

Quantum mechanics II, Problems 7- Perturbation Theory

TA : Achille Mauri, Behrang Tafreshi, Gabriel Pescia, Manuel Rudolph, Reyhaneh Aghaei Saem, Ricard Puig, Sacha Lerch, Samy Conus, Tyson Jones

Problem 1 : Confined quantum Stark effect

We consider an electron of mass m in a one-dimensional potential well of width L , whose infinite barriers are located at $x = \pm L/2$, described by the Hamiltonian $\hat{H}_0$. A constant electric field of intensity E is applied to the system, which subjects the electron to the Coulomb force $F = -eE$, resulting in a perturbation $\hat{V} = F\hat{x}$.

1. Sketch the total potential experienced by the electron for $F > 0$ (i.e. $E < 0$).
2. Provide the Hamiltonian $\hat{H}_0$. Recall the eigenenergies ϵ_n and wavefunctions $\varphi_n(x)$ ($n = 1, 2, \dots$) of the unperturbed electron, i.e., when $F = 0$, distinguishing between even and odd n cases.
3. Calculate the first-order energy correction $E_1^{(1)}$ for the ground state in the case $F \neq 0$. What do you notice?
4. Derive the first-order energy corrections $E_n^{(1)}$ for excited states with $n > 1$.
5. Now calculate the second-order energy correction $E_1^{(2)}$ for the ground state (exploit the wavefunction parity). For the sums over intermediate states, consider only the states $\varphi_1(x)$ and $\varphi_2(x)$, and denote by V_{21} the matrix element of the perturbation, computed between these two states.
6. Intuitively and qualitatively depict the shape of the wavefunction of the ground state in the total potential.

Problem 2 : Interacting particles in a potential well

We consider two indistinguishable (spinless) particles of mass m confined in a one-dimensional square potential well $V(x)$. We assume that the barrier height is such that only states associated with the wavefunctions $\varphi_1(x)$ and $\varphi_2(x)$ are confined in the well. The Hamiltonian of the system is given by :

$$\hat{H}^{(0)} = \hat{H}_1 + \hat{H}_2, \quad (1)$$

with

$$\hat{H}_1 = \frac{\hat{p}_1^2}{2m} + V(\hat{x}_1), \quad \hat{H}_2 = \frac{\hat{p}_2^2}{2m} + V(\hat{x}_2). \quad (2)$$

1. Suppose that the two-particle states are even under permutations. Determine a basis of two-particle states, starting from φ_1 and φ_2 .
2. Now assume that the particles can interact when they are precisely at the same location (contact interaction), which is represented by the perturbation $\hat{V}_{\text{int}} = V_0 \delta(\hat{x}_1 - \hat{x}_2)$, where $\delta(\hat{x}_1 - \hat{x}_2)$ is the Dirac delta function. Calculate the first-order energy correction for each of the two-particle states established earlier (as a function of φ_1 and φ_2). Discuss the relative values and signs of these corrections.
3. Repeat the previous calculations assuming the two-particle states are odd under permutations. Compare the results obtained with the symmetric case and draw conclusions.