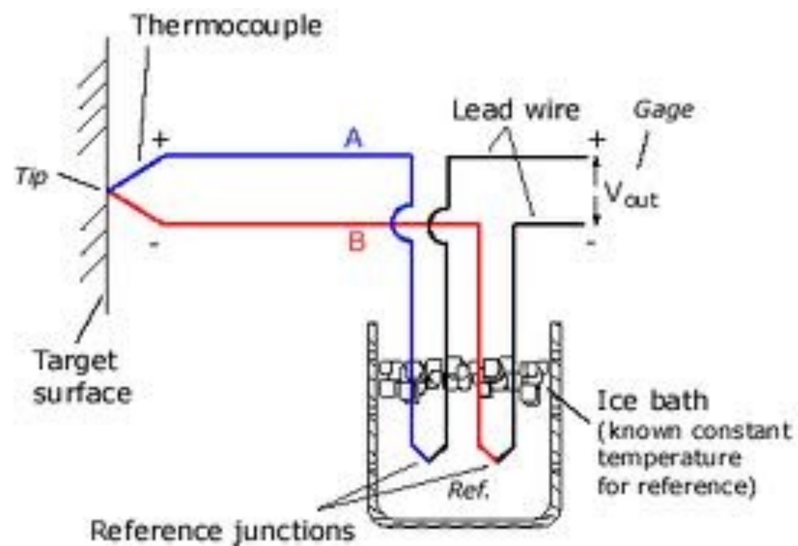
$$V = \int_{T_1}^{T_2} (S_A(T) - S_B(T))dT$$

- If the Seebeck coefficients S_A,S_B are temperature-independent, we can simplify: $V = (S_A - S_B)(T_1 - T_2)$

- This is the basis of the thermocouple thermometer

- Typical values: 1-40 μV/K

- The problem: if we have different metals between our sample and the preamplifier (incl. soldering joints!), any temperature gradient will generate a thermoelectric voltage!

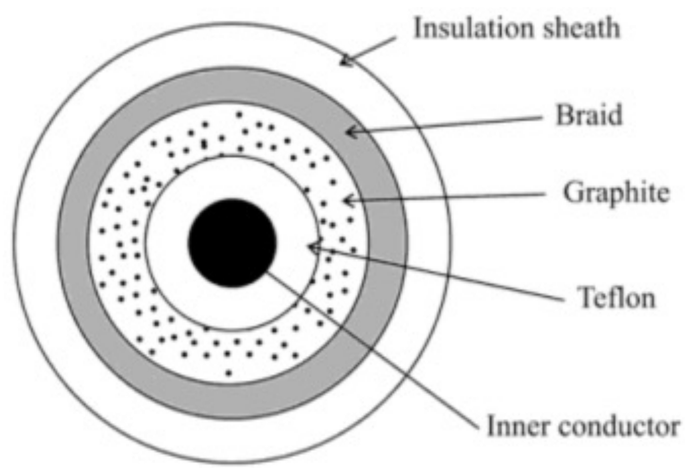


Interference: DC - chemical

- When two metals are linked by an ionic channel (water), they generate an electrochemical cell, so their different electrode potentials can generate a voltage
- This voltage depends on the metals, can easily reach tens to hundreds of mV

Interference: DC/AC - triboelectric

- When polarized dielectric material moves, the dipoles can generate a voltage.
- These can be quite high in piezoelectric materials
- Typical example: coaxial cable's insulation generates a voltage when bent.



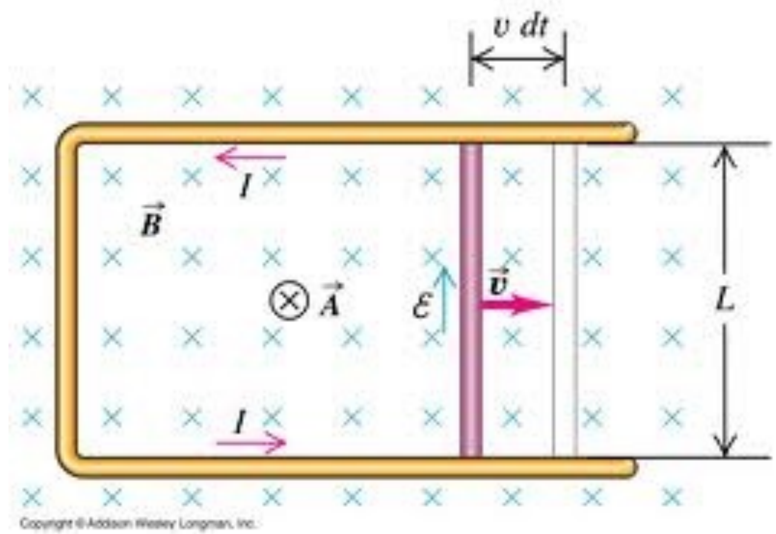
LABORATORY OF nanoSTRUCTURES

Interference: DC/AC - Magnetic

- Faraday's law of magnetic induction:

