General Physics: Electromagnetism, Problem Set 4

Exercise 1 :

Calculate the Electric Field within and outside a charged cylinder of radius R and length L with a charge density obeying the law $\rho(r) = \rho_0(a - br)$, where a and b are arbitrary parameters. Consider the case where $L \gg R$, which means that the direction of the electric field lines are radial.

- **Hint 1**: remember that the electric field is radial, so try to take a proper Gaussian surface to easily compute the electric field flux (left part of Gauss's law);
- **Hint 2**: to compute the integral of the charge density, move to cylindrical coordinates.

Exercise 2 :

Consider a spherical cavity of radius a at the center of a non-conductive sphere of radius R. The volume charge density in the rest of the sphere varies according to $\rho = A/r$, where A is a positive constant. Determine the electric field for $a < r < R$.

- **Hint**: the charge inside a shell of thickness dr is $dq = \rho dV = \rho(4\pi r^2)dr$.

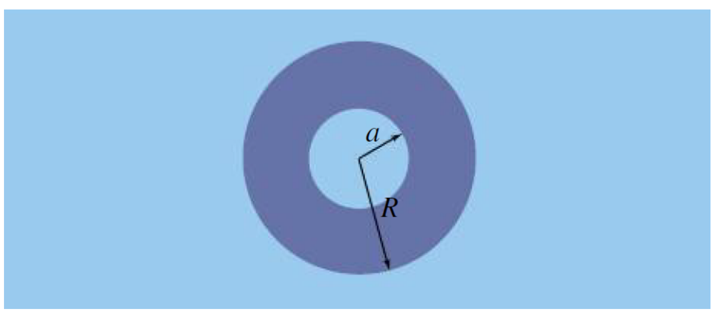

Figure 1: Spherical cavity of radius a at the center of a non-conductive sphere of radius R.

Exercise 3 :

Two non-conducting spheres of radii R_1 and R_2 are uniformly charged with charge densities ρ_1 and ρ_2, respectively. They are separated at center-to-center distance a (see below). Find the electric field at point P located at a distance r from the center of sphere 1 and is in the direction θ from the line joining the two spheres assuming their charge densities are not affected by the presence of the other sphere.

- **Hint**: Work one sphere at a time and use the superposition principle.

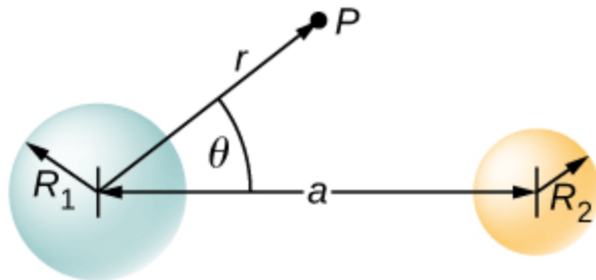

Figure 2: Two non-conducting spheres of radii R_1 and R_2, uniformly charged with charge densities ρ_1 and ρ_2, respectively.

Exercise 4 :

A wire having a uniform linear charge density λ is bent into the shape shown in Fig.1. Find the electric potential and the electric field at point O.

- **Hint 1**: use the superposition principle and compute the total potential as the sum of three separate contributions in the three different regions. Remember that the electric potential at distance r generated by a continuous charge distribution is $V(r) = k \int \frac{dq}{r}$.
- **Hint 2**: express the infinitesimal charge dq in the curved region as a function of the infinitesimal angle $d\theta$;
- **Hint 3**: for the electric field computation, use the symmetry of the wire to cancel out some contributions.

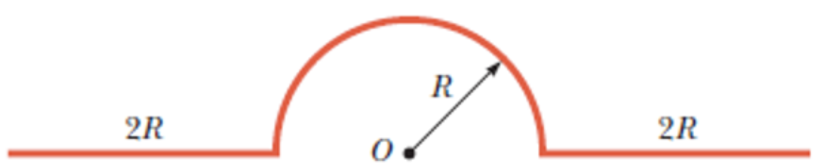

Figure 3: A bent wire with uniform charge density λ.

Exercise 5 :

Time of flight mass spectrometer (TOF MS) determines mass-to-charge ratio of ions by measuring time of their flight in a field free region (see Fig.3). It consists of a metal plate **A**, two metal grids **B** and **C**, which are transparent for ions, and a Detector. The grids **B** and **C** are grounded (i.e potential V=0), the plate **A** can be put at the fixed positive potential $V_0 = +1000\ V$ by closing the switch S. The ion Detector, which is very close to **C**, is at some high negative potential.
Positively singly charged ions are initially very close to the plate **A** (they do not interact with each other). At a well-defined time $t = 0$ the switch S is **on** and V_0 is being applied to **A** and ions begin to move toward the detector.

- **Hint**: the potential energy of a charge q in a potential V_0 is given by $E_{\text{potential}} = qV_0$.

(a) Show that ions of different masses ($q = +1$) will arrive to the detector at different time. Derive an expression for time-of-flight.

(b) Estimate the time-resolution of the detector, required to distinguish masses of uranium isotopes: ^{238}U ($m_1 = 3.95 \cdot 10^{-25}$ kg) and ^{235}U ($m_2 = 3.90 \cdot 10^{-25}$ kg).

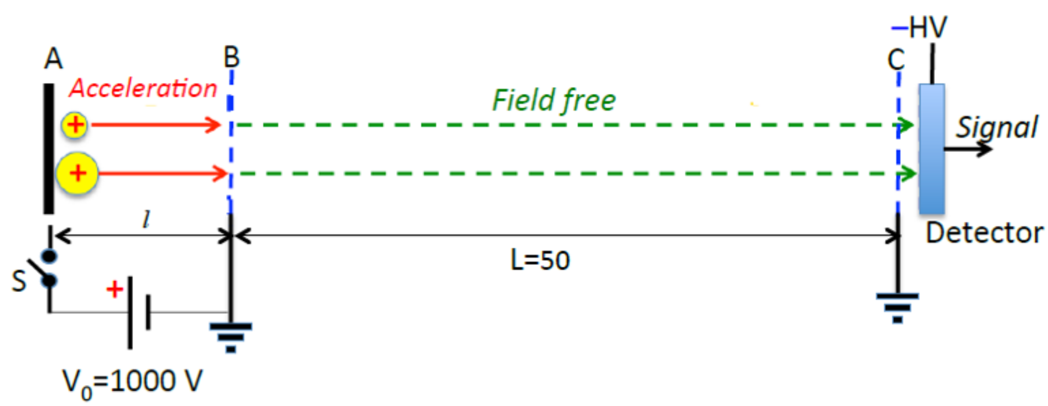

Figure 4: Time of flight-based mass spectrometer schematic.