General Physics: Electromagnetism, Problem Set 8

Exercise 1 :

A conductive plate of width w has current flowing in the $-\hat{x}$ direction. The plate is placed in an homogeneous magnetic field $\overrightarrow{B} = -B\hat{z}$, as shown in the figure below. Suppose the current is carried by electrons that move with a velocity $\overrightarrow{v} = v\hat{x}$.

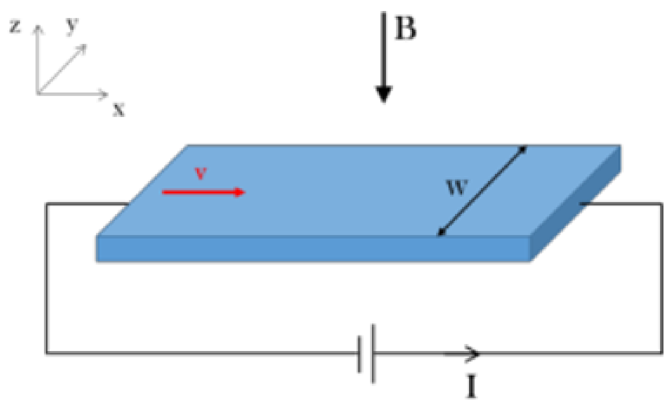


Figure 1: Circuit with a conductive plate in a magnetic field.

a) In which direction will the electrons be deviated ?

b) The deviation of electrons polarises the plate, therefore creating a potential difference between the two sides of the plate and hence, an electric field. The equilibrium is reached when the force generated by the electric field on each electron is equal and of opposite direction to the force on each electron generated by the magnetic field. The electric field at this equilibrium is called *Hall field* E_H. For $B = 0.1$ T and $v = 1.3 \cdot 10^6$ ms^{-1} find the value and direction of E_H.

c) Is it possible, by measuring the potential difference between the two sides of the plate, to determine whether the charge carriers in the conductor are electrons or positive ions?

d) Suppose now that the magnetic field is unknown. Would it be possible to determine the magnetic field by adding a component to the circuit? What are the limitations of this measurement system?

Exercise 2 :

1. A rectangular copper strip $d = 1.5$ cm wide and $t = 0.10$ cm thick carries a current of 5.0 A. Find the Hall voltage for a magnetic field of $B = 1.2$ T applied in a direction perpendicular to the strip. For finding the charge carrier density, assume that one electron per atom is available for conduction and use the molar mass $M = 0.0635$ kg/mol and density $\rho = 8920$ kg/m^3 of copper.

   Hint: To find the drift velocity v_d of the charge carriers with charge q, you can consider a uniform charge carrier density $n = N/V$. The current through the strip is then given by $I = qnAv_\mathrm{d}$, where $A = d \cdot t$ is the cross-sectional area of the strip.

2. Now, consider a strip of the same dimensions, but made of silicon with a typical charge carrier density of $n = 1.0 \cdot 10^{20}$ electrons/m^3. Find the Hall voltage for a magnetic field of $B = 1.2$ T. Which material is better suited for measuring magnetic fields?

Exercise 3 :

Mass spectrometers, which separate ions based on mass, are often used by chemists to determine the composition of a sample. The aim of the exercise is to explore the working principle of mass spectrometers.

1. The first section of the mass spectrometer is the **accelerator**. A particle of mass m and charge q is released from rest near one plate of a charged parallel-plate capacitor. The particle accelerates towards the other plate of the capacitor. The potential difference between the plates is ΔV.

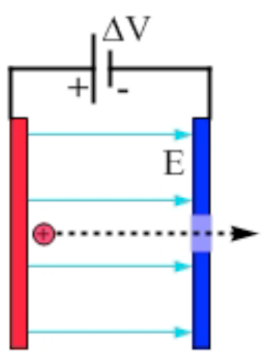


Figure 2: Charged particles are released from rest near the left-hand plate of a parallelplate capacitor, accelerate across the gap, and emerge via a hole cut in the right-hand plate.

   a) Apply energy conservation to obtain a relation between, the potential difference across the capacitor, and the speed of the particle when it emerges from a small hole in the second plate.

2. After leaving the accelerator, the particle passes through the **velocity selector**. The velocity selector is a second parallel-plate capacitor in which the plates are parallel to the particle's

velocity. In addition to the electric field E inside the capacitor there is also a magnetic field B, directed perpendicular to both the electric field and the velocity v of the particle. The combined effect of the two fields is such that a particle with just the right speed experiences no net force and passes undeflected through the velocity selector.