$$\oint_c E \cdot d\ell = -\frac{d}{dt}\int_s B \cdot ds$$

- An emf is formed by a change in magnetic flux: either a change of field or of path will do!
- The field decreases as $1/r^3$.
- Even movement of a big metallic object (elevator, metro, …) in the earth's magnetic field (0.5 Gauss) can perturb the field and create emf in a circuit.
- This emf is not DC, but usually of low frequency (0.1-10 Hz).



Interference: DC/AC – Ground loops

Ground loops are the manifestation of common resistance path between the signal and other (power) paths, or as differences in ground voltages.

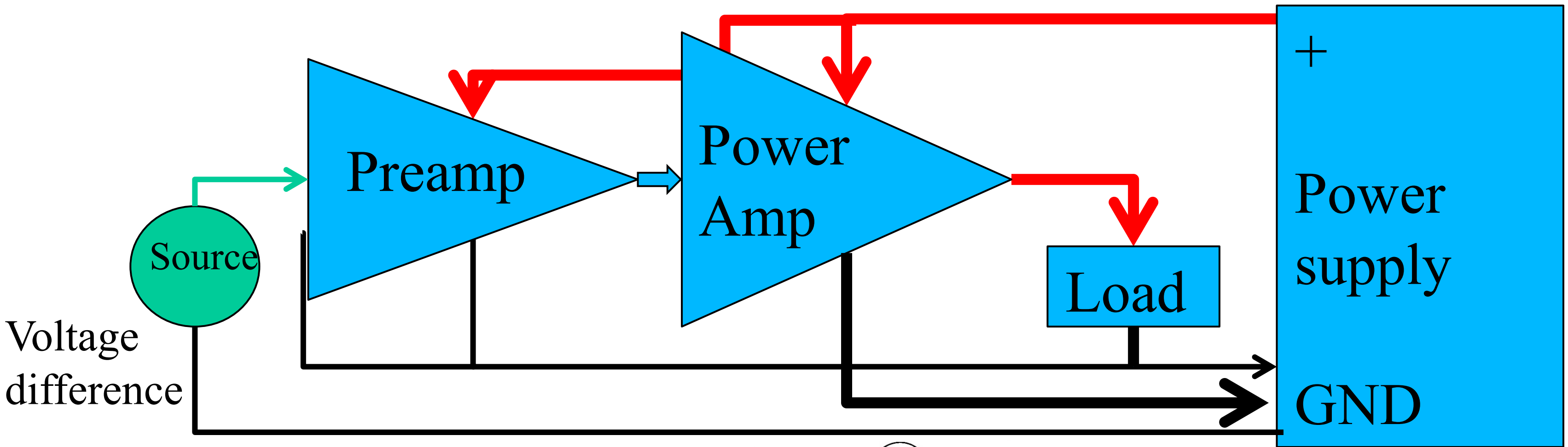
They can happen as the result of:

- Bad design of electronic circuits
- Bad connections of the input cables

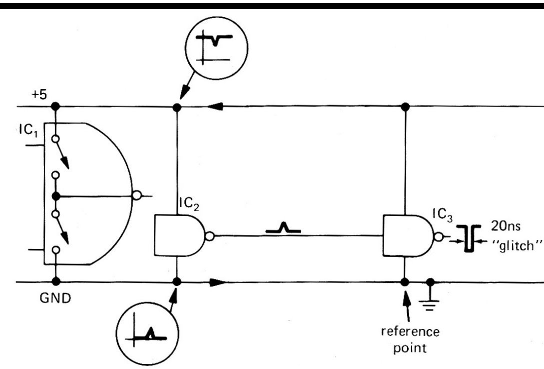
The result is interference in the signal path (DC, AC or both) and sometimes oscillations

