Classical Electrodynamics

Week 1

1. Consider an infinite cylinder of radius R with a uniform charge density ρ within its volume. Denote its linear charge density by κ.

a) Express κ in terms of ρ and give the units of these two quantites.

b) Using the symmetry of the problem, compute the electric field $\mathbf{E}$ using Gauss's law and deduce the scalar potential ϕ.

c) Find the scalar potential ϕ by solving Poisson's equation.

2. Consider a sphere of radius R with constant electric charge density $\rho > 0$.

a) Show that, both inside and outside of the sphere, the electric field is a power law function of the distance from the origin, i.e. it is proportional to r^n for some n. Find n for each region. Compute the electric potential, and plot the radial dependence of the electric field and potential.

b) Now assume there is a very narrow tunnel inside the sphere passing through the centre of the sphere. At time $t = 0$, a single point-like charge with electric charge $-q < 0$ and mass m is placed at rest in the tunnel at $r = a < R$. Neglecting the effect of radiation ($v \ll c$), find the equation of motion of this particle and solve it.

c) Two spheres, each of radius R and carrying uniform charge densities ρ and $-\rho$, respectively, are placed so that they partially overlap. Call $\mathbf{d}$ the vector from the positive centre to the negative centre, with $|\mathbf{d}| < 2R$. Show that the field in the region of overlap is constant and find its value.

3. A mass spectrometer is an instrument used to analyse the chemical composition of a material. The essential parts of a mass spectrometer are depicted in figure 1. At one end of the spectrometer, the material is heated so that some atoms are ionized, then these ions are accelerated by an electrostatic potential V, deflected by a magnetic field B and detected at the other end of the spectrometer. In practice, one varies the magnetic field B and measures the electric current carried by the ion beam that hits the electrode X.

a) For an ion of charge q and mass m, compute the magnetic field B necessary for the ion to hit the electrode X. Neglect the initial thermal velocity of the ion.

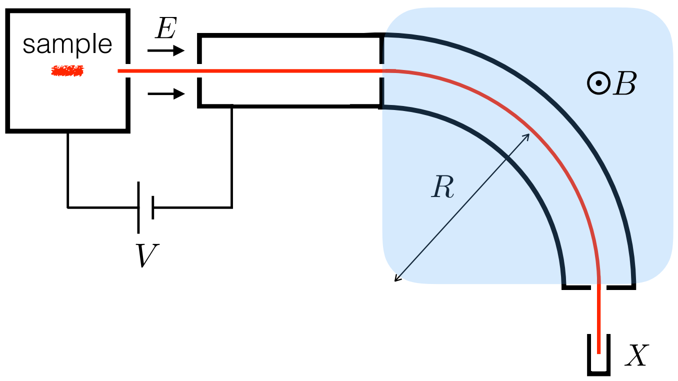

Figure 1: Schematic representation of a mass spectrometer. The ions are deflected by the magnetic field B perpendicular to the plane of the image. In order to hit the electrode X the ions have to follow a circular trajectory of radius R in the region with magnetic field (light blue region).

b) In the analysis of a pure substance in a spectrometer with potential $V = 1\,kV$ and $R = 35\,cm$, the magnetic field required to observe an electric current at X was $B = 98\,mT$. What was the substance?

c) **Challenge**: The ions created by thermal heating have random initial velocities. This will introduce some uncertainty in the measurement of the ratio q/m using the mass spectrometer. Can you estimate this uncertainty? Can you think of a strategy to select the initial velocity of the ions and reduce this uncertainty?

4. Scalar and vector potentials.

a) Show that a vector field $\mathbf{V}$ obeying $\nabla \times \mathbf{V} = 0$ can be written as $\mathbf{V} = -\nabla\phi$, for some scalar potential ϕ. Check that this applies to the electric field $\mathbf{E}$ in electrostatics.

b) Similarly, show that a vector field $\mathbf{V}$ obeying $\nabla \cdot \mathbf{V} = 0$ can be written as $\mathbf{V} = \nabla \times \mathbf{A}$, for some vector potential $\mathbf{A}$. Check that this applies to the magnetic field $\mathbf{B}$ in magnetostatics.

c) Show that a general vector field $\mathbf{V}$ can be written as a sum $\mathbf{V} = \mathbf{V}_{\parallel} + \mathbf{V}_{\perp}$, with $\nabla \times \mathbf{V}_{\parallel} = 0$ and $\nabla \cdot \mathbf{V}_{\perp} = 0$.

Hint: Use Fourier space.