Classical Electrodynamics

Week 10

1. *Velocity's transformations in Special Relativity*
In this exercise, you will study how velocities v_1 and v_2 transform from one reference frame $\mathcal{R}_1$ to another $\mathcal{R}_2$ in the context of Special Relativity, in different scenarios.

a) The reference frame $\mathcal{R}_1$ is moving along the x-axis with speed v_0 in the reference frame of the laboratory $\mathcal{R}_0$, and a particle is moving with speed v_1 along the x-axis in the reference frame $\mathcal{R}_1$. What is the velocity of the particle in the reference frame $\mathcal{R}_0$?

b) Two particles (with the same mass) are moving in the same direction with velocities v_1 and $v_2 > v_1$ in the reference frame $\mathcal{R}_1$. At what speed should the reference frame $\mathcal{R}_2$ move with respect to $\mathcal{R}_1$ so that the center of mass condition $v_1' + v_2' = 0$ is obeyed? What is the value of v_1'? Is your result compatible with your non-relativistic intuition?

c) The trajectory of a particle moving at constant velocity makes an angle θ with the x-axis of a reference frame $\mathcal{R}_1$. Compute the corresponding angle θ' in a reference frame $\mathcal{R}_2$ moving with speed v along the x-axis of $\mathcal{R}_1$.

d) Consider two particles (with the same mass) moving at the same speed v. The angle between their trajectories is θ. Find a reference frame in which $\mathbf{v}_1' + \mathbf{v}_2' = 0$.

2. *Synchrotron radiation*
Consider a non-relativistic electron ($v \ll c$) in circular movement due to a magnetic field $\mathbf{B}$ orthogonal to the plane of the movement.

a) Calculate the Poynting vector $\mathbf{S} = \epsilon_0 c^2 \mathbf{E}_e \times \mathbf{B}_e$, using the radiative part of the electromagnetic fields $\mathbf{E}_e$, $\mathbf{B}_e$ produced by the accelerated electron. You can use the formulas of Liénard-Wiechert in the non-relativistic limit ($v \ll c$).

b) Calculate the time average of the Poynting vector

$$\langle \mathbf{S} \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, \mathbf{S}(t)\,, \tag{1}$$

and determine the total radiated power.

c) Study the angular distribution of the radiation.

3. *Classical atom*

Consider the classical model of the hydrogen atom:

- The proton, of charge $e = 1.60 \times 10^{-19}$ C and mass $m_p = 1.67 \times 10^{-27}$ kg, is at rest at the center of the atom.
- The electron, of charge $-e$ and mass $m_e = 9.11 \times 10^{-31}$ kg $\ll m_p$, moves around the proton in a circular orbit of radius $r_0 = 5.29 \times 10^{-11}$ m.

a) Calculate the frequency ν of this rotation.

b) Calculate the total power radiated by the system. Recall the formula

$$\mathbf{S}(t) = \frac{e^2}{16\pi^2\varepsilon_0 c^3} \frac{|\mathbf{a}|^2 \sin^2 \alpha(t)}{r^2} \mathbf{e}_r \tag{2}$$

for the Poynting vector of a non-relativistic electron. Here, $\alpha(t)$ is the angle between the acceleration vector $\mathbf{a}$ of the electron and the observation direction $\mathbf{e}_r$.

c) Estimate the life time of the classical atom. Why are you still alive?