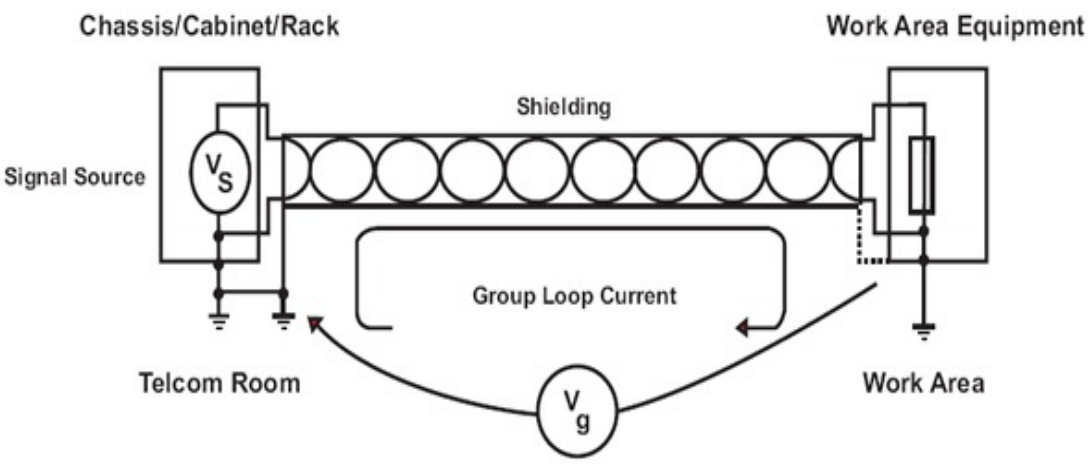
Ground loops: Examples

- Common resistance path between input and power:

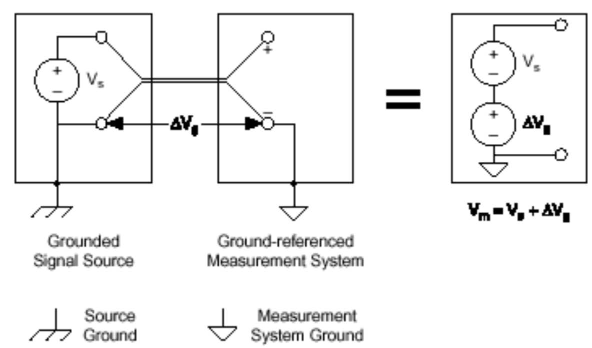


- Ground potential differences:





Ground Potential Difference - Ground Loop Source

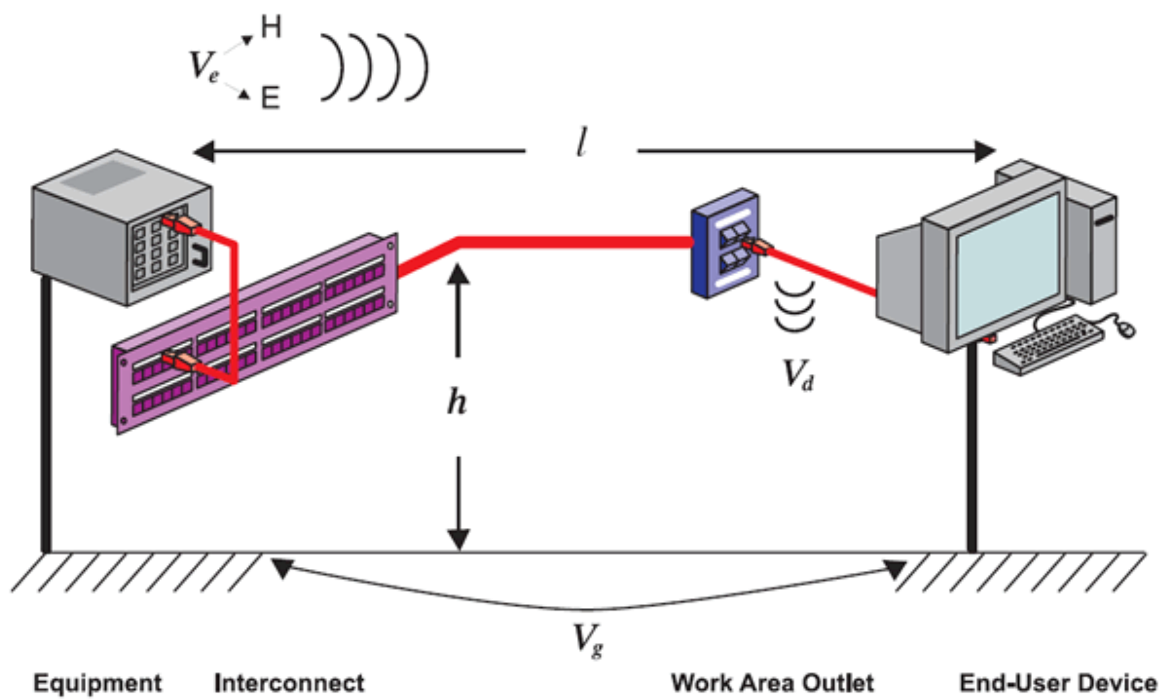


Signal S

Interference: AC "EMI"