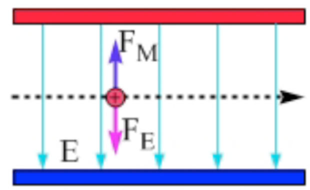


Figure 3: A charged particle with just the right velocity passes undeflected through the velocity selector because the magnetic force balances the electric force.

a) Determine the speed of the undeflected particle.

b) Is the magnetic field in figure above directed into or out of the page?

c) What happens to particles traveling faster than the undeflected particles? What happens to particles traveling slower than the undeflected particles?

Exercise 4 :

Particles that pass undeflected through the velocity selector of the mass spectrometer in the exercice above are sent into the **mass separator**, which consists of a uniform magnetic field that is perpendicular to the velocity of the particles. Suppose that we want to separate the fissile Uranium isotope U_{235} from the heavier non-fissile U_{238}, such that a mixed beam of U_{235} and U_{238} ions is sent into the mass spectrometer. The ions exit the velocity selector into a homogeneous magnetic field region, with $B = 0.5$ T perpendicular to the beam, as shown in the figure below. The charge of both U isotope ions is $q = 3.2 \cdot 10^{-19}$ C, and the acceleration voltage is 100 kV ($m_{U_{235}} = 3.903 \cdot 10^{-25}\ kg$, $m_{U_{238}} = 3.953 \cdot 10^{-25}\ kg$).

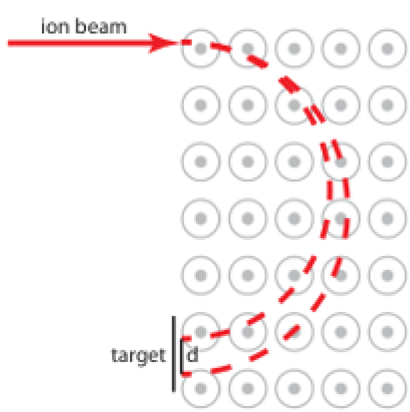


Figure 4: Ion beam in the magnetic field.

a) Evaluate the different speeds of the two ion species $v_{U_{235}}$ and $v_{U_{238}}$ when they enter the magnetic field region.

b) Evaluate the spatial separation d between the two isotopes when they exit the mass spectrometer after the semicircular path in the magnetic field region.

c) What acceleration voltage would be needed in order to get separation of $d = 2\ cm$?

Exercise 5 :

A circuit hangs vertically on the hook of a balance as shown in Figure 5. The balance is calibrated to zero (i.e. we subtract the weight of the circuit). The circuit is partially suspended in a magnetic field perpendicular to the page. The upper part of the circuit is out of the field. The circuit consists of a 1 V battery and a resistor of $0.5\,\Omega$. The width of the circuit is $a = 0.1\,m$ and a length $h = 0.07\,m$ is suspended in the magnetic field. If the balance measures a 'weight' of 3.5 g, what is the magnitude of the magnetic field?

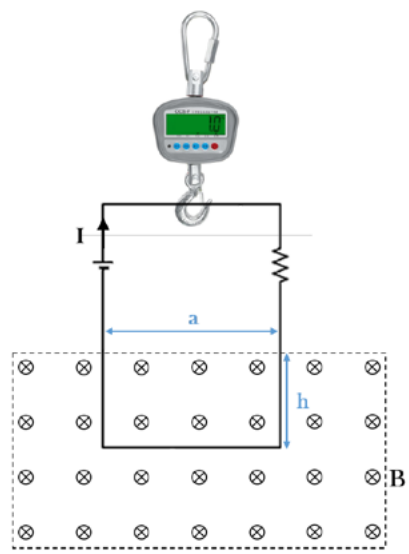
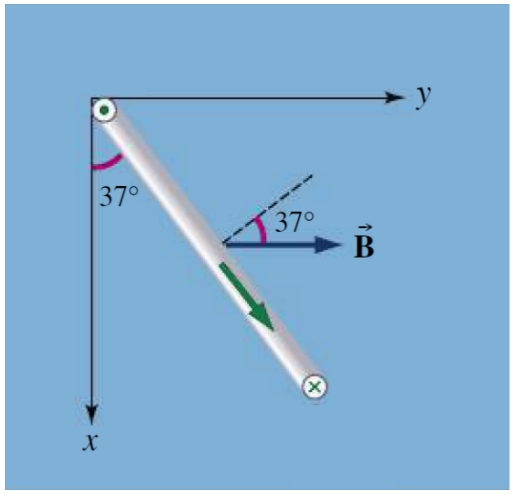


Figure 5: Circuit partially suspended in a magnetic field

Exercise 6 :

A rigid wire loop of square shape has sides of length 20 cm. It has fives turns and is carrying a current of 2 A (indicated by green arrow). The normal to the loop makes an angle of 37° with a uniform magnetic field $B = 0.5\hat{y}$ T.



a) Find the magnetic dipole moment.

b) Find the magnitude and direction of the torque acting on the loop.

c) Find the work that an external agent must provide to rotate the frame from its position of minimum energy to the given position.