General Physics: Electromagnetism, Problem Set 5

Exercise 1 :

Two particles, with charges 20.0 nC and -20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, -4.00 cm), as shown in Fig.1. A particle with charge 10.0 nC is located at the origin.

1. Find the electric potential energy of the configuration of the three fixed charges.
2. A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC, is released from the rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance.

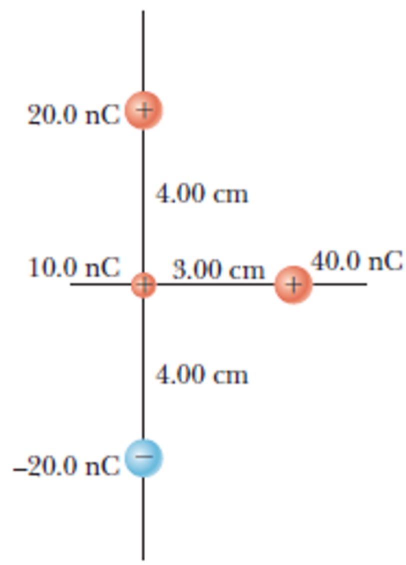

Figure 1: Schematic of the charges and their positions.

Exercise 2 :

A nonconducting sphere of radius r_0 carries a total charge Q distributed uniformly throughout its volume. Determine the electric potential as a function of the distance r from the center of the sphere for: (a) $r > r_0$ and (b) $r < r_0$. Take $V = 0$ at $r = \infty$; (c) Plot V versus r and E versus r.

- **Hint**: calculate the electric field first, and then the potential from it.

Exercise 3 :

An electric transmission line in a house consists in two cables of a length l and radius $R = 1$ mm, spaced by a distance $d = 10$ cm. Suppose that $l \gg d \gg R$. Compute the capacitance per unit length of the transmission line. The capacity proves to have a negative effect in the transmission of the signal. What can you do to reduce this effect?

- **Hint**: remember that the capacitance is given by $C = \frac{Q}{\Delta V}$

Exercise 4 :

Refer to Fig.2 and consider the three following cases.

(a) An insulating spherical shell, centered at the origin O of the Cartesian axes, is uniformly charged (with internal radius a, external radius b and total charge $+Q$). Calculate at any point in space the electric potential $V(\overrightarrow{r})$ and the electric field $\overrightarrow{E}(\overrightarrow{r})$. Graphically represent the functions $V(\overrightarrow{r})$ and the radial component of $\overrightarrow{E}(\overrightarrow{r})$.

(b) A conductive spherical shell, centered at the origin O of the Cartesian axes, is electrically neutral and floating (internal radius a and external radius b). A $+Q$ charge is placed in the center of the shell (in O). Calculate at any point in space the electric potential $V(\overrightarrow{r})$ and the electric field $\overrightarrow{E}(\overrightarrow{r})$. Graphically represent functions $V(\overrightarrow{r})$ and the radial component of $\overrightarrow{E}(\overrightarrow{r})$.

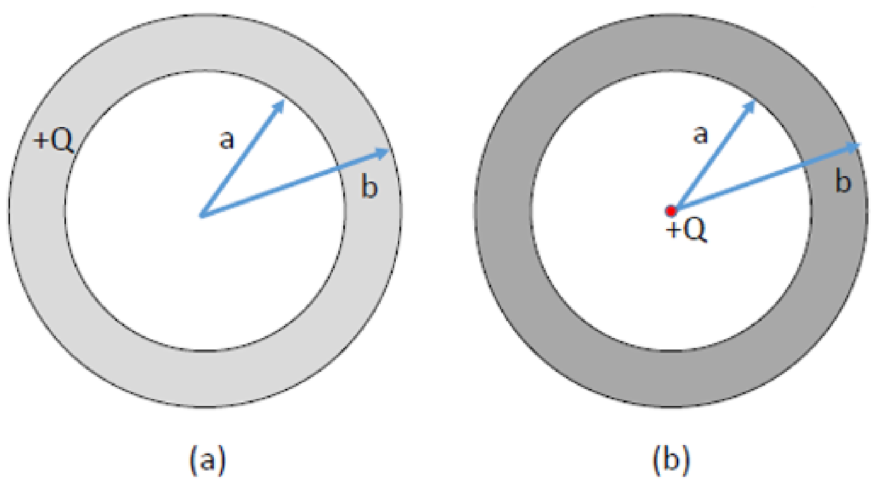

Figure 2: a) Insulating spherical shell uniformly charged with a total charge +Q. b) Neutral conductive spherical shell with a +Q charge in the centre. The shell is floating, i.e. electrically disconnected from the environment.

- **Hint 1**: use Gauss's law to compute the electric field first and then the potential by integrating the electric field.

Exercise 5 :

Diagnostic imaging techniques, such as for example ultrasounds, use coaxial cables to transmit instruments data. A coaxial cable with a length h is made up of a conductive cylinder with radius R_1 inside a second empty cylinder of radius R_2, separated by a dielectric, as shown in Fig.2. We consider here a coaxial cable with air as a dielectric. An advantage of this kind of cables is that the external electric noise is reduced, because the outer cylinder acts as a Faraday cage which shields the inner conductor. On the other hand, this type of cable acts as a cylindrical capacitor. A huge capacitance can result in a delay in the transmission, which can cause interference of the signal. It is also important that the capacitances are low for diagnostic imaging techniques because the images are high resolution.

- **Hint**: remember that the capacitance is given by $C = \frac{Q}{\Delta V}$

1. Find an expression for the capacitance per unit length of such a coaxial cable (consider $h \gg R_2$).

2. A coaxial cable factory propose you two cables for your diagnostic ultrasounds. The first cable has dimensions $R_1 = 0.25$ mm and $R_2 = 0.76$ mm. The second $R_1 = 0.08$ mm and $R_2 = 1.20$ mm. Which one do you choose?

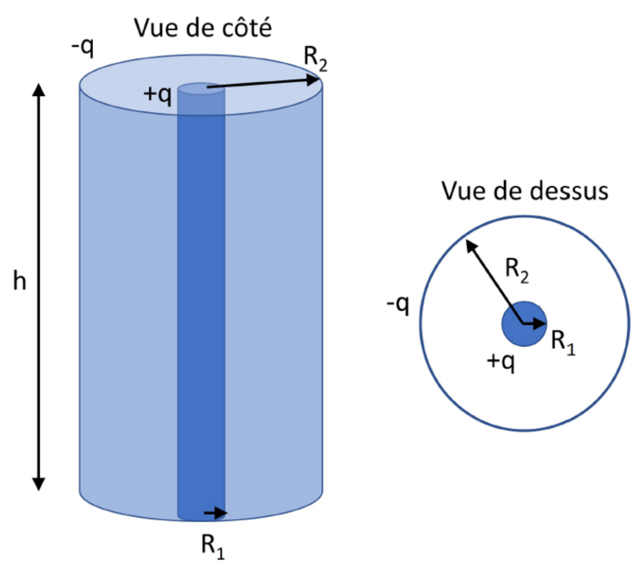

Figure 3: Schematic of a coaxial cable. Side and top view.