Unwanted AC voltage/current can flow in the sample or arrive at the input of preamp:

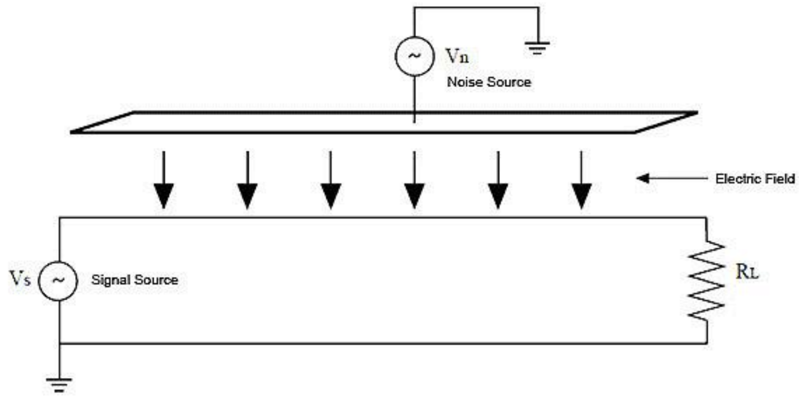
- Capacitive coupling: usually effective at high frequencies, pulses
- Inductive coupling: usually effective at high frequencies, pulses
- Ground coupling: usually related to 50Hz line interference, also pulses (see above)
- Magnetic EMF due to movement: usually low frequencies (see above)

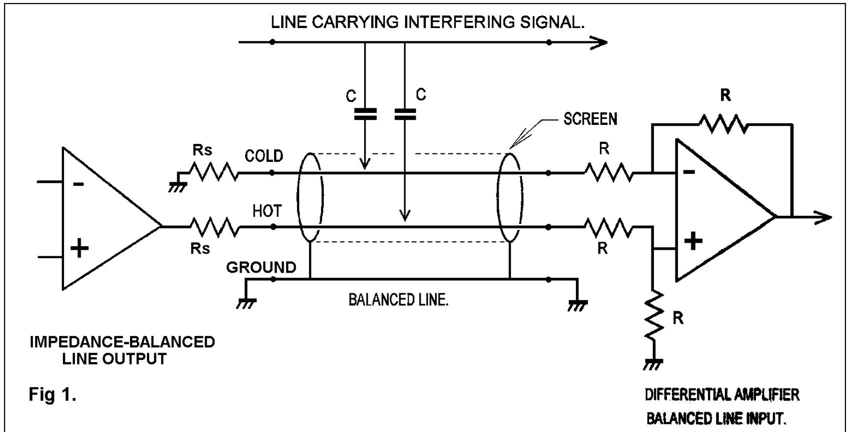


Interference: AC - capacitive

Parasitic capacitance between input and interference source can couple AC voltages.

- Capacitive impedance $Z = 1/2\pi f C$ decreases with increasing frequency.
- Example: a capacitance of 0.1 pF has a low impedance of 160 kΩ at f = 10 MHz
- The input capacitance of the preamp forms a voltage divider with the coupling capacitance: in the previous example, an input capacitance of 10 pF will divide the coupled voltage by 100 (still a lot: a nearby pulse of 3V will generate 30 mV input)
- High frequencies come not only from "known" RF sources (radio/TV equipment, plasma sources, motors …), but also from high-speed logic (especially PCs).





Interference: AC - inductive

Parasitic inductive coupling between input and interference source couple AC currents.

- Any AC current flowing in a nearby w[illegible]e can couple inductively to the input leads of the preamp. This will generate a c[illegible] to the preamp's input resistance.
- High-power transformers, heate[illegible]nerate 50 Hz magnetic fields
- Example: an induced current of [illegible]ate 1 mV interference in a preamp with 1 MΩ input resistance.
- High frequencies come not o[illegible]F sources (RF plasma sources,), but also from high-speed switc[illegible] Hz currents (motors, dimmers).