Classical Electrodynamics

Week 3

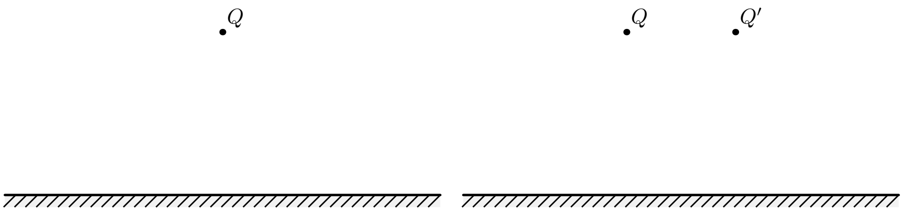
1. In order to get familiar with the method of image charges it is better to warm up with some examples. For each of the following configurations draw the position and the value of the image charge and check that the scalar potential ϕ is zero on the surface of the conductors.

a) A single point–like charge and an infinite conducting plane (figure 1(a)).

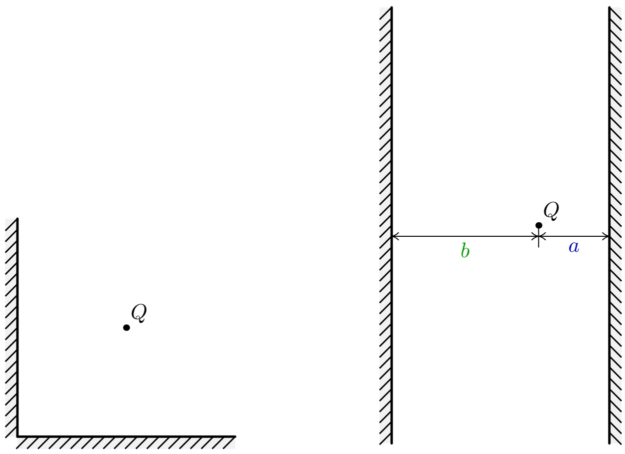
b) Two point–like charges of different values and an infinite conducting plane (figure 1(b)).

c) A single charge placed inside a 90 degrees corner of conducting materials (figure 1(c)).

d) A single charge between two parallel metallic planes (figure 1(d)). For this last one it is not required to compute the full solution but only to discuss qualitatively the configuration of image charges that realizes it.



(a) Single charge (b) Two charges



(c) A corner (d) Parallel planes

Figure 1: Configurations of charges and conducting planes.

2. Consider a conducting sphere of radius R and a point-like charge Q placed at a distace d from the surface of the sphere.

a) Find the position and the value of the image charge inside the sphere such that the potential ϕ is zero on the surface of the sphere.

b) Find how to place an additionnal image charge such that the sphere is neutral and has constant potential on its surface (not necessarily zero).

3. Consider a point-like charge Q hanging from an insulating string as shown in figure 2(a).

a) Using Maxwell equations, compute the electric field produced by the charge,

(i) neglecting the effect of all nearby objects.

(ii) assuming that the ground under the charge is a flat infinite conductor at a distance h. Determine also the surface charge density induced on the conducting floor. **Hint**: use the method of image charges.

b) Suppose that the floor is an insulator with the electrical permittivity of the vacuum and we place a neutral metallic sphere on the ground exactly below the hanging charge as depicted in figure 2(b). The sphere has radius R and the distance from the top of the sphere to the hanging charge is d.

(i) Using the method of image charges, show that the surface charge density induced on the conducting sphere is

$$\sigma(\theta) = \frac{Q}{4\pi}\left[\frac{1}{R(d+R)} - \frac{d(d+2R)}{R\left(d^2 + 4R(d+R)\sin^2\frac{\theta}{2}\right)^{3/2}}\right], \tag{1}$$

where θ is the polar angle from the top of the sphere. **Hint**: use the result of exercise **2.b**.

(ii) Study the limit of the charge density for a very large sphere ($R \to \infty$) keeping the distance d fixed. Compare your result with the case of a flat conducting floor.

(iii) What is the minimal weight of the sphere so that it remains on the ground?

c) Challenge: Assume the ground in figure 2(a) is a water pond. What is the shape of the surface of the water due to the presence of a small hanging charge?

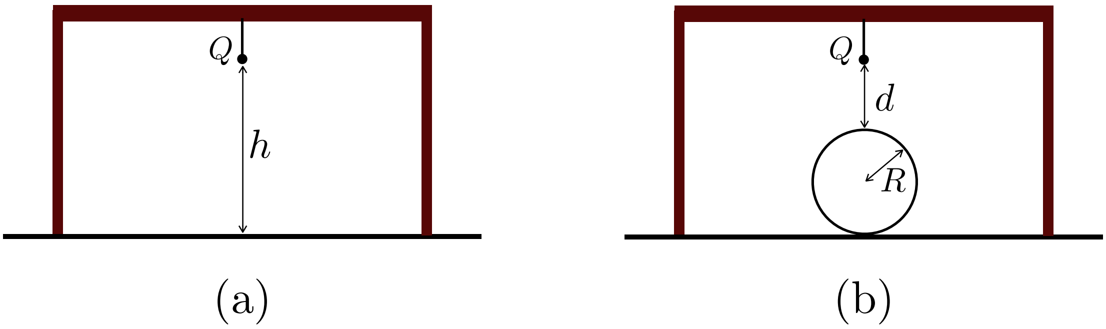


Figure 2: Charge hanging from an insulating string.